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THE

# ARITHMETICAL MANUAL:

CONTAINING

## EXERCISES FOR PRACTICE;

AND

DEMONSTRATIONS OF THE RULES

OF

## WRITTEN ARITHMETIC.

PREPARED FOR THE USE OF THE STUDENTS IN THE

**SEMINARY FOR TEACHERS,**

ANDOVER, MASS.

BY S. R. HALL.

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# PREFACE.

THE history of the following work is soon told. It has been prepared at the repeated request of those placed under the Author's instruction.

The design has been to furnish all the directions and examples for exercise, needed by the learner, to enable him to acquire a thorough knowledge of Arithmetic.* The labour of the teacher will thus be diminished, while the progress of the scholar is hereby secured.

The selection of examples has been made principally by F. A. BARTON, A. B. and JAMES DASCOMB, M. D.

Several of the rules and some of the demonstrations have been taken from Lacroix, Bonnycastle, and others. Whenever they have expressed intelligibly the views of the Author, he has used their language.

The peculiarities of the work are considerable. Before these are condemned, it is hoped their character may be tested in the *school-room*.

As the MANUAL has been wholly prepared under the pressure of the labours, demanded by a large school, it

* The *k* is omitted in this word for the purpose of conforming to the wishes of some of the friends of education, though the Author has in some of his other works heretofore used it.

can hardly be hoped that some errors should not escape detection.

The answers are generally given as far as to Section 30th of Part II. The remaining Sections will furnish the learner with opportunity to prove the answer for himself.

Seminary for Teachers,
Andover, Mass. 1832.

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## PART II.



## PART III.

# INTRODUCTION.

I. NUMBER is one of the early subjects of attention. It is impossible for any one to be entirely ignorant of it, after he is capable of contemplating the succession of objects around him. LACROIX justly observes, that "by comparing the objects which come within the reach of our senses, we perceive they have magnitude. When they appear in the form of a collection of similar things, or distinct parts, the term number is applied; and numbers in particular are the objects of Arithmetic.

Few sciences are so constantly necessary as this for the purpose of intelligibly communicating ideas, or the ready transaction of business. Number enters into almost every subject, whether of philosophy or religion,—and he, who would either receive or communicate instruction, must have some of its principles constantly in mind.

But, while it needs no argument to establish the claims of this science upon the attention of the young, it needs little time to prove that it *may be* pursued in such a way as to furnish very little improvement to them. Such evidence is furnished fully to every one, who has taken pains to examine any considerable number of those, who, having "ciphered through" their book, suppose themselves very well acquainted with the whole science. The natural consequence of the manner adopted by many teachers, and the character of the books used by many of the members of our schools, could not well lead to different results. Little more has been attempted than to commit rules to memory and then perform operations, according to the directions therein given. Ability to obtain the "answer" has satisfied the scholar, and he is willing to think his knowledge complete. After all, however, in a majority of cases, he has no more knowledge of the principles—the reasons—why the rule furnishes the given directions, than he would have of a proposition in Euclid, written in a language he does not understand. And, indeed, how should he, when in general the text-book with which he is furnished is without demonstra-

tions, and his teacher is well satisfied if the rule is recited and the operation carried through in the manner directed?

Ask a scholar of this description, whether he can give you a true answer to a sum, and he is ready to reply, at once, in the affirmative, if the *rule* to do it is specified. But if he is directed to ascertain the *rule for himself*, his resort is to the arithmetic, and if the sum corresponds in character to those, which he finds there under any particular rule, he tries to perform it by the aid of that rule. If in this way he obtains a satisfactory result, he is at once, in his own opinion, a master of the science. If however, unsuccessful, he tries another rule—and if this will not do, another—till presently, discouraged in his ill-directed effort, he abandons it in despair, and is ready to say "it can't be done."

Another prominent fault, which must have been noticed by every careful observer, is this—*the knowledge thus acquired is retained but a very short time.* Few, unless engaged in business, which constantly requires their use, retain any considerable degree of their former arithmetical acquisitions —save that of the fundamental rules and interest, perhaps— for any great length of time. The *practice* is discontinued, and therefore nearly *all* is lost. What benefit has been derived, then, from all the time devoted to the subject in the district school or elsewhere, year after year?

But let the principles on which every rule is founded, be thoroughly understood; let a portion of the time spent in the *mere* practical part, be devoted to acquiring a thorough acquaintance with the reasons on which rules are founded, and we have every reason to apprehend these *principles* will remain in the memory, when the details shall have been forgotten. Rules may be originated for himself, by any one, who is able to look over the whole ground—and when, from the conditions of the question proposed for solution, the learner is able to form a correct rule, he possesses that kind of knowledge, which is adapted to the wants and exigencies of life. He, who forms the key, can surely apply it, and unlock the door at his will.

It has been asserted by some, that the time allowed to the young, is merely adequate to the full acquisition of the *practical* part of arithmetic, and hence, the reason why the *theory* should be nearly disregarded. In reply, we beg leave to say, we have yet to learn that the right way is longer than the wrong, and that a path, lighted at every step, requires

more time to traverse than the same path, with every light extinguished. Let the reasons of every part of a process be fully apprehended, and instead of groping in darkness, we find all the connexions plain, simple and intelligible.

If any one shall complain that the illustrations of the following rules are too minute and unnecessarily extended, we remark that no one is obliged to follow them *all* throughout. We are, however, persuaded that every one, who is disposed to neglect them will do himself an injury, and our greatest fear is, that scholars generally will not be sufficiently attentive to them, and will not fully understand their subjects, after having gone over with the whole.

II. The importance of this science is presented forcibly by the following thoughts, selected from various authors. "Frequent exercise in computation, has a happy influence on the mind, by inducing habits of attention, by strengthening the memory, and by producing a promptness of recollection."—(*Thompson.*) "Few exercises strengthen and mature the mind so much as arithmetical calculations, if the examples are made sufficiently simple to be rightly understood by the pupil; because a regular, though simple, process of reasoning is requisite to perform them, and the results are attended with certainty."—(*Colburn.*) "The utility of arithmetic is so very great in the every day transactions of life, that to be ignorant of it argues no ordinary degree of neglect."—"Without a knowledge of it a person is unable to transact business easily or correctly in any occupation; and is liable to be defrauded by others."—(*Pierce.*) "Would you have a man reason well, you must accustom him to it; exercise his mind in observing the connexion of ideas, and following them in train. Nothing does this better than Mathematics, which should, therefore, be taught all who have time and opportunity; not so much, to make them mathematicians, as to make them reasonable creatures. Converse much, says Dr. Watts, with those friends and those parts of learning and those books, where you meet with the greatest clearness of thought and of reasoning. The mathematical sciences, and especially Arithmetic, Geometry, and Mechanics, abound with those advantages, and if there were nothing in them valuable for the uses of human life, yet the very speculative parts of this sort of learning are well worth our study; for, by perpetual examples, they teach us to conceive with clearness, and to connect our ideas in a train of dependence."—(*Jour. Education.*)

III. The following suggestions, on the manner of studying this science may be of importance to some, if not to every one. The learner ought to commence with intellectual Arithmetic, and continue in this, till he is able, not only to solve the various questions proposed, but also to give the reasons by which he arrives at a conclusion, in a given instance. He ought to regard the demonstration, or mode of reasoning, as more important far, than merely ability to ascertain the correct answer. When he is able to give a reason for every operation in COLBURN'S FIRST LESSONS, I consider him prepared to proceed to written Arithmetic. He may now write his operations on a slate. The first thing after taking a sum for solution is to make such inquiries as the following: What are the conditions of the question? What principles are involved? What preparations are necessary for commencing the operation? &c. The following general directions may be important.

1. Endeavour to obtain a definite idea of the nature of the question.

2. Ascertain what principles must be involved in obtaining a solution.

3. Inquire in how many ways a correct answer may be obtained; and by comparison, ascertain which is most natural and most useful.

4. Seek to learn the practical value of every rule.

5. Ask yourself, before leaving a given rule, can I teach this, and make others comprehend it?

6. Inquire in what particular business of life, *particular* rules may be most needed and valuable.

IV. Instructers will ask: What is the best mode of teaching this science? The inquiry is highly proper, and by all, who desire the most valuable advancement of their scholars, it will be continued so long as any improvements may be secured. Remarks have already been made on the manner of learning Arithmetic. From those suggestions, the general mode of teaching may be inferred. It will, however, be proper to present some more definite suggestions, and adapt them to the wants of those who use the following work.

1. The scholar is expected to have a thorough knowledge of "INTELLECTUAL ARITHMETIC," before he can most profitably use this book. From Mr. COLBURN'S invaluable treatise, he will be able to acquire a knowledge of inductive Arithmetic. In that work, he will be able to discover the principles on which many of the rules are founded, and will

readily understand the explanations and demonstrations in Part III. of *this book*.

2. The teacher should put these questions to the pupil, which will present the nature of the required operation distinctly to his view; and will prove, when he is able to answer them, that he has a distinct perception of the principles, on which the rule is founded. He should not rest satisfied that the scholar understands his lesson before *proof* of it is furnished. No fault is more common than to suppose the learner understands, because he is ready to *say he does*. Experience furnishes abundant proof that this belief, however sincere, is often erroneous.

3. Let the previous lesson be reviewed, at every recitation. This direction is very important and should constantly be observed.

4. Require each scholar to present at least one question on a previous lesson, to be answered by the class. The influence of this course will be highly salutary to those who propose, and to those who answer questions.

5. If each question thus proposed is recorded in a *class-book*, together with the answer, it will be found beneficial.

6. Whenever time will admit of it, let each one in the class be required to propose a sum to be wrought by the others, who will be expected to return an answer at the time of the next recitation.

### *Manner of Using this Book.*

After the learner has given proper attention to the Introduction, Notation, and Numeration, he should be required to perform the sums in Sec. 1. Part I, and then answer the questions which may be proposed on the theory as explained in Sec. 1. Part III. In the latter he will find all the explanations, which are needed to enable him to perform the given exercises in the former. The questions found under the exercises are designed to direct the attention of the learner to the nature of the given sum, *rather* than to be answered by him at the time of his recitation. Of some, it may be important, especially at first, to require an answer in their own language.

When sufficient time has been devoted to Sec. 1. Part I, the learner will commence the next, and proceed thus till he has performed all the exercises in Part I, and has given attention to the theory in the corresponding sections of Part III.

The learner will then be prepared to commence Part II, and the instructer will exercise his own judgment with regard to reviewing such parts of the previous lessons, as may, under different circumstances, be necessary.

When all the exercises of Part II have been performed, the way will be prepared to commence the Introduction to Algebra as given in the Sequel.

THE

# ARITHMETICAL MANUAL.

## PART I.

### DEFINITION.

ARITHMETIC is the science, which considers the powers and properties of numbers. It teaches how to calculate or compute with correctness and ease.

The prominent divisions of this science are, Intellectual, Written, Theoretical, and Practical Arithmetic.*

Intellectual and Written Arithmetic differ only in the manner of performing operations. The former is wholly omitted in this work.

Theoretical Arithmetic explains the properties and relations of numbers—and the reasons upon which rules are founded. Its great importance to the scholar, who desires to become a master of the subject, should lead him to give it a particular attention.

Practical Arithmetic shows how to apply principles, so as to be most expeditious in solving problems and performing practical operations; or "from certain numbers given, how to find others, whose relation to the former is known."

QUESTIONS.—What is Arithmetic? What does it teach? How divided? Are other divisions ever made? Are they important? What is the difference between Intellectual and Written Arithmetic? Define the theory of Arithmetic. What is Practical Arithmetic? Which is the more important? On a knowledge of which, does a thorough knowledge of the other depend?

* Other divisions are common, such as "instrumental, logarithmetical, specious, numerous, dynamical, duodecimal, sexagesimal, &c. &c. These are not, however, important to learners generally.

## SECTION I.

## NOTATION.

NOTATION teaches the mode of expressing any number by certain signs, or characters. Numbers can be expressed in words; but this would be entirely impracticable for the purpose of performing arithmetical operations.

Two methods have been used for writing numbers. One is by using Roman letters—the other by Arabic figures.

*Roman Numerals.*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| I. | One. | V. | Five. | IX. | Nine. | XX. | Twenty. |
| II. | Two. | VI. | Six. | X. | Ten. | XXX. | Thirty. |
| III. | Three. | VII. | Seven. | XI. | Eleven. | L. | Fifty. |
| IV. | Four. | VIII. | Eight. | XII. | Twelve. | C. | Hundred. |

The Arabic figures are signs of numbers, each having a definite value when written alone, and a relative value when used in connexion with others.

*Arabic figures.*

1, one; 2, two; 3, three; 4, four; 5, five; 6, six; 7, seven; 8, eight; 9, nine; 0, cipher. By the repetition and arrangement* of these figures, any number whatever can be easily expressed, as will be seen in the next section.

QUESTIONS.—What are Arabic figures? Can any number whatever be expressed by them?

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* "It is a fundamental law of notation, that a removal of one place towards the left hand increases the value of a figure ten times. With two figures we can express all as far as to nine tens and nine units, or 99. After this comes the hundred, expressed by the figure 1 placed one place farther to the left than it would be, if used to express tens only—as 100. Hence we see that the same figure expresses units ten times greater, in proportion as it is removed from right to left, and by a simple change of place acquires the power of representing successively all the different collections of units, which can enter into the expression of a number. In writing these characters so as to express numbers, it is important to keep in mind the order in which the collections succeed each other, and put ciphers in room of those which are wanting in the numeration, or enunciation of the numbers written. Thus in writing the number three hundred and twenty-four thousand, nine hundred and four, we

## SECTION II.

## NUMERATION.

NUMERATION is the art of reading figures. There are two modes, which have been adopted in different countries. They may be distinguished into the English and French modes. Both are used to some extent in this country, though the former more generally. Several of the first figures are alike in both.

### 1. *French Mode.*

By examining the annexed table, the mode of enumeration is easily understood. The first or top figure, 5, standing alone, has merely its own simple value, i. e. five units. The left hand figure in the second line, 7, being removed one place towards the left, expresses not seven units, but seven tens, or seventy, and with the six at the right hand, seventy-six. The left hand figure in the third line, 1, being removed two places to the left, has its value increased to a hundred, and with the figures at the right hand, is read read one hundred and ninety-eight. The fourth line extends one place farther to the left, or to the place of thousands; the fifth line to tens of thousands, and so on. The lower line is read thus: Five billions, three hundred and ninety-seven millions, six hundred and forty-two thousands, one hundred and forty-five.

| Billions | Hundreds of millions | Tens of millions | Millions | Hundreds of thousands | Tens of thousands | Thousands | Hundreds | Tens | Units |
|---|---|---|---|---|---|---|---|---|---|
| . | . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . | 5 |
| . | . | . | . | . | . | . | . | 7 | 6 |
| . | . | . | . | . | . | 1 | 9 | 8 | |
| . | . | . | . | . | 5 | 4 | 3 | 2 | |
| . | . | . | . | 9 | 0 | 8 | 7 | 6 | |
| . | . | . | . | 2 | 1 | 3 | 4 | 6 | 5 |
| . | . | . | 4 | 3 | 2 | 1 | 9 | 8 | 7 |
| . | . | 6 | 5 | 4 | 3 | 2 | 1 | 0 | 9 |
| | 4 | 8 | 1 | 3 | 2 | 4 | 6 | 7 | 8 |
| 5 | 3 | 9 | 7 | 6 | 4 | 2 | 1 | 4 | 5 |

QUESTIONS.—What is the first figure called in the French mode? What is the second? Third? Fourth? Fifth? Sixth? Seventh? Eighth? Ninth?

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put 3 for the hundreds of thousands, 2 for the twenty thousands, or two tens of thousands, 4 for the thousands, 9 for the hundreds; and as the tens come next after the hundreds, and are wanting in this instance, we put a cypher in their place and then write 4 for the units,—thus we have 324904."—(*Lacroix.*)

### 2. *English Mode.*

It will be seen that this table is in all respects like the preceding till we come to thousands of millions, which are in the *French* mode called *billions*;—whereas in this mode the place of billions is considered three figures farther to the left, as is seen in this table.

This method is the one used in the following work.

| Billions | Hundreds of thousands of millions | Tens of thousands of millions | Thousands of millions | Hundreds of millions | Tens of millions | Millions | Hundreds of thousands | Tens of thousands | Thousands | Hundreds | Tens | Units |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | 5 |
| | | | | | | | | | | | 7 | 6 |
| | | | | | | | | | | 1 | 9 | 8 |
| | | | | | | | | 5 | 4 | 3 | 2 | |
| | | | | | | | 9 | 0 | 8 | 7 | 6 | |
| | | | | | | 2 | 1 | 3 | 4 | 6 | 5 | |
| | | | | | 4 | 3 | 2 | 1 | 9 | 8 | 7 | |
| | | | | 6 | 5 | 4 | 3 | 4 | 1 | 0 | 8 | |
| | | | 4 | 8 | 1 | 3 | 2 | 2 | 6 | 7 | 9 | |
| | | 5 | 3 | 9 | 7 | 6 | 4 | 2 | 1 | 4 | 5 | |
| | 9 | 0 | 4 | 6 | 7 | 8 | 9 | 4 | 3 | 2 | 1 | |
| 8 | 7 | 6 | 3 | 5 | 7 | 9 | 8 | 2 | 4 | 0 | 7 | |
| 5 | 2 | 1 | 3 | 0 | 7 | 2 | 1 | 7 | 2 | 3 | 4 | 5 |

The general rule for the enunciation of any sum is this: "*To the simple value of each figure join the name of its place, beginning at the left and reading towards the right.*"—(Thompson.)

#### APPLICATION.

Write in figures the following numbers.

1. Twenty-seven.
2. Two hundred and forty-three.
3. Five thousand, four hundred and twenty-nine.
4. Seventeen thousand, two hundred and five.
5. One hundred and twenty-five thousand, three hundred and ninety-two.
6. Three millions, seven hundred thousands, four hundred and eleven.

Write in words the following numbers. 39,402.—1600.—79,411.—927,500.—2840701.

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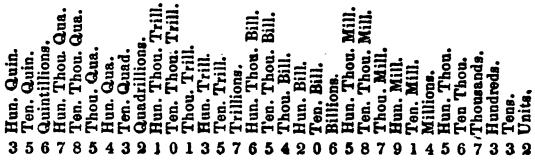
Numbers may be considered in two ways. One is when no particular denomination is mentioned, to which their units belong; in this they are called *abstract numbers*. The other is, when the denominations of their units are specified,

as when we say, two men—five miles—six hours; these are called *concrete numbers*. "The formation of numbers by the successive union of units is evidently independent of the nature of their units, and this is also the case with the properties resulting from this formation, by which properties we are able to compound and decompound numbers, which is called calculation."—(*Lacroix.*)

There are no limits to the numbers which may be enumerated. As far as names can be applied, numbers can be read; and as names may be applied indefinitely, figures may be read indefinitely.

The learner may read the following numbers.




Write 23 sextillions in figures—15 heptillions—178 octillions—25 nonillions—1687 decillions.

A RULE in Arithmetic, is the statement of a principle in its practical form,—so as to furnish the necessary information to the learner, with regard to the course he is to pursue in solving a given example.

A DEMONSTRATION shows the reasons on which a rule is founded. The rule gives the directions, and the demonstration shows the *reasons* for such directions. When the rule, and the reasons on which it is founded, are understood, the practical application is easy.

The simple or fundamental rules, are *four*. All those which follow, are merely combinations and changes of these. The learner will be repaid for a very particular and faithful attention to them, before he proceeds to those which follow.

QUESTIONS.—What is the first method in the English mode called? What the second? Third? Fourth? Fifth? Sixth? &c. &c. How do the modes agree? How differ? In how many ways may numbers be considered? What is said of the formation of numbers by successive units? How far can you enumerate? What do you mean by a rule? What is a demonstration? Are these of any use? How many fundamental rules? For what will you be repaid?

## SECTION III.

### ADDITION.

By Simple Addition, is meant the putting together of units, or collections of units, so that the whole may be included in one number. Three dollars, five dollars, and eight dollars, when added make sixteen, and of course may be written or spoken of as one number.

1. A man sold at one time 14 bushels of wheat, and at another time 27; how many bushels did he sell? Ans. 41.

2. What is the amount of 20, 44 and 173, when added together? Ans. 237.

3. A Bookseller receives four boxes of books; the first box contains 115, the second 146, the third 98, and the fourth 225; how many books does he receive in all? Ans. 584.

4. Pennsylvania has 53 counties, New York 56, and Virginia 115; how many counties in these three states? Ans. 224.

5. A man killed an ox, the meat of which weighed 642 pounds, the hide 105 pounds, and the tallow 92 pounds; what did they all weigh? Ans. 839.

6. A farmer gathered 12 bushels of apples from one tree, 16 from another, 19 from another, 11 from another, 22 from another, and 5 from another; how many bushels did he gather from all the trees? Ans. 85.

7. What is the sum of 412, 27, 1632, and 4? Ans. 2075.

8. According to the last census, New Hampshire has 269533 inhabitants, Vermont has 280679, and Massachusetts has 610014; what then is the whole number of inhabitants in these three states? Ans. 1160226.

9. A man has four flocks of sheep; in the first flock are 425 sheep and 217 lambs, in the second 300 sheep and 199 lambs, in the third 911 sheep and 881 lambs, in the fourth 218 sheep and 90 lambs; how many sheep and lambs has he? Ans. 3241.

10. A tree was broken off by the wind 12 feet from the ground, the part broken off was 37 feet long; what was the length of the tree before it was broken? Ans. 49 feet.

Questions.—What is Simple Addition? Can you give a rule for Addition?

11. What is the sum of 999, 0012, 7, 50005, 11 and 01?
Ans. 51035.

12. Sir Isaac Newton was born in the year 1642, and was 85 years old when he died; in what year did he die?
Ans. 1727.

13. In an orchard 14 trees bear peaches, 27 bear plums, 30 bear cherries, and one hundred and sixteen bear apples; how many trees are there in the orchard? Ans. 187.

14. A merchant after selling 216 bushels of salt had 112 left; how many bushels had he at first? Ans. 328.

15. How many days are there in the twelve calendar months? Ans.

16. What is the sum of 8, 1000, 0001, 2, 444444, 20, and 18? Ans. 445493.

17. What is the sum of three billions, two hundred and twenty-six thousand, five hundred and forty millions, nine hundred and twenty-seven thousand, one hundred and one; six hundred and seventy three thousand, nine hundred and eleven; one million, two thousand and two; and five hundred and ninety-seven, when added together?
Ans. 3226542603014.

18. A man bought a farm for 1515 dollars, and sold it for 217 dollars more than he gave for it; what did he sell it for? Ans. $1732.

19. What is the amount of 24180, 29, three thousand, four hundred and nine, 50, 99, and seventy-five?
Ans. 27842.

20. New York contains 1913508 inhabitants, Pennsylvania 1347672, New Jersey 320779 and Delaware 76739; what is the whole number of inhabitants in the four states?
Ans. 3658698.

21. A merchant bought 55 barrels of beef for 320 dollars; 70 barrels for 505 dollars; and 120 barrels for 975 dollars; how many barrels did he buy, and how many dollars did he pay for the whole?
Ans. { Bought 245 bbls. / Paid 1800 dolls.

22. A man has four horses: the first is worth 125 dollars; the second is worth 112 dollars; the third is worth 100 dollars; and the fourth is worth 90 dollars; what are all worth?
Ans. 427 dollars.

23. Add together 2346721, 900200, 40, 234719.
Amount 3481680.

24. A merchant bought a quantity of salt for 325 dollars,

and sold it for 78 dollars more than the cost; what did he sell it for? Ans. 403 dolls.

25. Add 2; 19; 817; 4298; 50916; 730205; 9180634; together. Ans. 9966891.

26. Add 509267; 235809; 72920; 8292; 420; 21; 9; together. Ans. 826738.

27. Add 456934000; 5903568; 89034511; 21; together. Ans. 551872100.

28. Add 32; 451; 89; 1734; 5; 89103; together. Ans. 91414.

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## SECTION IV.

### SUBTRACTION.

1. A man having 79 sheep, sold 15; how many had he left? Ans. 64.

2. If 432, be taken from 612; how many will remain? Ans. 180.

3. A man bought a chaise for 175 dollars, and to pay for it gave a colt worth 40 dollars, and the rest in money; how much money did he pay? Ans. 135 dolls.

4. In 1830 the number of inhabitants in a certain town was 2014 of whom 1042 were under 20 years of age; how many were over 20 years of age? Ans. 972.

5. A man sells his farm for 2720 dollars to pay his debts which amount to 1946 dollars; how much has he left, after paying his debts? Ans. 774.

6. What number is that which taken from 365 leaves 159? Ans. 206.

7. A merchant bought a quantity of butter for 117 dollars and sold it for 141 dollars; how much did he gain in the bargain? Ans. 24 dolls.

8. The population of Boston in the year 1700 was 7000, in the year 1800 it was 24937; what was the increase during this century? Ans 17937.

9. What number must be added to 436 that it shall become 3524? Ans. 3088.

10. A man having been born in the year 1797, what was his age in 1831? Ans 34 years.

11. What is the difference between 8000 and 2999? Ans. 5001.

12. The number of inhabitants in the United States in

1820 was 9638166, in 1830 there were 12856165; what was the increase between these two dates? Ans. 3217999.

13. From four billions, seven hundred millions sixty-eight thousand and nine; take three billions, eight hundred and eighty eight thousand, four hundred and ninety-seven millions, one thousand nine hundred and ninety-nine.

112203066010 remainder.

14. A minuend is 4002, and a subtrahend is 999; what is the difference? Ans. 3003.

15. The sum of two numbers is 64892, and the greater number 46234; what is the smaller number? Ans. 18658.

16. Peace between the United States and Great Britain took place in 1783, and war was again declared in 1812; how long did the peace continue? Ans. 29 years.

17. What is the difference between 45678912 and 37899048? Ans. 7779864.

18. If I lend a friend 9480 dollars, and receive in part payment 1987 dollars; how much remains due? Ans. 7493.

19. The population of Virginia in 1830 was 1211272, of whom 469724 were slaves; how many free persons?

Ans. 741548.

20. The imports of the United States during the year ending Sept. 30, 1830 amounted to 70876920 dollars; the exports during the same time amounted to 73849508 dollars; how much did the exports exceed the imports?

Ans. 2972588 dolls.

21. A farmer has 7 bags of hops weighing 1428 pounds; he sells 6 bags weighing 1195 pounds; how many pounds has he left? Ans 233 pounds.

22. If you buy 20 sheep for 40 dollars, and sell 15 of them for 35 dollars how many sheep will you have left?

Ans. 5.

23. From 24791 less 3218 take 9110; what number remains? Ans. 12463.

24. From 5331806 take 5073918. Ans. 257888.

25. What is the difference between 7020974 and 2766809?

Ans. 4254165.

26. If from 8503602—574271 be taken—what will remain? Ans. 7929331.

27. From 834000900 take 9909093.

28. What is the difference between 987654321 and 123456789?

29. From 9000000 take 0000001.

## SECTION V.

## MULTIPLICATION.

1. What will 167 barrels of flour come to, at 9 dollars a barrel? Ans. $1503.

2. What will be the price of 8 hogsheads of wine, at 129 dollars per hogshead? Ans. $1032.

3. In an orchard there are 134 rows of trees, and 9 trees in each row; how many trees are there in the orchard? Ans. 1206.

4. If 23679 be repeated ten times, what will be the product? Ans. 236790.

5. What is the product of 9763421110 multiplied by 7? Ans. 68343947770.

6. If a book has 256 pages, and 100 lines on each page, how many lines are there in the book? Ans. 25600.

7. How many days are there in 20 years, there being 365 in one year? Ans. 7300.

8. What is the product of 4000 multiplied by 400? Ans. 1600000.

9. A certain army was composed of 204 battallions consisting of 824 men each; required the whole number of men. Ans. 168096.

10. In dividing a sum of money among 352 men, each received 17 dollars; what was the sum divided? Ans. $5984.

11. If the pendulum of a clock swing 60 times in one minute, how many times will it swing in 3500 minutes? Ans. 210000.

12. If a man's income be 235 dollars per annum, what will it amount to in 99 years? Ans. $23265.

13. If a water-wheel revolve 36 times in a minute, how many times will it revolve in an hour? Ans. 2160.

14. If 2114 men receive 48 dollars each, how many dollars do all receive? Ans. 101472.

15. A garrison of 3000 is to be paid, and each man receives 128 dollars; how many dollars will they all receive? Ans. 384000.

16. Multiply 79246817 by twenty-seven thousand, four hundred and thirteen. Prod. 2172392994421.

17. How many are 2000 times 14? Ans. 28000.

18. If 4 bushels make a barrel of flour, and the price of

wheat be 2 dollars per barrel, what will 225 barrels of flour cost? Ans. $1800.

19. A divisor is 29, and the quotient the same; required the dividend. Ans. 841.

20. A certain city is divided into 12 wards, each ward contains 2000 families, and the average number in a family is 5; what is the number of inhabitants in the city?
Ans. 120000.

21. Multiply 74218 by 999. Prod. 74143782.

22. What number, divided by 24, will give a quotient of 48? Ans. 1152.

23. If the earth in her annual revolution move 68000 miles in an hour, what distance will she move in 16 hours?
Ans. 1088000 miles.

24. Required the product of 935 multiplied by itself.
Ans. 874225.

25. In a certain building there are 64 apartments, in each apartment are 4 windows, and in each window 24 lights; how many lights are there in the house? Ans. 6144.

26. Multiply six hundred and fifty-nine thousand four hundred and twenty-six millions, eight hundred and ninety-four thousand, seven hundred and twenty-seven, by one hundred and nine. Prod. 7187531525243.

27. What number, divided by 12, will give a quotient of 144? Ans. 1728.

28. Multiply 99999 by 3468.
29. " 000919 by 20000.
30. " 82164973 by 99003.
31. " 9999 by 99.

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## SECTION VI.

### DIVISION.

1. At 8 dollars a barrel how many barrels of flour can be bought for 576 dollars? Ans. 72.

2. If 1380 dollars be distributed equally among 12 men, how many will each receive? Ans. 115.

3. A man having 5520 bushels of corn, wishes to put it into bins holding 16 bushels each; how many bins will it take? Ans. 345.

4. A man dies leaving an estate of 7875 dollars to his 7 sons; what is each son's share? Ans. 1125 dolls.

5. Suppose 729 men formed into 9 ranks; how many will there be in each rank? Ans. 81.

6. Forty-five horses were sold for 9900 dollars; what was the average price of each? $220.

7. A prize valued at 32832 dollars is divided among 72 seamen; what is each man's share? Ans. $456.

8. Required a number which multiplied by 48 shall become 4656. Ans. 97.

9. Divide 24986 by 62. Quotient 403.

10. A gentleman's annual income is 38325 dollars; how much is that per day, there being 365 days in a year? Ans. 105 dolls.

11. How many times are 66 contained in 132132? Ans. 2002.

12. In 31755 days how many years, allowing 365 days to a year? Ans. 87.

13. A man raised 64562 bushels of grain on 1565 acres; how many bushels was that per acre? Ans. 41.

14. Divide 1020096 by 13241. Quotient 77.

15. How many times can a cask which is capable of containing 54 gallons be filled from a cistern containing 2496 galls.? Ans. 46 times, and 12 gallons remain in the cistern.

16. Divide 351 by 18. Quotient 19, Rem. 9.

17. How many times are 94 contained in 717? Ans. 7 times, and 59 remain.

18. The President of the United States has a salary of 25000 dollars per annum; how much is that per day allowing 365 days to a year? Ans. $68 per day, and $180 over.

19. If 40000 persons send one representative to Congress, how many will Massachusetts send; her population according to the census of 1830 being 610014? Ans. 15, and there is an excess of 10014 persons.

20. Divide 1827000 by 900. Ans. 2030.

21. Four hundred and sixty-four thousand dollars are sufficient to pay a garrison consisting of 8000 men; how much can each man receive if they share equally? Ans. $58.

22. Divide 58724 by 2600. Quo. 22, Rem. 1524.

23. A general has disposed his army consisting of 36000 men into companies of 100 men each; how many companies has he? Ans. 360.

24. What number multiplied by 10 will become 600? Ans. 60.

25. Connecticut has a population of 297711; if she support 1 minister to 1000 persons, how many ministers does the state support? Ans. 297 ministers; and there is an excess of 711 persons.

26. Divide 30114 bushels of corn equally among 63 men; how many will each man receive? Ans. 478.

27. A manufacturing company employed a number of workmen, and gave them 27 dollars per month; at the expiration of one month, it took $10125 to pay them. How many men were there? Ans. 375.

28. One factor of 1872 is 24; what is the other factor? Ans. 78.

29. How many times are 6942 contained in 13884? Ans. 2.

30. What number must you multiply by 47 to produce 2346894000?

31. In 783954 barley-corns, how many inches? Ans. 261318.

32. Divide 409876960 by 125.

33. Divide 280208122081 by 912314. Ans. $307140_{9\,1\,2\,3\,14}^{1\,4\,1}$

34. How many gallons, in 218363 pints?

35. With 152 dollars how many yards of cloth can you buy at $1,75 per yard?

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## SECTION VII.

### COMPOUND ADDITION.

1. A man bought one load of cider for 8£ 4s; and another for 6£ 8s. 4d.; what did both amount to? Ans. 14£ 12s. 4d.

2. What is the amount of the following sums?

| £ | s. | d. |
|---|---|---|
| 2 | 4 | 6 |
| 13 | 9 | 4 |
| 16 | 11 | 7 |
| 20 | 8 | 11 |

3. What is the sum of 1£ 2s. 9d.; 2£ 5s. 6d.; 3£ 8s. 3 d.; 4£ 11s. and 5£ 13s. 9d.? Ans. 17£ 1s. 3d.

4. A man bought two loads of hay, one weighing 18 cwt. 2 qrs., the other 17 cwt. 1 qr. 24 lbs.; what was the weight of both? Ans. 35 cwt. 3 qrs. 24 lb.

5. What quantity of coffee is there in 5 bags weighing, viz. 2 qrs. 14 lbs.; 3 qrs. 5 lbs.; 3 qrs. 17 lbs.; 1 cwt. 1 qr. 14 lb.; 1 cwt. 2 qrs. 27 lbs.? Ans. 5 cwt. 1 qr. 21 lbs.

6. A butcher kills an ox, one quarter of which weighs 1 cwt. 3 qrs. 24 lbs; another 2 cwt. 19 lbs.; another 1 cwt. 3 qrs. 27 lbs.; the other 1 cwt. 2 qrs. 25 lbs; the hide 1 cwt. 10 lbs.; the tallow 3 qrs. 7 lbs; what is the whole weight of the ox? Ans. 9 cwt. 3 qrs.

7. What is the total weight of the following sums?

| | *Cwt.* | *qr.* | *lbs.* | *oz.* | *dr.* |
|---|---|---|---|---|---|
| | 20 | 8 | 27 | 15 | 13 |
| | 12 | 0 | 0 | 6 | 2 |
| | 10 | 1 | 3 | 9 | 4 |
| | 6 | 2 | 8 | 1 | 15 |
| | 4 | 2 | 20 | 13 | 3 |
| | 27 | 0 | 21 | 2 | 6 |
| | 8 | 2 | 2 | 0 | 0 |
| | 0 | 2 | 13 | 12 | 11 |
| Ans. | 90 | 2 | 13 | 13 | 6 |

8. A goldsmith has two sets of silver spoons, one weighing 2 lbs. 10 oz. 16 pwts. 20 grs., the other 3lb. 8 oz. 12 pwts; what is the weight of both? Ans. 6 lbs. 3 oz. 8 pts. 20 grs.

9. What is the amount of the following sums?

| *Lbs.* | *oz.* | *pwts.* | *grs.* |
|---|---|---|---|
| 14 | 6 | 12 | 13 |
| 17 | 5 | 3 | 12 |
| 15 | 0 | 9 | 16 |
| 2 | 7 | 15 | 20 |
| 13 | 2 | 10 | 19 |

Ans.

10. What is the total weight of 3 gold watches, one weighing 10 oz. 18 pwts.; another 1 lb. 2 oz. 7 grs.; the other 7 oz. 11 pwts. 14 grs.?

Ans. 2 lb. 8 oz. 9 pwts. 21 grs.

11. A physician purchased at one time medicines to the amount of 4 lb. 6 oz. 5 dr. 2 scr., at another 11 oz. 6 dr. 19 gr.; how much at both times?

Ans. 5 lb. 6 oz. 3 dr. 2 scr. 19 gr.

12. Required the total weight of 5 parcels of drugs, the several weights of which are as follows:

| *Lb.* | *oz.* | *dr.* | *scr.* | *gr.* |
|---|---|---|---|---|
| 4 | 9 | 7 | 2 | 2 |
| 12 | 11 | 6 | 1 | 14 |
| 10 | 8 | 4 | 0 | 9 |
| 7 | 10 | 5 | 2 | 0 |
| 18 | 0 | 3 | 1 | 19 |

Ans.

13. An apothecary made a composition of 3 ingredients; the first weighed 7 oz. 2 dr. 1 scr. 10 gr.; the second 1 lb. 0 oz. 2 dr. 2 scr.; and the third 7 dr. 18 gr.; what was the weight of the compound? Ans. 1 lb. 8 oz. 4 dr. 1 scr. 8 gr.

14. There are two pieces of road; the length of one piece is 3 mi. 6 fur. 30 rods; that of the other is 10 mi. 18 rods 14 ft.; what is the length of both?

Ans. 13 mi. 7 fur. 8 rods 14 ft.

15. Add together the following—

| *Deg.* | *mi.* | *fur.* | *rds.* | *ft.* | *inch.* | *bar.* |
|---|---|---|---|---|---|---|
| 16 | 24 | 6 | 32 | 14 | 7 | 2 |
| 9 | 17 | 4 | 16 | 9 | 11 | 0 |
| | 40 | 1 | | 4 | | 1 |

Ans.

16. The distance from Washington to Dumfries is 37 mi. 7 fur. 39 rods,—thence to Richmond, 92 mi. 6 fur. 38 rods,—thence to Suffolk, 110 mi. 1 fur. 3 rods,—thence to Wilmington, N. C., 240 mi. 7 fur. 21 rods,—thence to Charleston, S. C., 190 mi. 19 rods. What is then the distance from Washington to Charleston? Ans. 672 miles.

17. A man has 3 fields; the first contains 12 acres 2 roods 28 rods,—the second, 9 acres 15 rods,—the third, 11 acres 3 roods 37 rods; how much in all? Ans. 33 acres 3 roods.

18. What is the amount of the following sums?

| *A.* | *roods* | *rods* | *yds.* | *ft.* | *in.* |
|---|---|---|---|---|---|
| 44 | 2 | 28 | 10 | 9 | 27 |
| 97 | 1 | | 11 | 2 | |
| 8 | | 21 | | 1 | 9 |

Ans.

19. A landholder sells 4 house-lots; the first contains 2 roods 17 rods 9 feet,—the second, 1 rood 12 rods,—the third, 3 roods 12 rods 97 feet,—the fourth, 1 rood 33 rods 127 feet; how much in all? Ans. 2 A. 0 roods, 34 rods 233 feet.

20. In one pile of wood are 4 cords 27 feet,—in another, 10 cords 15 feet 1124 inches,—in another, 10 cords 40 feet; and in another 5 cords, 23 feet, 708 inches how much in all?

Ans. 34 cords 106 ft. 104 in.

21. Add the following sums.

| | | |
|---|---|---|
| 47 | 111 | 299 |
| 18 | 24 | 176 |
| 9 | 3 | 202 |
| 16 | 124 | 4 |
| 49 | 50 | 51 |

Ans.

22. A man carries to market 4 loads of bark; the first containing 1 cord 64 feet 864 inches,—the 2nd, 1 cord 44 inches; the 3rd. 1 cord, 63 ft. 20 inches; the 4th, 1 cord 60 feet 931 inches; how much in all? Ans. 5 cords 60 ft. 131 in.

23. A merchant bought 3 pieces of cloth; one containing 20 yards three quarters 1 nail,—another 26 yards,—and the other 30 yards 3 quarters 3 nails; how many yards did he buy? Ans. 77 yds. 3 qrs.

24. Required the amount of the following sums.

| *Yds.* | *qr.* | *na.* | *in.* |
|---|---|---|---|
| 24 | 2 | 3 | 1 |
| 60 | 1 | 1 | 2 |
| 94 | 3 | 2 | 0 |
| 49 | 0 | 1 | 1 |

Ans.

25. How much calico in 2 pieces, one measuring 13 yds. 2 na.—the other 17 yds. 3 qrs. 1 na.?

Ans. 30 yds. 3. qrs. 3 na.

26. A man bought 3 bushels 3 pecks of wheat at one time, 6 bu. at another, 7 bu. 2 pks. 7 qts. at a third, and 4 bu. 1 pk. 6 qts. at a fourth; how many bushels did he buy in all?

Ans. 21 bu. 3 pks. 5 qts.

27. What is the amount of the following quantities?

| *Ch.* | *qr.* | *bu.* | *pk.* | *qt.* | *pt.* |
|---|---|---|---|---|---|
| 4 | 3 | 6 | 2 | 2 | 1 |
| 9 | 2 | 7 | 3 | 1 | 0 |
| 10 | 1 | 5 | 1 | 7 | 1 |
| 2 | 0 | 4 | 0 | 4 | 0 |

Ans.

QUESTIONS.—Is there any deficiency in the 21st sum? What is it? What must you do in order to perform the operation?

28. A merchant receives corn; from one man, 141 bushels, from another, twenty-seven bushels 3 pecks and six quarts, from another, 19 bu. 2 pks. 4 qts., and from another, 6 bu. 1 pk.; how much does he receive?

Ans. 194 bu. 3 pks. 2 qts.

29. A wine merchant has in his cellar 2 pipes 1 hogshead 44 gallons of Madeira,—1 pipe 27 gall. 3 qts. Port,—and 5 pipes 1 hhd. 39 gall. 1 qt. Champaigne; how much has he in all? Ans. 9 pipes 1 hhd. 48 galls.

30. A brewer sells to different persons the following quantities of beer: 1 bar. 28 gall.—1 bar. 17 gall.—5 bar.—2 gall. —1 gall 2 qts. 1 pt.—7 gall. 2 qts. 1 pt.—18 gall.; what is the amount? Ans. 9 bar. 3 gall. 1 qt.

31. A was born in Plymouth, where he resided till he was 14 years 6 months of age,—he then removed to Boston, where he lived 7 years 8 months 21 days;—he afterwards lived in Salem 8 yrs. 2 mo.—in New-York 1 yr. 9 mo. 11 days;—from New-York he went to Philadelphia, where he has now lived just 9 mo.; how much time has he spent in all these places? Ans. 32 yrs. 11 mo. 2 days.

32. A man's hired labourers worked as follows:

| | *Yrs.* | *mo.* | *wk.* | *ds.* | *hs.* |
|---|---|---|---|---|---|
| The 1st | 2 | 9 | 3 | 4 | |
| " 2d | | 11 | 1 | 2 | |
| " 3d | 1 | 6 | | | |
| " 4th | 1 | 7 | 2 | 1 | |
| " 5th | | | 3 | | |
| " 6th | | | | 1 | 6 |

For how much time is he indebted to all?

Ans. 6 yrs. 11 mo. 2 wks. 1 day 6 hours.

33. There is a certain family, of which the eldest child is 30 yrs. 7 mo. 11 days of age,—the second, 25 yrs. 5 mo.,—the third, 23 yrs. 11 mo. 7 days,—the fourth, 21 yrs. 11 mo. 23 days,—and the youngest 13 yrs. 9 mo. 22 days. What is the sum of their ages? Ans. 115 yrs. 6 mo. 3 days.

34. What is the sum of 2 days 10 hours 15 minutes 12 seconds,—20 h. 35 min. 45 sec., and 7 min. 19 sec.?

Ans. 3 days 6 hours 58 min. 16 sec.

35. Add together

| | *Signs.* | *deg.* | *min.* | *sec.* |
|---|---|---|---|---|
| | 4 | 20 | 45 | 18 |
| | 1 | 2 | 3 | 4 |
| | 2 | 12 | 13 | 14 |
| | | 6 | 7 | 3 |

36. What is the amount of 13 deg. 11 min. 29 sec.—2 S. 12 deg. 50 min. 50 sec.—8 S. 24 min. 41 sec., and 1 S. 10 deg. 9 sec.? Ans. 12 S. 6 deg. 27 min. 9 sec.

## SECTION VIII.

### COMPOUND SUBTRACTION.

1. A having a note against B of 24£ 16s. 11d., received 9£ 18s. 9d. 2 qrs.; how much remains due, exclusive of interest? Ans. 14£ 18s. 1d. 2 qr.

| | *Ton.* | *cwt.* | *qr.* | *lb.* | *oz.* | *dr.* |
|---|---|---|---|---|---|---|
| 2. From | 3 | 7 | 1 | 18 | 10 | 8 |
| take | 1 | 15 | 3 | 8 | 12 | 14 |
| Remainder | | | | | | |

3. A silversmith bought an ingot of silver weighing 8 lbs. 2 oz. 12 pwts.,—from which he made a set of spoons weighing 2 lb. 4 oz. 7 pwts. 19 grs.; how much silver had he left? Ans. 5 lb. 10 oz. 4 pwts. 5 grs.

| | *Deg.* | *mi.* | *fur.* | *rods.* | *ft.* | *in.* | *bar.* |
|---|---|---|---|---|---|---|---|
| 4. From | 10 | 44 | 5 | 27 | 10 | 4 | 2 |
| take | 4 | 50 | 7 | 16 | 9 | 6 | 1 |
| Rem. | | | | | | | |

5. A man has two farms; the first contains 475 acres 3 roods 4 rods,—the other 180 acres 1 rood 27 rods; how much more land in the first than in the second? Ans. 295 A. 1 roods 17 rods.

6. From a pile of wood containing 27 cords 48 feet 1612 inches, were sold 3 loads measuring 4 cords 119 feet; how much remained? Ans. 22 C. 63 ft. 1612 in.

| | *Yd.* | *qr.* | *na.* |
|---|---|---|---|
| 7. From | 45 | 2 | 1 |
| take | 17 | 3 | 2 |
| Rem. | | | |

8. A man having 43 bu. 2 pks. of wheat, consumed 11 bu. 3 pks.; how much had he left? Ans. 31 bu. 3 pks.

9. A man drew from a hogshead of beer, 1 bar. 7 galls. 3 qts.; how much remained? Ans. 28 galls. 1 qt.

10. From a pipe of wine 1 hhd. 18 galls. 2 qts. were sold; how much remained? Ans. 44 galls. 2 qts.

11. A's age is 74 years 9 months 15 days,—B's age is 12 years 10 months 25 days; how much older is A than B?

Ans. 61 yrs. 10 mo. 20 dys.

| | *S.* | *deg.* | *min.* | *sec.* |
|---|---|---|---|---|
| 12. From | 8 | 16 | 40 | 30 |
| take | 7 | 20 | 19 | 29 |

Rem.

13. A note, bearing date November 4th, 1824, was paid April 6th, 1827; how long was the note at interest?

Ans. 2 yrs. 5 mo. 2 dys.

## SECTION IX.
## DECIMAL NUMBERS.
(See Section 9. of Part III.)

## SECTION X.
## FEDERAL MONEY.
(See Section 10. of Part III.)

## SECTION XI.
## *Exchange from one CURRENCY to another.*
(See Section 11. Part III.)

## SECTION XII.

### COMPOUND MULTIPLICATION.

1. A man bought 5 yards of cloth at 9 s. 6 d. per yard; what was the cost? Ans. 2 £. 7 s. 6 d.

2. A merchant buys 6 casks of raisins, each weighing 1 cwt. 2 qrs. 15 lb.; what is the weight of the whole?

Ans. 9 cwt. 3 qrs. 6 lb.

3. A man owns 8 house-lots, each measuring 2 roods 20 rods; what do all measure? Ans. 5 acres.

4. Bought 6 loads of bark, each measuring 1 cord 16 feet; how much in all? Ans. 6 c. 96 ft.

5. In 9 casks, each containing 24 galls. 3 qts., how many barrels of wine? Ans. 7 bar. 2 galls. 1 qt.

6. If a man spend 12 £. 4 s. 9 d. each week, what will his expenses amount to in a year? Ans. 636 £. 7 s.

7. What is the weight of 36 silver spoons, each weighing 1 oz. 9 pwts. 14 grs.? Ans. 4 lb. 5 oz. 5 pwts.

8. How much sugar in 24 hogsheads, each containing 9 cwt. 3 qrs. 21 lb.? Ans. 11 tons 18 cwt. 2 qrs.

9. How much wine in 84 casks, each containing 22 galls. 3 qts. 1 pt. 3 gills? Ans. 30 hhds. 39 galls. 1 qt. 1 pt.

10. The distance from A to B is 5 miles 6 fur. 27 rods; but the distance from C to D is 81 times as much; what is the latter distance? Ans. 472 mi. 4 fur. 27 rods.

11. What will 47 yards of cloth cost, at 17s. 9d. per yard? Ans. 41£ 14s. 3d.

12. If 14 men build 12 rods 6 feet of wall in one day, how much will they build in 35 days? Ans. 432 rods 12 ft.

13. How much land in 97 lots, each containing 48 acres 2 roods 20 rods? Ans. 4716 A. 2 roods 20 rods.

14. In 241 barrels of flour, each containing 2 cwt. 9 lb., how many cwt.? Ans. 501 cwt. 1 qr. 13 lb.

15. How many bushels of wheat in 376 bags, each containing 2 bu. 3 pks.? Ans. 1034 bu.

16. In 19 years of 365 days 5 hours 48 minutes 48 seconds, how many days? Ans. 6939 d. 14 h. 27 min. 12 sec.

17. If it be noon 2 hours 30 minutes 20 seconds sooner at A than at B, what is their difference of longitude, the earth moving 15 degrees each hour? Ans. 37 deg. 35 min.

18. How much silver in 115 ingots, each weighing 7 lb. 9 oz. 16 pwts. 20 grs.? Ans. 899 lb. 3 oz. 15 pwts. 20 grs.

19. How much wood in 87 loads, each measuring 117 ft. 80 in.? Ans. 79 c. 71 ft. 48 in.

20. What will 7 cwt. of sugar cost, at 2 £. 7 s. 6 d. per hundred weight? Ans. 16 £. 12 s. 6 d.

21. If a ship sail at the rate of 2 deg 34 min. 16 sec. each day, how far will she proceed in 20 days? Ans. 51 deg. 25 min. 20 sec.

22. Eleven ships are so situated, that the first is in longitude 5 deg. 25 min. 33 sec. west, and the difference of longitude between each is equal to the longitude of the first; what is the longitude of the eleventh ship? Ans. 59 deg. 41 min. 3 sec.

## SECTION XIII.

### COMPOUND DIVISION.

1. If 4 £. 6 s. be divided equally between 2 men; how much will each receive? Ans. 2 £. 3s.

2. If 8 casks of nails weigh 15 cwt. 1 qr. 14 lbs., what is the weight of each cask? Ans. 1 cwt. 3 qrs. 19 lbs. 4 oz.

3. If 9 silver cups weigh 5 lbs. 9 oz. 2 pwts. 15 grs.; what is the weight of each? Ans. 7 oz. 13 pwts. 15 grs.

4. If a man travel 122 miles 4 fur. 20 rods in 5 days; how far does he travel each day? Ans. 24 mi. 4 fur. 4 rods.

5. If 9 fields of equal dimensions contain 113 acres 3 roods 25 rods; how much land is there in each field?
Ans. 12 acres 2 roods 25 rods.

6. If 12 loads of wood equal in size contain 14 cords 56 feet; how much wood in each load? Ans. 1 cord 26 feet.

7. Bought 16 barrels of flour for 35 £. 16 s. 10 d.; how much was that per barrel? Ans. 2 £. 4 s. 9 d. $2\frac{1}{2}$ qrs.

8. If 24 teams be loaded with 29 tons 12 cwt. 2 qrs. of hay; how much is that to each team?
Ans. 1 ton 4 cwt. 2 qrs. 21 lbs.

9. A township containing 64236 acres 3 roods 36 rods, was divided equally among 96 settlers; how much land did each receive? Ans. 669 acres 21 rods 170 ft. $22\frac{1}{2}$ in.

10. Bought 84 pipes of wine, containing 9468 galls. 1 qt. 1 pt.; how much in a pipe? Ans. 112 galls. 2 qts. 1 pt. 3 gills.

11. If it take 593 yards 3 qrs. of cloth to make 144 coats, how much is that to each coat? Ans. 4 yds. 1 na. $2\frac{5}{36}$ in.

12. If 15 cwt. 3 qrs. 3 lb. of provisions be divided equally among 2356 soldiers; how much will each man's share be? Ans. 12 ounces.

13. If a man spend 364 £. 12 s. 4 d. 3 qrs. in a year, what is that per day, calling the year 365 days?
Ans. 19 s. 11 d. 3 qr.

14. If 2 men build 642 rods of wall in 99 days, how much is that per day? Ans. 6 rods. 8 ft.

15. A privateer with a clear of 412 men took a prize valued at 97890 £.; what was each man's share?
Ans. 237 £. 11 s. 11 d. $1\frac{21}{103}$ qr.

16. A capital of 223 £. 16 s. 8 d. being divided into 192 shares, what is the value of each share?
Ans. 1 £. 3 s. 3d. $3\frac{1}{6}$qr.

17. Divide 375 miles 2 fur. 7 rods 2 yds. 1 ft. 2 in. by 39. Ans. 9 mi. 4 fur. 39 rods 2 ft. 8 in.

18. A piece of cloth containing 48 yards, cost 31£ 12s; what was it per yard? Ans. 13s. 2d.

19. Divide 1061 cwt. 2 qrs. by 28.
Ans. 37 cwt. 3 qrs. 18 lbs.

20. If a vessel sail 52 leagues 1 mi. 5 fur. 23 rods in 21 hours; at what rate does does she sail per hour?
Ans. 2 leagues 1 mi. 4 fur. 3 rods.

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## SECTION XIV.

### REDUCTION.

1. In 12 £. 4 s. 6 d. how many pence? Ans. 2934.

2. Reduce 36 £. 12 s. 6 d. 2 qrs. to farthings.
Ans. 35162 qrs.

3. If I receive 2£ 4s. 6d. from one man, and 12s. 9 d. from another; how many pence do I receive from both?
Ans. 687.

4. In 324 panes of glass, 8 inches wide and 12 long, how many square feet? Ans. 216.

5. In 23400 cubic feet, how many inches?
Ans. 40435200.

6. In 44800 pounds, how many drams and tons?
Ans. 11468800 dr. and 20 tons.

7. In 12613⅓ threepences, how many French crowns, at 6s. 8d. each? Ans. 473.

8. How many half inches will reach round the globe, it being 360 degrees? Ans. 3170534400.

9. How many barleycorns from Andover to Salem it being 15 miles? Ans. 2851200.

10. In 9 tons of round timber how many inches?
Ans. 622080.

11. How many seconds in a solar year, or in 365 days, 5 hours, 48 minutes, 48 seconds? Ans. 31556928.

12. In 25 cords of wood how many inches?
Ans. 5529600.

13. In 89763 square yards how many acres, &c.?
Ans. 18 a. 2 r. 7 p. 101 ft. 36 in.

14. How often will a wheel, of 16 feet and 6 inches circumference, turn round in the distance from Newburyport to Cambridge, it being 42 miles? Ans. 13440.

15. How many minutes since the discovery of America, it being the 1st day of January, at noon, 1832?

16. In 24 lbs. 11 oz. 9 dr. how many drams? Ans. 6329.

17. In 50 eagles, how many dollars? Ans. 500.

18. In 28 dollars, 17 cents, 5 mills, how many mills? Ans. 28175.

19. In 3 eagles, 2 dollars, how many mills? Ans. 32000.

20. Bring 345358 halfpence into pence, shillings, and pounds.

21. In 9758 pints of wine, how many gills? Ans. 39032.

22. In 9376 quarts how many bushels? Ans. 293.

23. How many feet are there in 72 miles? How many inches? Ans. 380160 ft. 4561920 in.

24. How many steps of one foot each must a man take in walking one rod? How many in a mile? How many in walking 20 miles?

25. How many times will a wheel 15 feet in diamater revolve in going 30 feet? How many in going 90 feet? How many in going 40 rods? How many in going 40 miles?

26. How many steps, of two feet 8 inches each, must you take in going from Andover to Boston, the distance being 20 miles?

27. How many square feet in a square yard? How many square feet in a square rod? How many square rods in a square mile? How many square inches in a square mile?

28. In 1 yard of cloth 4 quarters wide, how many yards 1 quarter wide? How many yards 1 quarter wide are there in 16 yards 4 quarters wide?

29. How many yards of cloth, 1 quarter wide, must you have to line 1 yard 4 quarters wide? How many to line 24 yards?

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## SECTION XV.

## DECIMAL FRACTIONS.

1. What cost 8 yards of cloth, at $7.875 per yard? Ans. 63.000.

2. How many cwt. of hay in 14 loads, containing 23.25 cwt. each? Ans. 325.5 cwt.

3. Multiply .3 by .2 Ans. .06.

4. At $5.37 per yard what cost 7.4 yards of cloth?
Ans. $39.738.

5. Multiply 1.340068 by 1.003084.
Ans. 1.344200769712.

6. If an orange is worth $.06, what is .3 of an orange worth? Ans. $ 0.018.

7. What cost 13 cwt. 1 qr. 15 lbs. of iron, at $4.27 per cwt.? Ans. $57.145.

8. What will 23 lbs. 7 oz. of sugar cost, at $11.43 per cwt.? Ans. $2.391.

9. How many times is 1.468 contained in 473.75?
Ans. 322.718.

10. How many times is .002 contained in .01? Ans. 5.

11. .1 is .001 of what number? Ans. 100.

12. Required the sum of 973—19—1.75—93.7164—9501.
Ans. 10588.4664.

13. From 480 take 245.0075. Ans. 234.9925.

14. Multiply .84179 by .0385. Ans. .032402415.

15. Divide 1 by .005643. Ans. 177.211.

16. How many times is .3 contained in 3? Ans. 10.

17. Goliah the Philistine is said to have been 6.5 cubits high, each cubit being 1 foot 7.168 English inches; what was his height in English feet? Ans. 10.383 feet.

18. If a traveller perform a journey in 35.3 days, when the days are 11.374 hours long; in how many days will he perform it, when the days are 9.13 hours long?
Ans. 43.976.

19. 1.5064 is what part of 8.944783? Ans. $\frac{15064}{89447830}$

20. If .35 of a ton of iron cost 10 £. 3 s. 5 d., what cost a ton at that rate? Ans. 29 £. 1 s. 2 d. 2 qr.

21. Divide .003 by 0.00001. Ans. $\frac{300}{1}$

22. Required the sum of 5.3, 11.973, 49, .9, 1.7314, 34.3.
Ans. 103.2044.

23. From 209.1384 take 195.91. Ans. 13.2284.

24. Divide .8297592 by .153. Ans. 5.4232.

25. Multiply 21.31 by 10000. Ans. 213100.00.

26. Multiply .0107 by .00103. Ans. .000011021.

27. Multiply .04 by .2. Ans. .008.

28. Multiply 1.0007 by .0003. Ans. .00030021.

29. Divide .4 by 13. Ans. .03077.

30. Divide 13.84783 by 137648. Ans. .0001006.

## SECTION XVI.

### VULGAR FRACTIONS.

1. What is the sum of $\frac{1}{2}$ and $\frac{1}{2}$? Ans. 1.

2. I received from one man $\frac{1}{4}$ of a cord of wood; from another $\frac{2}{4}$; what part of a cord did I receive? Ans. $\frac{3}{4}$.

3. How many feet are there in $\frac{3}{4}$ of a cord? Ans. 96.

4. What is the value of $\frac{1}{8}^4$ and $\frac{2}{8}$? Ans $2\frac{7}{8}$.

5. Bought 3 loads of hay—the first weighed $19\frac{3}{4}$ cwt; the second $20\frac{1}{5}$ cwt; and the third $22\frac{2}{3}$ cwt.; what was the weight of the whole? Ans. $62\frac{37}{60}$.

6. What is the sum of $\frac{3}{5}$ and $\frac{1}{7}$? Ans. $\frac{26}{35}$.

7. A man having $\frac{12}{24}$ of a quire of paper, received $\frac{6}{24}$ more; what part of a quire had he then? Ans. $\frac{3}{4}$.

8. Add $\frac{1}{3}$ of a week, $\frac{1}{4}$ of a day, and $\frac{1}{2}$ an hour together. Ans. 2 dys. $14\frac{1}{2}$ hrs.

9. What is the amount of $\frac{1}{7}$ of a pound, $\frac{4}{5}$ of a shilling, and $\frac{5}{12}$ of a penny? Ans. 3 s. 1 d. $1\frac{19}{21}$ qr.

10. Add 4, $3\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{8}$ and $\frac{1}{8}^2$ together. Ans. $9\frac{1}{8}$.

11. Required the sum of $\frac{3}{4}$ of 15 £., $3\frac{3}{7}$ £., $\frac{1}{5}$ of $\frac{4}{7}$ of $\frac{3}{5}$ of a pound, and $\frac{2}{3}$ of $\frac{3}{7}$ of a shilling. Ans. 7 £. 17 s. $5\frac{1}{2}$ d.

12. What is the difference between $5\frac{3}{8}$, and $\frac{3}{7}$ of $4\frac{1}{6}$? Ans. $4\frac{31}{136}$.

13. What is the difference between $\frac{5}{12}$ and $\frac{7}{13}$? Ans. $\frac{19}{156}$.

14. From $\frac{7}{8}$ of a £. take $\frac{3}{4}$ of a shilling. Ans. 16. s. 9 d.

15. Reduce $\frac{2}{4}\frac{1}{7}\frac{8}{5}\frac{0}{2}\frac{0}{0}\frac{0}{0}$ to its lowest terms. Ans. $\frac{5}{11}$.

16. Reduce $\frac{5}{11}$ of a mile to rods. Ans. $145\frac{5}{11}$.

17. What is the value of $\frac{2}{5}$ of 4s. 10d.? Ans. 1s. $11\frac{1}{5}$d.

18. What is the value of $\frac{5}{16}$ of a cwt.? Ans. 1 qr. 7 lb.

19. Reduce 17 s. 7 d. $3\frac{3}{5}$ q. to the fraction of a pound. Ans.

20. Required the product of $3\frac{3}{7}$ and $4\frac{1}{3}\frac{4}{3}$. Ans. $14\frac{1}{2}\frac{1}{3}\frac{4}{1}$.

21. What number is that, which if multiplied by $\frac{7}{9}$, will be $10\frac{8}{9}$? Ans. 14.

22. What is $\frac{4}{5}$ of $130\frac{2}{3}$? Ans. $81\frac{7}{3}$.

23. A man, having $\frac{7}{8}$ of a dollar, paid away $\frac{3}{8}$ of a dollar; how much had he left? Ans. $\$\frac{1}{2}$.

24. What is the difference between $\frac{7}{8}$ and $\frac{4}{12}$ of a mile? Ans. $\frac{13}{24}$ of a mile.

25. A merchant, having 8 cwt. of sugar, sold $2\frac{9}{10}$ cwt.; how much had he left? Ans. $5\frac{1}{10}$ cwt.

26. Add together $\frac{1}{5}$m. $\frac{4}{8}$c. $\frac{3}{16}$c. and $\frac{7}{8}$m. Ans. 20c. 9m.
27. From $13\frac{1}{3}$ take $\frac{3}{4}$ of 15. Ans. $2\frac{1}{12}$.
28. Divide $\frac{3}{7}$ by 9. Ans. $\frac{1}{21}$.
29. Reduce 2 qrs. $2\frac{2}{3}$ na. to the fraction of a yard. Ans. $\frac{4}{3}$ yds.

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## SECTION XVII.

### *Changing Vulgar to Decimal Fractions.*

1. Reduce $\frac{1}{8}$ to a decimal. Ans. .125.
2. Reduce $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{2}{3}$ to decimals. Ans. .375, .625, .666+.
3. Reduce $\frac{9}{1129}$ and $\frac{5}{1875}$ to decimals. Ans. .00797+, .00266+.
4. Reduce 1 farthing to the decimal of a shilling. Ans. .02083.
5. Reduce 13s. $5\frac{1}{2}$d. to the decimal of a pound. Ans. .6729+.
6. Reduce 7 cwt. 3 qrs. 17 lb. 10 oz. 12 dr. to the decimal of a ton. Ans. .39538+.
7. What is the value of .569 of a year? Ans. 20 dys. 16 h. 26 mi. 24 sec.
8. What is the value of .397 of a yard? Ans. 1 qr. 2.352 na.
9. Change $\frac{1}{2}$ dollar to decimals. Ans. .5.
10. How many tenths in one? Then how many tenths in $\frac{1}{2}$?
11. Change $\frac{1}{2}$ £. to decimals. Ans. .5.
12. Change $\frac{1}{4}$ to decimals. Ans. .25.
13. Change $\frac{3}{4}$ to decimals. Ans. .75.
14. Change $\frac{4}{5}$ to decimals. Ans. .4.

REM.—The student will find it convenient, in changing vulgar fractions to decimals, first to divide 1 dollar, 1 £., or 1 foot, as the case may be, into ten equal parts, and if necessary, into 100 equal parts, or 1000, and so on. In 1 there are 10 tenths; of course $\frac{1}{2}$ of 1 is equal to $\frac{5}{10}$, commonly written .5. Also in 1, there are 100 hundredths; consequently $\frac{3}{4}$ of one are equal to $\frac{75}{100}$, written .75.

15. Change $\frac{2}{3}$ of a dollar to decimals. Ans. .6666+.
16. Change 1 s. to the decimal of a £. Ans. .05.

REM.—One shilling is $\frac{1}{20}$ of a pound. Change 1 shilling to hundredths, then take $\frac{1}{20}$ of them;—$\frac{1}{20}$ of 100 hundredths is equal to 5 hundredths, written .05.

17. Change 15 s. to the decimal of a £. Ans. .75.

18. Change 8 d. to the decimal of a £. Ans. .0333+.

19. Add together $\frac{1}{4}$, $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{2}{12}$ of a foot, when changed to decimals. Ans. 1.4999+.

20. Subtract 4 inches from $\frac{1}{3}$ of a foot when reduced to decimals. Ans. .00.

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## SECTION XVIII.

### SIMPLE PROPORTION.

1. If 4 bls. of flour cost $10, what will 12 bls. cost at the same rate? Ans. $30.

2. Sold 75$\frac{3}{4}$ chaldrons of lime, at 11 s. 6 d. per chaldron; how much did it come to? Ans. 43 £. 11 s. 1$\frac{1}{2}$ d.

3. If 23$\frac{1}{3}$ yds. of riband cost $5$\frac{1}{2}$, what will 34$\frac{3}{4}$ yds. cost? Ans. $8.191$\frac{5}{70}$.

4. If 47.25 bushels of barley cost 15 £. 17 s. 5 d., what is that per bushel? Ans. 6 s. 8$\frac{7}{15}$ d.

5. At 3 s. 8 d. per bushel, how many bushels of corn may be bought for 3 £. 5 s. 7 d.? Ans. 17.886 bush.

6. If .845 of a yard of cloth cost $5.37, what is that per yard? Ans. $6.355.

7. If $11.02 will buy 9$\frac{1}{2}$ yds. of cloth, what will $195.75 buy? Ans. 168 yds. 3 qrs.

8. If a ship cost 537 £., what are $\frac{3}{8}$ of her worth? Ans. 201 £. 7 s. 6 d.

9. A person having $\frac{4}{5}$ of a vessel, sells $\frac{2}{3}$ of his share for $1080$\frac{2}{3}$; what is the whole vessel worth? Ans. $2026.25.

10. What number of men must be employed to finish in 9 days, what 15 men would perform in 30 days? Ans. 50.

11. If 100 yds. of cloth cost $66, what will 1 yd. cost? Ans. 66 cents.

12. If a piece of board be 30 inches in length, what breadth will make 1$\frac{1}{2}$ square feet?

13. Suppose 450 men are in a garrison, and their provisions are calculated to last but 5 months; how many must leave the garrison, that the same provisions may be sufficient for those who remain 9 months? Ans. 200.

14. There is a cistern, having a pipe which will empty it in 6 hours; how many pipes of the same size will empty it in 20 minutes? Ans. 18.

15. How much will 75 chaldrons 7 bushels of coal come to, at the rate of 1 £. 13 s. 6 d. per chaldron?
Ans. 125 £. 19 s. $0\frac{1}{6}$ d.

16. How many yds. of stuff, of 3 qrs. wide, will line a cloak that is $1\frac{3}{4}$ yds. long, and $3\frac{1}{2}$ yds. wide?
Ans. 8 yds. 0 qrs. $2\frac{2}{3}$ na.

17. What is the half year's rent of 547 acres of land, at 15s. 6d. the acre? Ans. 210 £. 19 s. 3 d.

18. What will be the charge of keeping 20 horses for a year, at the rate of $14\frac{1}{2}$d. per day for each horse?
Ans. 441 £. 0 s. 10 d.

19. If $3\frac{4}{7}$ cwt. of iron cost $\$14\frac{5}{7}$, what is that per cwt.?
Ans. $\$4.07\frac{1}{3}$.

20. If $\frac{7}{8}$ of a ship be worth $\frac{3}{7}$ of her cargo, the cargo being valued at 2000 £., what is the whole ship and cargo worth?
Ans. 2653 £. 1 s. $2\frac{3}{4}\frac{4}{5}$ d.

21. There is a pole, standing so that $\frac{2}{7}$ of it is in the water, $\frac{2}{5}$ as much in the mud as in the water, and $7\frac{3}{8}$ feet of it is above the water; what is the length of the pole?
Ans. $12\frac{7}{2\frac{7}{4}}$ ft.

22. When corn is $\frac{7}{8}$ of a dollar a bushel, what part of a bushel may be bought for $\frac{2}{3}$ of a dollar? Ans. $\frac{1}{3}\frac{6}{5}$ of a bu.

23. If a man can build $4\frac{5}{7}$ rods of wall in a day, how many days will it take him to build $84\frac{4}{1\frac{5}{3}}$ rods?
Ans. $19\frac{8}{1}\frac{4}{3}\frac{7}{8}\frac{7}{7}$.

24. At $25\frac{3}{2\frac{2}{2}}$ dollars a cask, how many casks of claret wine may be bought for $387\frac{5}{1\frac{5}{8}}$ dollars? Ans. $15\frac{2}{4}\frac{0}{5}\frac{2}{7}\frac{5}{7}$.

25. At $95\frac{6}{2\frac{6}{5}}$ dollars a ton, how much iron may be bought for $2956\frac{7}{8}$ dollars? Ans. $30\frac{5}{6}\frac{8}{3}\frac{4}{7}\frac{5}{6}$.

26. A man failing, can only pay 17 s. on a £.; how much can he pay on 147 £. 14 s.? Ans. 125 £. 10 s. 10 d. $3\frac{1}{5}$ q.

27. Divide the number 360 into parts which shall be to each other as 2, 3, and 4. Ans. 80, 120, 160.

28. If 745 soldiers are to be clothed, and each suit is to contain $3\frac{1}{2}$ yds. of cloth $1\frac{3}{8}$ yds. wide, and to be lined with shalloon $\frac{7}{8}$ yd. wide,—how many yds. shalloon will be required? Ans. $4097\frac{1}{2}$ yds.

29. How many yards of carpet, $2\frac{3}{4}$ ft. wide, will cover a floor which is 16 ft. long and 11 ft. wide? Ans. $21\frac{1}{2\frac{1}{1}}$ yds.

30. How many yards of cloth, 3 quarters wide, are equal to 30 yards 5 quarters wide? Ans. 50.

31. If 100 men can finish a piece of work in 24 days, how many are sufficient to do the same in 3 days? Ans. 800.

32. If 16 men can do a piece of work 12 days, in what time will 8 men do it? Ans. 24 days.

33. If a piece of land be 20 rods in length, how wide must it be to contain an acre? Ans. 8 rods.

34. How much land, at $2.50 per acre, must be given for 360 acres at $3.65 per acre? Ans. $525\frac{1}{25}$ acres.

35. If 52 yards of cloth cost 156 dollars, how much will 4 yards cost? Ans. $12.

36. If 40 men can build a house in 180 days, how long will it take 80 men to build it? Ans. 90 days.

37. If 5 pieces of linen, each containing 12 yards, cost 13 £. 10 s., what is it per yard? Ans. 4 s. 6 d.

38. A merchant bought a piece cloth for $60, at $1.50 per yard; how many yards were there in the piece? Ans. $40.

39. If 4 cords of wood be worth $24, what are 3 cords of the same wood worth? Ans. $18.

40. A man engaged to labour 8 months for $88; but being obliged to leave at the end of 5 months, how much ought he to receive? Ans. $55.

41. If the rent of a house be $40 for 3 months, what will the rent of the same house be for 2 years? Ans. $320.

42. How many tiles, 8 inches square, will lay a floor 20 feet long and 16 feet wide? Ans. 720.

43. The ship Emily left Boston and sailed east at the uniform rate 24 leagues per day; 15 days after, the sloop Esther sailed from the same port and in the same direction; how many leagues per day must she sail, to overtake the Emily in 60 days? Ans. 30.

44. A commissary has 24000 lbs. of meat for 400 soldiers; if each man eat 4 lbs. per week, how many weeks will it last? Ans. 15.

45. If I am taxed 10 dollars for 50 acres of land, how much is the tax of the town in which the land lies for 23040 acres? Ans. $4608.

46. If a man perform a journey in 15 days when the days are 12 hours long, how long ought the days to be, that he may perform the same journey in 12 days? Ans. 15 hrs.

47. If 8 men can perform a piece of work in 24 days, how many men would it require to perform twice as much in 12 days? Ans. 32.

48. If $480 gain $28.80 in one year, how much will it gain in 87 days? Ans. $6.864.

49. If 1 and 2 make 7, what will 3 and 6 make? Ans. 21.

50. If a man earn 64 dollars in 4 months, how long must he work at the same wages to pay a debt of 300 dollars?
Ans. $18\frac{3}{4}$ months.

51. If the inventory of a certain town be 358400 dollars, upon which there is assessed a tax of 850 dollars, what will a man's tax be, whose estate in that town is valued at 1792 dollars? Ans. $4.25.

52. If a man travel 5 miles per day, how far does he travel per hour? Ans. $\frac{5}{24}$ of a mile.

53. The hour and minute hand of my watch were together at 12 o' clock; at what time will they be together again?
Ans. 5 min. $27\frac{3}{11}$ sec. past 1.

54. If 60 yards of matting will cover a room which is 18 feet wide and 30 feet long, what is the width of the matting?
Ans. 1 yard.

55. A building was completed in 18 months by 80 workmen, but it having been destroyed by fire, it is required that it be rebuilt in 12 weeks; how many workmen must be employed? Ans. 480.

56. If a suit of clothes can be made from $4\frac{1}{2}$ yards of cloth, $1\frac{3}{4}$ yard wide, how many yards $\frac{7}{8}$ of a yard wide will it require for the same person? Ans. 9.

57. If a man travel 60 miles per day, how far is that per hour,—provided he travel uniformly and without interruption? Ans. $2\frac{1}{2}$ miles.

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## SECTION XIX.

### COMPOUND PROPORTION.

1. If 60 dollars' worth of provision will serve 20 men 30 days, how long would 120 dollars' worth serve 10 men? How long would it serve 15 men? How long 40 men?
Ans. 120 days. 80 days. 30 days.

2. If 1464 quarters of wheat be used by 27816 soldiers in a month, in what time will 950 soldiers consume 350 quarters? Ans. 7 months.

3. If 20 men can do a piece of work in 30 days, how many men will it take to do a piece 4 times as large in 4 days?
Ans. 600.

4. If 7 men build 36 rods of wall in 3 days, how many rods can 20 men build in 14 days? Ans. 480.

5. If 100 £. in 1 year gain 5 £. interest, what will be the the interest of 750 £. for 7 years? Ans. 262 £. 10 s.

6. If 3000 lbs. of beef serve 340 men 15 days, how many lbs. will serve 120 men 25 days? Ans. 1764 lbs. $11\frac{1}{51}$ oz.

7. How many men can complete a trench 135 yds. long in 8 days, when 16 men can dig 54 yds. in 6 days?

Ans. 30 men.

8. If 8 men spend 32 dollars in 13 weeks, what will 24 men spend in 52 weeks? Ans. $384.

9. If 7 men can reap 84 acres of wheat in 12 days, how many men can reap 100 acres in 5 days? Ans. 20.

10. A hare is 50 leaps before a hound, and takes 4 leaps while the hound takes 3. But 2 of the hound's leaps are equal to 3 of the hare's. How many leaps must the hound make to overtake the hare? Ans. 300.

11. If a family of 9 persons spend 120 dollars in 8 months; how much will serve a family of 24 persons 16 months? Ans. $640.

12. A wall to be built at the height of 27 feet, was raised to the height of 9 feet by 12 men in 6 days; how many men must be employed to finish the wall in 4 days, at the same rate of working? Ans. 36.

13. If a man travel 130 miles in 3 days, when the days are 12 hours long; in how many days of 10 hours each, may he travel 360 miles. Ans. $9\frac{2}{65}$ days.

14. If 40 men in 15 days, of 12 hours each, can build a wall 200 feet long, 12 feet high, and 5 feet thick; how many men in 12 days of 15 hours each, can build a wall 100 feet long, 10 feet high, and 6 feet thick? Ans. 20 men.

15. A man put out 450£. at interest, and at the end of 9 months received for principal and interest 473£. 12 s. 6 d; at what rate per cent did he receive interest?

Ans. 7 per cent.

16. If a regiment of soldiers, consisting of 939 men consume 351 quarters of wheat in 7 months; how many soldiers will consume 1464 quarters in 5 months, at that rate?

Ans. $5483\frac{23}{195}$.

17. If 100 dollars gain 6 dollars in 12 months, what will 400 dollars gain in 7 months? Ans. $14.

18. If 20 bushels of wheat are sufficient for a family of 8 persons 5 months, how much will be sufficient for 4 persons 12 months? Ans. 24 bushels.

19. If 30 men perform a piece of work in 20 days; how many men will accomplish another piece of work 4 times as large, in a fifth part of the time? Ans. 600.

20. What principal at 6 per cent. per annum, will gain 14£. in 7 months? Ans. 400£.

21. If the freight of 10 hogsheads of sugar each weighing 12 cwt., cost 12 dollars for 50 miles; what must be paid for the freight of 40 tierces, each weighing $3\frac{1}{2}$ cwt. 150 miles? Ans. $42.

22. If the interest of 700 dollars in half a year be 14 dollars; what will be the interest of 400 dollars for 5 years? Ans. $80.

23. If 20 acres of grass feed 12 oxen 16 days; how many days will 30 acres feed 18 oxen? Ans. 16.

24. If $9.375 be the interest gained in 15 months by 150 dollars; what sum at interest will gain 5 dollars in 12 months? Ans. $100.

25. A farmer sells 204 dollars worth of grain in 5 years, when it is sold for 60 cents per bushel; what then is it per bushel when he sells 1000 dollars worth in 18 years, provided he sells the same quantity yearly? Ans. $0.8169.+

26. If a family of 9 persons spend 450 dollars in 5 months; how much will be sufficient to maintain them 8 months, if 5 more persons are added to the family? Ans. $1120.

27. If 10 bushels of oats supply 18 horses 20 days; how many bushels will supply 60 horses 36 days. Ans. 60.

28. If 248 men in 5 days of 11 hours each, can dig a trench 230 yards long, 3 wide and 2 deep; in how many days of 9 hours long, will 24 men dig a trench 420 yards long, 5 wide, and 3 deep? Ans. $288\frac{52}{207}$.

NOTE.—A few examples given in this and some of the other sections, are repeated in part II.

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## SECTION XX.

## CONJOINED PROPORTION.

1. If 20 lbs. at Boston make 23 lbs. at Antwerp, and 155 at Antwerp make 180 at Leghorn; how many at Boston are equal to 144 at Leghorn? Ans. $107\frac{18}{23}$.

2. If 12 lbs. at Boston make 10 lbs. at Amsterdam, and 10 lbs. at Amsterdam 12 lbs. at Paris; how many pounds at Boston are equal to 80 lbs. at Paris? Ans. 80.

3. If 20 lbs. in America be equal to 19 lbs. Flemish, and 95 lbs. Flemish be equal to 125 lbs. at Bologna; how many lbs. in America are equal to 250 lbs. at Bologna? Ans. 200.

4. If 10 braces at Leghorn be equal to 5 vares at Lisbon, and 10 vares at Lisbon be equal to 20 braces at Lucca; how many braces at Lucca are equal to 200 braces at Leghorn? Ans. 200.

5. Five pounds New Jersey currency are equal to 4 pounds New England currency, and 3 pounds New England currency are equal to 4 pounds New York currency, and 8 pounds New York currency are equal to 5 pounds Canada currency, and 10 pounds Canada currency are equal to 9 pounds sterling. How many pounds sterling are equal to 100 pounds New Jersey currency? Ans. 60 pounds.

6. If 25 lbs. at Boston be equal to 22 lbs. at Nuremburg, and 88 lbs. at Nuremburg be equal to 92 lbs at Hamburg, and 46 lbs. at Hamburg be equal to 49 lbs. at Lyons; how many pounds at Boston are equal to 98 lbs. at Lyons? Ans. 100 lbs.

7. If 100 acres of meadow in Concord be equal in value to 95 acres in Dunstable, and 19 acres in Dunstable are equal to to 25 at Hanover; how many acres at Concord are equal to 50 at Hanover? Ans. 40.

8. If 30 boys will perform as much labour as 27 men; and 270 men will perform as much as 336 women; how many women will be required to do a piece of work in one day, which 150 boys will do in the same time? Ans. 168.

9. If 10 calves are worth as much as 9 colts; and 90 colts are worth as much as 112 sheep? how many sheep are equal in value to 50 calves? Ans. 56.

10. If 20 barrels of cider will pay for 10 cords of wood; and 40 cords of wood will buy 8 tons of plaster; how many tons of plaster will be equal in value to 100 barrels of cider? Ans. 100.

11. If 25 pounds of cheese are equal in value to 22 pounds of butter; and 88 pounds of butter are equal to 92 pounds of tallow; and 46 pounds of tallow are equal to 49 pounds of sugar; how many pounds of cheese ought I to exchange for 98 pounds of sugar? Ans. 100.

## SECTION XXI.

### INTEREST AND DISCOUNT.

1. What is the interest of 316 dollars for 1 year and 10 months? Ans. $34.76.

2. What is the interest of two hundred and forty dollars, and sixteen cents, for 3 years, 5 months, and 1 day? Ans. $49.672.

3. If a broker sells goods to the amount of 508£. 17 s. 10 d.; what does his allowance amount to at 1½ per cent.? Ans. 7£. 12 s. 8 d.

4. What is the interest of 93 cents for a year, at 6 per cent.? Ans. $0.055.

5. What is the interest of one hundred and twenty four dollars, and eighteen cents for 2 years and 8 months? Ans. $19.868.

6. What is the compound interest of 246 dollars for 2 years? Ans. $30.40.

7. What is the difference between the discount and interest of 100£. for 10 years at 6 per cent.? Ans. 22£. 10 s.

8. What is the interest of seven hundred and thirty-six dollars, twenty one cents and three mills, for three years, at 7 per cent. per annum? Ans. $154.604.

9. A merchant in England writes to his friend in Boston that he has purchased goods on his account to the amount of 754£. 16 s.; what does his commission amount to at 2½ per cent.? Ans. 18£. 17 s. 4¾ d.

10. What is the interest of $162,14 for 2 years and 4 months, at 7 per cent.? Ans. $26.482.

11. A note of 1500 dollars dated July 10th, 1806, payable on demand, with interest till paid, has the following payments endorsed on it, viz:

| | | |
|---|---|---|
| Received March 1st, 1807, | - - - - | $50 |
| " July 20th, 1807, | - - - - | $300 |
| " Sept. 1st, 1807, | - - - - | $400 |

How much was due January 1st, 1808, Ans. $868.

12. What will 1256 dollars amount to in 8 years at 6 per cent. per annum compound interest? Ans. $2001.863.

REMARK. As the interest of 1 dollar (sum 1st,) must be 11 cents for the time specified (1 year and 10 months) the interest of 316 dollars, must be 316 times as many cents.

13. What is the discount of 573£. 15 s. due 3 years hence, at 4½ per cent. per annum? Ans. 68£. 4 s. 10¼d.

14. What is the present worth of 874 dollars at 6 per cent. due 4 years hence? Ans. $704.838.

15. What is the interest of $327,825 for 3 years, 5 months and 27 days, at 4½ per cent. per annum?
Ans. $81.01.

16. A house is estimated at 3000 dollars; what is the amount of insurance against fire at 5½ per cent.? Ans. $165.

17. What is the interest of $1173.57 for 18 months, at 5 per cent.? Ans. $88.014.

18. What discount must be made for the present payment of 100 dollars due 90 days hence, at 6 per cent. per annum?
Ans. $1.457.

19. What is the interest of thirty-six dollars and twenty-six cents for 4 years, 11 months, and 6 days, at 5¼ per cent. per annum? Ans. $9.391.

20. What is due a commission merchant who has sold goods to the amount of 1834 dollars; his commission being 2½ per cent.? Ans. $45.85.

21. What is the interest of sixty-four dollars and eighteen cents, for one year and 10 months, at 4 per cent.?
Ans. $4.706.

22. What is the compound interest of $876,90 for 3½ years, at 6 per cent. per annum? Ans. $198.83.

23. What is now the amount of a bond which has been on interest 11 years, 9 months, 21 days; the principal being 550 dollars and the interest 5 per cent. per annum?
Ans. $874.729.

24. What is the amount of 15£. 10 s. for 9 years, at 3½ per cent. per annum, compound interest? Ans. 21 £. 2 s. 5¾ d.

25. What is the amount of $225 for 3 years, at 5 per cent. per annum, compound interest? Ans. $260.465.

26. What is the insurance of an East India ship and cargo valued at 35727£. 17 s. 6 d. at 17⅞ per cent.?
Ans. 6386£. 7 s. 1¾d.

27. A held a note against B of two hundred and forty-six dollars, and twenty-four cents, dated October 12th, 1822; March 12th, 1823, B paid 68 dollars; what was due September 24th, 1823? Ans. $190.296.

28. How much must be discounted for the immediate payment of 100 dollars, due 1 year hence, at 6 per cent. per annum? Ans. $5.66.

29. A gentleman gave his note for five hundred and seventy-three dollars and twenty-five cents, with interest at 6 per cent.; what sum will discharge the note 3 years after the date? Ans. $676.435.

30. What is the present worth of 800 dollars, payable in 10 months, discounting at the rate of 6 per cent. per annum? Ans. $761.904.

31. What is the interest of 256 dollars from June 1st to September 10th inclusive? Ans. $4.289.

32. How much was due June 1st 1807, on a note for 2000 dollars, dated June 1st, 1805, on which were endorsed the following payments?

| | | | |
|---|---|---|---|
| Received | Sept. 1st, 1805, | 96 | dollars. |
| " | Dec. 10th, 1805, | 15 | " |
| " | April 20th, 1806, | 36 | " |
| " | July 1st, 1806, | 200 | " |
| " | Jan. 10th, 1807, | 20 | " |
| " | March 25th, 1807, | 90 | " |

Ans. $1767.48.

## SECTION XXII.

### PRACTICE.*

1. What will 365 yards of calico cost, at 3 s. 10½ d. per yard? Ans. 70 £. 14 s. 4½ d.

2. What will be the cost of 6843 apples, at ¼ d. apiece? Ans. 7 £. 2 s. 6¾ d.

3. What cost 7654 cocoa-nuts, at 2 d. apiece? Ans. 63 £. 15 s. 8 d.

4. What will 9789 pounds of hops come to, at 9 d. per pound? Ans. 367 £. 1 s. 9 d.

5. What cost 5600 yards of cambric, at 17 d. per yard? Ans. 396 £. 13 s. 4 d.

6. What cost 543 barrels of cider, at 15 s. per barrel? Ans. 407 £. 5 s.

7. What will 5063 handkerchiefs cost, at 2 s. 6 d. apiece? Ans. 632 £. 17 s. 6 d.

8. What cost 942 pounds of tea, at 5 s. 2 d. per pound? Ans. 243 £. 7 s.

* This rule is given merely to exercise the learner in it, and not for its practical utility to business men in the United States.

9. What will 38 loads of hay cost, at 6 £. 10 s. per load?
Ans. 247 £.

01. What cost 25 barrels of wine, at 4 £. 3 s. 4 d. per barrel? Ans. 104 £. 3 s. 4d.

11. What cost 4678 lemons, at $\frac{3}{4}$ of a penny each?
Ans. 14 £. 12 s. 4½ d.

12. What is the value of 840¼ pounds of sugar, at 10 d. per pound? Ans. 35 £. 0 s. 7½ d.

13. What will 1250¼ cords of wood come to, at 18 s. per cord? Ans. 1125 £. 4 s. 6 d.

14. What cost 658 pounds of rice, at 6 d. per pound?
Ans. 16 £. 9 s.

15. What will 871 cwt. of iron cost, at 3 £. per cwt.?
Ans. 2613 £.

16. What cost 385 pounds of hyson tea, at 8 s. per pound?
Ans. 154 £.

17. What must I pay for 896 skeins of silk, at 3 d. per skein? Ans. 11 £. 4 s.

18. What cost 584 beaver hats, at 17 s. 6 d. each?
Ans. 511 £.

19. What will 2157 cords of bark cost, at 3 £. 15 s. 2¼ d. per cord? Ans. 8108 £ 19 s. 5¼ d.

20. What cost 458 bushels of rye, at 5 s. 7½ d. per bushel?
Ans 128 £. 16 s. 3 d.

21. How much will 345 yards of broadcloth cost, at 2 £. 6 s. 8d. per yard? Ans. 805 £.

22. Bought 5 quires of paper, at 1 £. 13 s. 4 d. per ream; what must I pay? Ans. 8 s. 4 d.

23. What cost 3 quarters 2 nails of cloth, at 4 £. 10 s. per yard? Ans. 3 £. 18 s. 9 d.

24. What will 32 gallons 3 quarts and 1 pint of wine cost, at 47 £. 5 s. per hogshead? Ans. 24 £. 13 s. 1 d.

25. What cost 100 bushels 4 quarts of wheat, at 6 s. 8 d. per bushel? Ans. 33 £. 7 s. 6 d.

26. What cost 372 oranges, at 1¾ d. each?
Ans. 2 £. 14 s. 3 d.

27. What will a salary of 200 £. per annum amount to in 5 years 3 months and 25 days? Ans. 1063 £. 17 s. 9 d.

28. What will 24 sheep come to, at 18 s. per head?
Ans. 21 £. 12 s.

29. What cost 752½ yards of calico, at 1 s. 10 d. per yard?
Ans. 68 £. 19 s. 7 d.

30. What will 67 yards and 2 quarters of cloth cost, at 6 s. 1 d. per yard? Ans. 20 £. 10 s. 7½ d.

31. A man bought a farm, containing 598 acres and 2 roods, at 44 £. 18 s. 6 d. per acre; what did it cost him?

Ans. 26887 £. 12 s.

## SECTION XXIII.

### FELLOWSHIP, OR COMPANY BUSINESS.

1. Three merchants traded in company; A put in 600 dollars, B 400 dollars, and C 250 dollars; they gained 500 dollars; what is the share of each partner, in proportion to his stock in trade?

Ans. { $240 A's gain. 160 B's gain. 100 C's gain.

2. Two merchants joined their stocks and bought a quantity of goods for 820 dollars; the first paid 350 dollars, and the second 470 dollars; they sold their goods so as to clear 250 dollars; what ought each to share?

Ans. { The first $106.707+. The second 143.292.

3. Two merchants traded in company; A put in 215 dollars for 6 months, and B 390 dollars for 9 months, but by misfortune they lost 200 dollars; how must they share the loss?

Ans. { A's share $ 53.75. B's do. 146.25.

4. Three men entered into copartnership with equal concern in the stock, they gained 1000 dollars; what is each man's share?

Ans. $333.333⅓.

5. Two merchants entered into trade; P put in stock to the amount of 2500 dollars for 3 months; E put in 1800 dollars for 5 months; they gained 875 dollars; what was each man's share of the gain?

Ans. { P's share $397.727. E's share 477.272.

6. G and W commenced business in company; G put in 9740 dollars, and W put in 8790 dollars; they gained 1874 dollars; what must each share of the gain?

Ans. { G's share $985.038. W's share 888.961.

7. Three partners, L, M, and N, shipped 108 mules for the West Indies, of which L owned 48, M 36, and N 24. In consequence of a storm 45 of them were thrown overboard; how much of the loss must each owner sustain?

Ans. { L 20. M 15. N 10.

8. Three ship carpenters built a vessel, which cost them 15500 dollars; S advanced 4000 dollars, H 7000 dollars, and B 4500 dollars. The ship sold for only 14850 dollars; what did each lose?

Ans. { S $167.741+. H 293.548+. B 188.709+.

9. A, B and C are to share 600 £.; A is to have a certain sum, B as much again as A, and C three times as much as B; what is each man's share?

Ans. { A's 66⅔ £. B's 133⅓ £. C's 400 £.

10. A gentleman bequeathed his estate to his four sons in the following manner, viz. to his first son, 5000 dollars, to his second, 4500 dollars, to his third, 4500 dollars, and to his fourth 4000 dollars. But his whole estate, after paying debts, amounted to but 12000 dollars; what must each son receive?

Ans. { 1st $3333.333⅓. 2d 3000. 3d 3000. 4th 2666.666⅔.

11. J and M commenced business in company; J put in stock to the amount of 240 £., and M the same; at the end of 6 months M took out his stock, but J continued his in trade 6 months longer, at which time they settled and found their gain to be 280 £.; what should each receive?

Ans. { J 186 £. 13 s. 4 d. M 93 £. 6 s. 8 d.

12. Three neighbours hired a pasture for 60 £. 10 s. A put in 5 sheep for 4½ months, B put in 8 for 5 months, and C put in 9 for 6½ months; how much of the rent must each pay?

Ans. { A 11 £. 5 s. B 20 £. C 29 £. 5 s.

13. Two merchants entered into partnership for 18 months. A at first put in 500 dollars, and at the end of 8 months put in 100 dollars more; B at first put in 800 dollars, and at the end of 4 months took out 200 dollars. At the expiration of the time they find they have gained 700 dollars; what is each man's share of the gain?

Ans. { A's $324.074. B's $375.925.

14. R, L and N were concerned in an adventure; R advanced 4740 dollars for 5 months; L 2700 dollars for 6 months; N 4100 for 4 months; by failure they were involved

to the amount of 1800 dollars; what must each sustain of the loss?

Ans. { R $757.726. L 517.939. N 524.333.

15. Two farmers hold a pasture in common, for which they are to pay 30 £. A put in 20 oxen for 10 weeks, and B 25 oxen for 12 weeks; what ought each to pay for the rent?

Ans. { A 12 £. B 18 £.

16. On the first of January, A began to trade with 380 dollars; and on the first of May following, he took in B with 270 dollars; on the first of August following, he took in C with 400 dollars; at the end of the year, they found there was gained 436 dollars; what is each man's share of the gain?

Ans. { A's $228. B's 108. C's 100.

17. R, S and T put their money into a joint stock; R put in 160 pounds; S and T together put in 680 pounds; they gained 504 pounds, of which T took 168 pounds; what did R and S gain, and S and T put in respectively?

Ans. { R gained 96£. and S 240£. S put in 400£. and T 280£.

18. Three merchants freight a ship with 340 tuns of wine, of which the first owned 110 tuns, the second 97 tuns, and the third the rest. In a storm 85 tuns were thrown overboard; how much must each sustain of the loss?

Ans. { The first 27½. The second 24¼. The third 33¼.

19. A copartnership was formed by A, B and C. A put in 140 dollars, B 250 dollars, and C put in 120 yards of cloth, at cash price; they gained 230 dollars of which C took 100 dollars for his share of the gain; how did C value his cloth per yard in common stock, and what was A's and B's part of the gain?

Ans. { C put in the cloth at $2½ per yd. A gained $46.666. B gained $83.333.

20. Three persons entered into trade for 16 months; A at first put in $7400, but at the end of 4 months took out $2000, at the end of 12 months he put in $3000, and at the end of 14 months took out $850; B at first put in $5900, and at the end of 3 months he put in $4300 more; but

at the end of 9 months took out $4000, and at the end of 12 months put in $1500, but withdrew $2000 at the end of 14 months; C at first put in $12000, at the end of 6 months took out $5000, and at the end of 9 months put in $3200, but at the end of 12 months took out $4000; they gained $8000; what must each have of the gain?

Ans. A $2219,395$\frac{3}{7}\frac{7}{7}\frac{9}{4}$.
B $2634,870$\frac{6}{3}\frac{2}{7}\frac{0}{7}\frac{}{4}$.
C $3145,733$\frac{3}{3}\frac{6}{7}\frac{5}{7}\frac{8}{4}$.

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## SECTION XXIV.

### LOSS AND GAIN.

1. If I buy serge at 90 c. per yard and sell it again at $1 02 c. per yard; what do I gain per cent. in laying out $100?
Ans. $13$\frac{1}{3}$ per cent.

2. A draper bought 60 yards of cloth at $4.50 c. per yard, and 38 yards of cloth at $2.50 c. per yard; and sold them one with another, at $4 25 c. per yard: did he gain or lose and what per cent.? Ans. $14.11 c. gain per cent.

3. If I buy a cwt. of sugar for 9 £. 6 s. and 8 d., and sell it again at 1 s. 10 d. per pound; do I gain or lose, and what per cent.? Ans. 10 per cent gain.

4. Bought sugar at 6$\frac{1}{2}$ d. per pound, and sold it at 2 £. 3 s. 9 d. per cwt.; what was the gain or loss per cent.?
Lost 0 £. 16 s. 11 d.

5. If I buy gloves at $1 25 c. per pair; how long credit must I have to gain $13 per cent., when I sell them at $1 36 c. per pair? Ans. 8 mo. 12 days.

6. If a barrel of powder cost 4 £. how must it be sold to lose 10£ per cent.? Ans. 3 £. 12 s.

7. If a bag of cotton weighing 8 cwt. 0 qrs. 20 lbs. cost $45 55 c.; how must it be sold per cwt. to lose $8 per cent.
Ans. $5.12 c. 3 mi.

8. Bought 50 gallons of wine, at 75 c. per gallon, but by accident 10 gallons leaked out; at what rate must I sell the remainder per gallon to gain upon the prime cost of the whole, at the rate of 10 per cent.? Ans. $1.03 c. 1$\frac{1}{4}$ m.

9. If 19$\frac{1}{2}$ cwt. of sugar be sold at $14.50 c. per cwt. and I gain $15 per cent.; what did it cost per cwt.
Ans. $12.60 c. 8 m.

10. If cloth, sold at 4 s. per yard, be 10 £. per cent. profit, what shall I gain or lose per cent. if sold at 3 s. 6 d. per yard? Ans. $3\frac{3}{4}$£ loss.

11. A gentleman bought 10 tons of steel for 200 pounds, the freight and duties came to 25 pounds, and his own charges to 8 pounds 6 shillings, 8 pence; how must he sell it per pound to gain 20 pounds per cent. by it?

Ans. 1£. 3 d. per pound.

12. If I sell a gallon of brandy for $1.50 c. and by the sale lose 12 per cent.; what shall I gain or lose per cent. if I sell 4 gallons of the same brandy for $6.75 c.?

Ans. 1 per cent. loss.

## SECTION XXV.

## INVOLUTION AND EVOLUTION.

Involution is the raising of Powers from any given number—which is called a root. A Power is a quantity produced by multiplying any number into itself a certain number of times. Thus, 2=2, is the root, or 1st power of 2.

3+3=9, is the 2nd power, or square of 3.

4+4+4=64 is the 3rd power, or cube of 4.

Evolution, is the opposite of Involution, or the extracting the root of any given power. The Root of any power is the number which being multiplied into itself a certain number of times will produce that power. Thus,

2 is the square root of 4.
2 is the cube root of 8.
2 is the fourth root of 16.

1. What is the second power of 45? Ans. 2025.
2. What is the square of $\frac{2}{3}$? Ans. $\frac{4}{9}$.
3. Involve 8 to the 3rd power? Ans. 512.
4. What is the fourth power of $\frac{1}{2}$. Ans. $\frac{1}{16}$.
5. What is the second power of 351? Ans. 123201.
6. What is the third power of $\frac{19}{18}$. Ans. $\frac{1000}{6359}$.

## SECTION XXVI.

## SQUARE ROOT.

1. What is the square root of 2035? Ans. 45.
2. What is the square root of 16? Ans. 4.

3. What is the square root of $\frac{3}{5}\frac{7}{0}\frac{5}{0}$? Ans. $\frac{5}{12}$.
4. What is the square root of $72\frac{1}{4}$? Ans. $8\frac{1}{2}$.
5. Extract the second root of 302500. Ans. 550.
6. What is the square root of 550? Ans. 23,4520788.
7. What is the square root of $17\frac{3}{8}$? Ans. 4,168333.
8. Extract the second root of $\frac{5}{12}$? Ans. 0.645497.
9. What is the square root of 000729? Ans. 027.
10. Extract the square root of 902500. Ans. 950.
11. A certain square pavement contains 889249 square stones, all of the same size; how many are contained in one of its sides? Ans. 943.
12. Suppose an army of 20449 soldiers, are to be drawn up in an exactly square column; how many men will be required to occupy one side of the square? Ans. 143.
13. The wall of a town which is besieged is 18 feet high, and is surrounded by a ditch 28 feet in width; what must be the length of the scaling ladders, in order to have them reach from the outside of the ditch to the top of the wall? Ans. 33.2+.

## CUBE ROOT.

1. What is the cube root of 64? Ans. 4.
2. What is the third root of 2803221? Ans. 141.
3. Extract the cube root of 262144. Ans. 64.
4. What is the cube root of 224755712? Ans. 608.
5. What is the cube root of $573\frac{9}{27}$? Ans. $8\frac{1}{3}$.
6. Extract the third root of $1\frac{1}{2}$? Ans. 1.1447.
7. What is the cube root of .0001357? Ans. .05138+.
8. What is the cube root of $13\frac{2}{31}$? Ans. 2.3908.
9. There is a stone which contains 68921 cubic inches; what is the length of one of its sides? Ans. 41 in.
10. Suppose a stone of a cubic form contains 474552 solid inches; what is the superficial contents of one of its sides? Ans. 6084 inches.
11. The diameter of a bushel measure being $18\frac{1}{2}$ inches, and the height 8 inches; what will be the length of the side of a cubic box which shall contain the same quantity? Ans. 12.908.
12. What is the cube root of 200? Ans. 5.848035.

## VARIOUS ROOTS.

1. What is the 3d root of 2? Ans. 1.259921.
2. What is the 4th root of 2? Ans. 1.189207.

3. What is the 4th root of 97.41? Ans. 3.1415999.
4. What is the 5th root of 2? Ans. 1.148699.
5. Extract the 6th root of 21035.8. Ans. 5.254037.
6. What is the 6th root of 2? Ans. 1.122462.
7. What is the 7th root of 21035.8? Ans. 4.145392.
8. What is the eighth root of 21035.8? Ans. 3.470323.
9. What is the 9th root of 2? Ans. 1.0580059.

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## SECTION XXVII.

### ARITHMETICAL PROGRESSION.

1. The first term of a series is 1, the last term 23, and the number of terms 14. Ascertain the sum of the series.
Ans. 210.

2. If the first term is 2, the last term 51, and the number of terms 18, what is the sum of the series? Ans. 477.

3. One hundred apples are a yard apart in a straight line; the first is a yard from the basket, into which they are to be put one at a time. How far must you travel to pick them up and carry them one by one to the basket?
Ans. 5 mi. 5 fur. 36 r. 2 yds.

4. How many strokes will a clock strike in 24 hours, if it is made so as to strike the number of hours from 1 to 24?
Ans. 300.

5. The first term is 3, the last term 58, and the number of terms 12. What is the common difference? What is the sum of the series?
1. Ans. 5.
2. Ans. 366.

6. If the extremes are 2 and 53, and the common difference 3, what is the number of terms? Ans. 18.

### GEOMETRICAL PROGRESSION.

1. If the first term of a series in geometrical progression is 1, the last term 2187, and the ratio 3, what is the sum of the series? Ans. 3280.

2. 1 and 65536 are extremes; the ratio is 4; what is the sum of the series? Ans. 87381.

3. If I buy a horse and pay 1 cent for the first nail in his shoes, 2 for the second, 4 for the third, &c., and there are 32 nails in his shoes, how much will he cost?
Ans. $9265100944259.20.

4. What is the 12th term of a series, whose first term is 3, and the ratio the same? Ans. 531441.

5. A agreed to work for B 40 years, on condition of receiving one kernel of corn for the first year, ten for the second, one hundred for the third, and so on. If one thousand kernels make a pint, and the corn was sold for half a dollar a bushel, how much did his wages amount to?

Ans. $8,680,555,555,555,555,555,555,555,555,555,555,555,555.

6. A man rode 252 miles. The first day he went 4, the next 8, and so on. How far did he ride the last day?

Ans. 128 miles.

7. A owes B 252 dollars. The first payment is to be 4 dollars, the last 128. In what ratio will the payments exceed each other? Ans. 2 (i. e. double.)

(For other sums in Progression, see Miscellaneous Examples.)

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## SECTION XXVIII.

### SINGLE POSITION.

1. A and B found a bag of money; A said the half, third and fourth of it made 130 £., and if B could tell how much there was, he should have all, otherwise none; how much was in the bag? Ans. 120 £.

2. A lent B a sum of money, to be paid at four payments; when three were made, and the fourth demanded, B would pay no more, except A would tell what was already paid; A said the first payment was a fourth, the second a fifth, and the third a sixth of the sum first lent, and all made 74 £.; what was the sum lent? Ans. 120 £.

3. A person after spending $\frac{1}{3}$ and $\frac{1}{4}$ of his money, has yet remaining 60 dollars; what had he at first? Ans. $144.

4. A gentleman had a certain number of dollars in his purse; the sum of the third, fourth, and sixth part of them made 54; how many were in the purse? Ans. 72.

5. Seven eighths of a certain number exceeds four fifths by 6; what is that number? Ans. 80.

6. A gentleman bought a chaise, horse and harness, for 500 dollars; the horse cost $\frac{1}{4}$ more than the harness, and the chaise $\frac{1}{3}$ more than the horse; what was the price of each?

Ans. { Harness $127.669$\frac{27}{47}$.
Horse 159.574$\frac{22}{47}$.
Chaise 212.765$\frac{15}{47}$.

## DOUBLE POSITION.

1. A and B laid out equal sums of money in trade; A gained a sum equal to $\frac{1}{4}$ of his stock, and B lost 225 dollars, when A's money was double that of B; what did each expend? Ans. $600.

2. What number is that, which, being increased by its half, its fourth and 5 more, will be doubled? Ans. 20.

3. A man dying, bequeathed 100 dollars to his three sons, A, B and C, in this manner: A was to have a certain part, B twice as much as A, wanting 8 dollars, and C thrice as much as A, wanting 15 dollars; how must it be divided?
Ans. A = $20.50; B = 33; C = 46.50.

4. Three persons talking of their ages—said A, I am 20 years old; said B, I am as old as A and half as old as C; said C, I am as old as you both; I demand their several ages.
Ans. A = 20 years; B, 60; C, 80.

5. A and B have the same income; A saves $\frac{1}{8}$ of his; but B, by spending 30 £. per annum more than A, at the end of 8 years finds himself 40 £. in debt; what is their income, and what does each spend per annum?
Ans. { Their income is 200 £. per ann.
{ A spends 175 £., and B 205 £. per ann.

6. Two men, A and B, lay out equal sums of money in trade; A gains 126 £. and B loses 87 £., after which A's money is double to B's; what did each lay out?
Ans. 300 £.

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## SECTION XXIX.

## ALLIGATION MEDIAL.

1. A farmer mixed 15 bushels of rye, at 64 cts. a bushel, 18 bushels of corn, at 55 cts. a bushel, and 21 bushels of oats, at 28 cts. a bushel; what is the worth of a bushel of this mixture? Ans. 47 cts.

2. A grocer mixed 2 cwt. of sugar at 56 s. per cwt., 1 cwt. at 43 s. per cwt., and 2 cwt. at 50 s. per cwt. together; I demand the price of 3 cwt. of this mixture?
Ans. 7 £. 13 s.

3. Suppose 5 lbs. of gold of 22 carats fine—2 lbs. of 21 carats fine—and 1 lb. of alloy, be melted together; what is the fineness of the compound? Ans. 19 carats fine.

4. A goldsmith melted together 5 lbs. of silver bullion, of 8 oz. fine—10 lbs. of 7 oz. fine—and 15 lbs. of 6 oz. fine; what is the quality of the mixture?

Ans. 6 oz. 13 pwt. 8 gr. fine.

5. A wine merchant mixes 15 gallons of wine at 4 s. 2 d. per gallon, with 24 galls. at 6 s. 8 d., and 20 galls. at 6 s. 8 d.; what is a gallon of the composition worth?

Ans. 5 s. 10 d. $2\frac{43}{50}$ qrs.

6. A grocer has several sorts of sugar, viz. one sort at 8 dols. per cwt.—another sort at 9 dols. per cwt.—a third sort at 10 dols. per cwt.—and a fourth sort at 12 dols per cwt.—and he would mix an equal quantity of each sort together; what is the price of $3\frac{1}{2}$ cwt. of this mixture?

Ans. $34.125.

## ALLIGATION ALTERNATE.

1. A grocer has two sorts of tea, viz. at 9 s. and at 15 s. per lb.; how must he mix them so as to afford the composition for 12 s. per lb.? Ans. An equal quantity of each sort.

2. A goldsmith would mix gold of 17 carats fine, with some of 19, 21, and 24 carats fine, so that the compound may be 22 carats fine; what quantity of each must he take?

Ans. 2 of each of the first three sorts, and 9 of the last.

3. How much water must be mixed with 100 gallons of brandy, worth 7 s. 6 d. per gallon, to reduce it to 6 s. 3 d. per gallon? Ans. 20 galls.

4. How much corn at 2 s. 6 d.—3 s. 8 d.—4 s.—and 4 s. 8 d. per bushel, must be mixed together, that the compound may be worth 3 s. 10 d. per bushel? Ans. 2 at 2 s. 6 d.; 2 at 3 s. 8 d.; 3 at 4 s.; and 3 at 4 s. 8 d.

5. How much sugar at 4 d.—at 6 d.—and at 11 d. per lb. must be mixed together, so that the composition formed by them may be worth 7 d. per lb.? Ans. 1 lb. or 1 stone, or 1 cwt. or any other equal quantity of each sort.

6. It is required to mix brandy at 12 s.—wine at 10 s.—cider at 1 s.—and water at 0 per gallon together, so that the mixture may be worth 8 s. per gallon. Ans. 8 gallons of brandy, 7 of wine, 2 of cider, and 4 of water.

## SECTION XXX.

### EXAMPLES INVOLVING PHILOSOPHICAL PRINCIPLES.

*Specific Gravity.*

1. A mineral weighs 960 grains in air, and 739 grains in water: what is its specific gravity? Ans. 4.343.

2. A Greenland whale sometimes has a surface of 3600 square feet: what pressure would it bear at the depth of 800 fathoms? Ans. 1080,000,000 lbs.

3. A stone weighs 15 lbs. in the air and 12 lbs. in water: what is its specific gravity? Ans. 5.

*Parallelograms.*

4. What is the area of a rectangular board, whose length is $12\frac{1}{2}$ feet, and breadth 9 inches? Ans. $9\frac{3}{8}$ feet.

5. What is the area of a piece of land, whose length is 6.20 chains, and perpendicular height 5.45? Ans. 3 A. 1 rood 20 rods.

6. What is the area of a parallelogram, the length being 12.25, and height 8.5? Ans. 104.125.

*Triangles.*

7. What is the area of a triangle, whose base is 49 feet, and height $25\frac{1}{4}$ feet? Ans. 68.7361 ft.

8. What is the area of a triangle, whose base is 150 yds. and perpendicular 60 ft.? Ans. 750 yds.

9. What is the area of a triangular field, whose length is $20\frac{1}{2}$ rods, and width $16\frac{1}{2}$ rods. Ans. $169\frac{1}{8}$ rods.

*Circles.*

10. What is the area of a circle, whose circumference is 22 ft. and diameter 7 ft.? Ans. $38\frac{1}{2}$ ft.

11. To find the area of a circle, whose diameter is 18, and its circumference 31.416. Ans. 78.54.

12. What is the area of a circular pond of water, whose circumference is 130 rods, and its diameter 43.2 rods? Ans. 2808 rods.

*Lever.*

13. A body weighs 10 lbs. 9 oz. at one end of a false balance, and $12\frac{1}{4}$ lbs. at the other: what is its real weight? Ans. 11 lbs. 6 oz.

14. If a man, weighing 160 lbs., rest on the end of a lever 10 ft. long, what weight will he balance on the other end, supposing the prop is one foot from the weight?

Ans. 1440 lbs.

15. A and B are of the same height, and sustain upon their shoulders a weight of 150 lbs. placed on a pole $8\frac{1}{2}$ feet long; the weight is placed $3\frac{1}{2}$ feet from A; what is the weight sustained by each person?

Ans. A sustains $88\frac{4}{17}$ lbs.; B $61\frac{13}{17}$ lbs.

*Axle.*

16. Suppose the diameter of the wheel 60 inches, and that of the axle 6 inches; what weight at the axle will balance 1 lb. at the wheel? Ans. 10 lbs.

17. Suppose the diameter of the wheel to be 60 inches; what must be the diameter of the axle so as that 1 lb. on the wheel may balance 10 lbs. on the axle? Ans. 6 in.

18. A power of 5 lbs. keeps in equilibrio a weight of 150 lbs. by means of a wheel whose diameter is 10 feet; what is the diameter of the axle? Ans. 4 in.

*Screw.*

19. A lever 6 feet long is fixed at right angles in a screw, whose threads are one inch apart, so that the lever turns just once round in raising or depressing the screw one inch. If then this lever be urged by a force of 50 lbs., with what force will the screw press? Ans. 2219.448.

20. Suppose the lever to be 30 inches, the threads of the screw 1 inch apart, and the power 11.88 lbs.—required the weight to be raised. Ans. 2240 lbs. nearly.

21. Suppose the weight is 2240 lbs., the power 11.88 lbs., and the lever 30 inches: required the distance between the threads? Ans. 1 inch.

*Cube.*

22. Required the solid content of a cube, whose side is 24 inches. Ans. 13824.

23. How many cubic feet are in a block of marble, its length being 3 feet 2 inches, breadth 2 feet 6 inches, and thickness 2 feet 6 inches? Ans. $21\frac{1}{3}$.

24. How many gallons of water will a cistern contain, whose dimensions are the same as in the last example, when 282 cubic inches are contained in one gallon?

Ans. $129\frac{17}{141}$.

*Cylinder.*

25. What is the content of a cylinder, whose length is 20 feet, and circumference 5 feet 6 inches? Ans. 48.1459 ft.

26. What is the content of a cylinder, whose length is 15 feet, and circumference 4 feet?

27. Required the content of a cylinder, whose length is 5 feet, and circumference 6 inches.

*Pyramid and Cone.*

28. What is the solidity of a square pyramid, each side of its base being 30 feet, and its perpendicular height 25 feet?
Ans. 7500.

29. Required the content of a triangular pyramid, whose perpendicular height is 30, and each side of the base 3.
Ans. 38.97117.

30. What is the content of a pentagonal pyramid, its height being 12 feet, and each side of its base 2 feet?
Ans. 27.5276.

31. Required the content of a cone, its height being $10\frac{1}{2}$ feet and the circumference of its base 9 feet.
Ans. 22.56093.

*Sphere.*

32. What is the content of a sphere whose axis is 12?
Ans. 904.7808.

33. Find the solid content of the earth, supposing its diameter to be 7918.7, and consequently its circumference 24877.4 miles. Ans. 260002677535.

# PART II.

## SECTION I.

### MISCELLANEOUS EXAMPLES.

1. What number is that which being multiplied by 21, the product will be 1365? Ans. 65.

2. The remainder of a division is 325, the quotient 467, and the divisor is 43 more than the sum of both; what is the dividend? Ans. 390270.

3. How many minutes from the creation to the commencement of the christian era, it being 4004 years? Ans. 2104840032.

4. What is the difference between 126 d. 12 h. 40 m. 02 sec. and 86 d. 18 h. 48 m. 58 sec.? Ans. 39 d. 17 h. 51 m. 4 sec.

5. Reduce $\frac{1}{2}$ of a farthing to the fraction of a pound. Ans. $\frac{1}{1920}$.

6. Reduce 7 s. 6 d. to the fraction of a pound. Ans. $\frac{37}{80}$.

7. What is the interest of 750 dollars for 2 years and 6 months, at 6 per cent? Ans. 112.50.

8. What is the compound interest of 12 £. 17 s. 6 d. for 6 years, at 6 per cent?

9. If 8 men can do a piece of work in 24 days, in what time will 16 men do it? Ans. 12 days.

10. If a man's income be two dollars per day, what is it per annum? Ans. 730 dollars.

11. Reduce 24 pounds to farthings. Ans. 23040 qrs.

12. What number is that to which if $\frac{2}{7}$ of $\frac{5}{8}$ be added the sum will be 1? Ans. $\frac{53}{63}$.

QUESTIONS.—What are the conditions of sum 3d? What is the first thing to be done in obtaining a true answer? What is the second thing? How do you reason on the 9th sum?—Ans. If 8 men can accomplish the work in 24 days, it must take one man 8 times 24 days to perform it = 192 days' work. It will take 16 men one sixteenth part as long = 12 days.

13. If 476 be the square root of a number, what will be the quotient of that number divided by half its root?
Ans. 952.

14. From $\frac{5}{7}$ of a league take $\frac{5}{8}$ of a mile.
Ans. 2 mi. 5 rods, $1\frac{1}{2}\frac{3}{2}$ feet.

15. There are two numbers the greater of which is 30 times 80, and their difference is 12 times 25; what is their sum?
Ans. 4500.

16. What number is that which multiplied by the 47th part of 23500 will produce 400000?
Ans. 800.

17. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of 4 hours, to the fraction of a day.
Ans. $\frac{1}{12}$.

18. Suppose a mast is erected so that $\frac{1}{5}$ of its length is in the ground, 12 feet of it in the water, and $\frac{2}{3}$ of its length above the water; what is the whole length of the mast?
Ans. 216 feet.

19. If 20 bushels of wheat at $1.35 a bushel be mixed with 10 bushels of rye at 90 cents a bushel; what will a bushel of the mixture be worth?
Ans. 1.20.

20. What is the sum of $\frac{3}{8}$ of a ton, and $\frac{1}{16}$ of a hundred weight?
Ans. 8 cwt. 1 qr. 21 lbs.

21. What will 9 cwt. of cheese cost at 1 £. 11 s. 5 d. per cwt.?
Ans. 14 £. 2 s. 9 d.

22. If 30 dollars be divided equally among 27 men, how much will each man receive?
Ans. 1.111.

23. What is the value of .857 of a day?
Ans. 20 h. 34 m. 4 sec.

24. What is the difference between 400 years and 98 years, 3 mo. 8 h. 10 sec.?
Ans. 301 yrs. 8 mo. 3 wks. 6 ds. 15 h. 59 m. 50 sec.

25. A man being asked his age answered that if its $\frac{1}{2}$ and $\frac{1}{3}$ were added to it, the sum would be 77; what was his age?
Ans. 42 years.

26. How much in length, that is $4\frac{1}{2}$ inches broad, will make a square foot?
Ans. 32 inches.

27. What will 7211 yards of cloth cost at 1 s. 3 d. per yard?
Ans. 450 £. 13 s. 9 d.

28. What will $218\frac{1}{2}$ bushels of beans come to at 12 s. 6 d. per bushel?
Ans. 136 £. 11 s. 3 d.

29. What is the value of 136 £. 11 s. 3 d. in Federal money?
Ans. 455.21.

QUESTION.—What is the first thing to be done in obtaining an answer to sum 25th?

30. If iron cost $6.50 per cwt.; what is it per pound?
Ans. 0.058.

31. If $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of a ship be worth $\frac{1}{3}$ of $\frac{5}{7}$ of $1\frac{1}{13}$ of the cargo valued at 40000 dollars; what is the value of the ship and cargo? Ans. 50744.81.

32. Reduce $\frac{4}{5}$ to a decimal fraction. Ans. .8.

33. A and B gained 1260 dollars, of which A is to have ten per cent more than B; what is the share of each?
Ans. { A's 660 dollars. B's 600 do.

34. A general has an army of 103041 men, and wishes to form them into a square; how many must he place in rank and file? Ans. 321.

35. What is the quotient of 8 divided by 12? Ans. $\frac{2}{3}$.

36. What will a board measure, which is 18 feet long and 1 foot 2 inches wide? Ans. 21 feet.

37. Two men depart from the same place, one goes north, the other south. One travels at the rate of 25 miles per day—the other at the rate of 32 miles per day; how far are they from each other 5 days after their departure?
Ans. 285 miles.

38. What part of 25 is $\frac{5}{8}$ of one? Ans. $\frac{1}{40}$.

39. A can do a piece of work in 10 days, and B in 13; in what time will both together finish it? Ans. $5\frac{5}{23}$ days.

40. What is the square root of 106929? Ans. 327.

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## SECTION II.

1. A person having spent 10 dollars more than $\frac{1}{3}$ of his yearly income, had 15 dollars more than $\frac{1}{2}$ of it left; what was his income? Ans. 150 dollars.

2. Let $5205\frac{1}{5}$ be divided by $\frac{4}{5}$ of 91. Ans. $71\frac{1}{2}$.

3. What is the least number which can be divided by the nine digits separately without a remainder? Ans. 2520.

4. What is the square root of 196? Ans. 14.

5. A young man received 210 dollars, which was equal to $\frac{2}{3}$ of his elder brother's portion, and three times the elder

QUESTIONS.—How do you reason on the 39th sum?
How will you prove that 327 is the true answer to the 40th sum?

brother's portion was equal to $\frac{1}{2}$ the father's estate; what was the estate. Ans. 1890 dolls.

6. The population in the several New England States in 1830, was as follows: Maine 399462; New Hampshire 269-533; Vermont 280679; Massachusetts 610014; Rhode Island 97210; Connecticut 297711; what was the whole population of New England at that time?

Ans. 1954609.

7. What is the cube root of 389017? Ans. 73.

8. What is the cube root of $\frac{4}{5}$? Ans. .703.

9. Divide 2345678964 by 6. Ans. 390946494.

10. What cost 846 yards of broadcloth at 18 s. 6 d. per yard? Ans. 782 £. 11 s.

11. If a gallon of wine cost 6 s. 8 d. and is sold for 7 s. 2 d.; what is gained per cent? Ans. $7\frac{1}{2}$ per cent.

12. Reduce 3 roods and 24 rods to the decimal of an acre? Ans. .9.

13. What decimal is equivalent to $\frac{3}{8}$? Ans. .375.

14. Reduce $3\frac{1}{3}$ bushels to the decimal of a quarter?

Ans. .416.

15. What is the interest of 325.41 dollars for 3 years and 4 months at 5 per cent.? Ans. 54.235.

16. A note for 730 dollars and interest at 6 per cent. was paid 5 years, 7 months, 12 days, after date; what was the amount at the time of payment? Ans. 975.99.

17. From 100046 take 10009. Ans. 90037.

18. What is the square root of 11.99998881?

Ans. 3.4641.

19. What number multiplied into itself will produce $42\frac{1}{4}$.

Ans. $6\frac{1}{2}$.

20. What number multiplied into itself will produce 123-201? Ans. 351.

21. If 37 £. 16 s. be divided equally among 24 men, how much will each man receive? Ans. 1 £. 11 s. 6 d.

22. How many tons are there in 6881280 drachms?

Ans. 12.

23. In $24\frac{1}{2}$ bushels, how many quarts? Ans. 784.

24. What will 25 bushels of corn cost at 92 cents per bushel? Ans. 23 dolls.

Questions.—What is the first step in solving the 8th example? After having reduced the fraction to a decimal, what is next to be done?

25. A person having $\frac{3}{5}$ of a vessel, sold $\frac{3}{4}$ of his share for 570 dollars; what was the whole vessel worth at this rate.
Ans. 1266.666.

26. A person having spent $\frac{1}{2}$ and $\frac{1}{3}$ of his money, had $180 left; what had he at first? Ans. 1080 dolls.

27. What number is that which being increased by its $\frac{1}{2}$ and its $\frac{1}{4}$ and 15 more, will be trebled? Ans. 20.

28. If the shadow of a staff 4 feet long, at a given hour is 7 feet long; what is the length of a steeple whose shadow at the same hour is 198 feet? Ans. 113$\frac{1}{7}$.

29. If a man travel 120 miles in 6 days, when the days are 12 hours long; how many days of 16 hours long will he require to travel 2880 miles? Ans. 108 days.

30. Two ships sailed from the same port; one sailed due north 60 leagues in 2 days—the other sailed due east 45 leagues in the same time; how far were they apart at the end of the second day? Ans. 75 leagues.

31. Divide 1673652552 by 4092. Ans. 409006.

32. Multiply 5032 by 304. Ans. 1529728.

33. Multiply 2486 by 1000. Ans. 2486000.

34. Divide 50 by $\frac{1}{2}$ of one. Ans. 100.

35. Bought 150 yards of cloth for 45 £.; for how much must it be sold to gain 25 per cent.? Ans. 56 £. 5 s.

36. A has 200 yards of broadcloth, worth $2.50 per yard, for which B gives him $250 in cash, and 500 gallons of molasses; at what does B value his molasses per gallon?
Ans. 50 cents.

37. Multiply .25 by .25. Ans. .0625.

38. Divide .56 by $.112. Ans. .5.

39. What will 140 reams of paper cost, at $2.35 per ream?
Ans. $329.

40. Divide 145260 by 108. Ans. 1345.

Questions.—What method of reasoning do you pursue in solving the 29th example?

How do you obtain the answer to the 30th sum? Why do you square 60 and 45 leagues, and then extract the square root of the sum of these two squares, in order to come to the true result? What general principle is involved in this process, which is true of all right angled triangles?

How can you prove that the quotient in Ex. 34, should be greater than the dividend?

## SECTION III.

1. What will 35 lbs. 8 oz. of cheese cost, at 8 cents per pound? Ans. $2.84.

2. A privateer of 65 men took a prize, which, being equally divided among them, gave to each man 119 dollars; what was the value of the prize? Ans. 7735 dolls.

3. What will $9\frac{3}{4}$ tons of hay come to at 14 dollars per ton? Ans. $136\frac{1}{2}$ dolls.

4. The war between England and the United States commenced April 19th, 1775, and peace took place January 20th, 1783; how long did the war continue? Ans. 7 yrs. 9 mo. 1 day.

5. What is the difference between .999 and 1? Ans. .001.

6. Divide 60000 by 100. Ans. 600.

7. Divide 34.72 by 1.24. Ans. 28.

8. Reduce 14 minutes to the decimal of a day. Ans. .009722, &c.

9. What is the value of .625 shil.? Ans. $7\frac{1}{2}$ d.

10. If $\frac{3}{8}$ of a yard of velvet cost $\frac{2}{5}$ of a pound, what will $\frac{5}{16}$ of a yard cost? Ans. 6 s. 8 d.

11. What part of a pound is one penny? Ans. $\frac{1}{240}$.

12. What will 2 cords of wood amount to, if $\frac{1}{4}$ of $\frac{4}{5}$ of $\frac{5}{8}$ of a cord cost $\frac{5}{6}$ of $\frac{9}{10}$ of a dollar? Ans. $12.

13. What sum, at interest for 2 years at $4\frac{1}{2}$ per cent., will amount to $272.50? Ans. $250.

14. At what rate per cent. will 300 dollars amount to 396 dollars in 8 years? Ans. 4 per cent.

15. Reduce 118 lbs. 1 oz. 6 pwts. 10 grs. to grains. Ans. 680314 grs.

16. What will be the expense of keeping 20 horses for a year, at 16 cents per day for each? Ans. $1168.

17. A drover agreed for 1 dozen lambs at the rate of 1 cent for the first, 2 cents for the second, 4 cents for the third, and so on, doubling the price at every lamb; what was the price of the last? Ans. $20.48.

18. The population of Sweden in 1826 was 2751582, that of Norway 1050132; how many more people in Sweden than in Norway? Ans. 1701450.

QUESTION.—What method of reasoning do you pursue in solving the 18th example?

19. A prize of 48726 dollars is owned by 270 men; what is each man's share? Ans. $180\frac{7}{5}$.

20. A labourer was employed for 20 days, on condition that he should receive 1 dollar for every day that he worked, and should pay 75 cents for each day that he was idle; at settlement he received $11.25; how many days did he work? Ans. 15.

21. In 1830 there were in New-York 46 slaves—in New Jersey 2246—in Pennsylvania 386—and in Delaware 3305; what was the whole number of slaves in these four states? Ans. 5983.

22. What will 750 barrels of beef come to, at 11 dollars per barrel? Ans. $8250.

23. What is the sum of .12345 and .987654321? Ans. 1.11104321.

24. Divide 39 £. 14 s. $5\frac{1}{4}$ d. by 57. Ans. 0 £. 13 s. $11\frac{1}{4}$ d.

25. From .2497 take .1221745. Ans. .1175255.

26. Multiply .425 by .5. Ans. 2125.

27. Divide .77 by .7. Ans. 1.1.

28. Sold one dozen of fowls for 9.17—a firkin of butter for 13.62—one cwt. of cheese for 7.25—and 18 lbs. of flax for 1.80; what did all amount to? Ans. 31.84.

29. Bought 3 cwt. of hops for 33.50, and sold them for 41.25; how much did I gain by the bargain? Ans. $7.75.

30. What will 17 barrels of cider come to at 2.92 per barrel? Ans. $49.64.

31. Divide 0.00078759 by 0.525. Ans. 0.00150.

32. What sum added to 17 £. 11 s. $8\frac{1}{4}$ d. will make 100 pounds? Ans. 82 £. 8 s. $3\frac{3}{4}$ d.

33. A merchant bought 17 cwt. 2 qrs. 14 lbs. of sugar, of which he sold 9 cwt. 3 qrs. 25 lbs.; how much had he left? Ans. 7 cwt. 2 qrs. 17 lbs.

34. A certain farm is divided into 6 fields—two of which contain 42 acres 3 roods 29 rods 28 yards—and each of the others 23 acres 3 roods 35 rods 25 yards; how much land in the farm? Ans. 138 a. 3 roods 13 rods 7 yds.

35. What will 96 bushels of corn come to at 7 s. 6 d. per bushel? Ans. 36 £.

36. Bought 1 doz. silver spoons, which together weighed 3 lbs. 2 oz. 13 pwts. 12 grs.; how much silver did each spoon contain? Ans. 3 oz. 4 pwts. 11 grs.

## SECTION IV.

1. In 100 £. how many shillings, ninepences, sixpences, fourpences, and pence, of each an equal number?
Ans. 750.

2. In 24 hogsheads of sugar, each 11 cwt. 25 lbs., how many pounds? Ans. 30168.

3. How many barley-corns in the circumference of the earth, it being 360 degrees? Ans. 4755801600.

4. What is the sum of $\frac{5}{6}$ of $\frac{1}{3}$,—$\frac{2}{5}$ of $\frac{1}{7}$ of 2,—$3\frac{1}{2}$,—$\frac{1}{3}$ of $\frac{3}{4}$, —and $\frac{3}{10}$? Ans. $4\frac{347}{1260}$.

5. A man bought at one time $\frac{1}{3}$ of a cord of wood, at another $\frac{3}{4}$ of a cord, and at another time $\frac{4}{5}$ of a cord; how much did he buy in all? Ans. $1\frac{53}{60}$ cord.

6. From $\frac{4}{5}$ of $\frac{7}{8}$, take $\frac{1}{4}$ of $\frac{5}{8}$. Ans. $\frac{101}{160}$.

7. Multiply $\frac{5}{8}$ of $15\frac{5}{6}$ by $\frac{3}{7}$ of $1\frac{1}{2}$. Ans. $6\frac{31}{224}$.

8. Divide $\frac{7}{8}$ by $13\frac{9}{10}$. Ans. $\frac{35}{556}$.

9. At $\frac{1}{5}$ of a dollar a pound, how many pounds of figs can be bought for $\frac{3}{4}$ of a dollar? Ans. $3\frac{3}{4}$.

10. Reduce $\frac{4}{5}$ to a decimal fraction. Ans. .8.

11. Reduce .825 to a vulgar fraction. Ans. $\frac{825}{1000}$.

12. A piece of cloth, 13 yds. long, was sold for 130 dollars; what will 18 yds. cost at the same rate? Ans. $180.

13. If 100 workmen can finish a piece of work in 12 days, how many can do the same in 3 days? Ans. 400.

14. If 120 bushels of oats will serve 14 horses 56 days, how many days will 94 bushels serve 6 horses?
Ans. $102\frac{16}{42}$.

15. If 6 lbs. at Portland make 5 lbs. at London, and 50 lbs. at London make 60 lbs. at Paris; how many pounds at Portland are equal to 160 lbs. at Paris? Ans. 160.

16. What is the interest of $984.50 for 7 years and 6 months, at 6 per cent.? Ans. $443.025.

17. What is the compound interest of 760 £. 10 s. for 4 years, at 4 per cent.? Ans. 129 £. 3 s. $6\frac{1}{4}$ d.

18. What is the present worth of $600, due 4 yrs. hence, at 5 per cent.? Ans. $500.

19. What will $129\frac{1}{2}$ yards of cloth cost, at 2 s. 6 d. per yard? Ans. 16 £. 3 s. 9 d.

20. What will 8765 eggs cost, at $\frac{3}{4}$ d. apiece?
Ans. 27 £. 7 s. $9\frac{3}{4}$ d.

21. A and B gained by trading $182. A put in stock to

the amount of $300, and B $400; what is each one's share of the gain?

Ans. { A's $ 78. B's 104.

22. How many times 3 in $462\frac{1}{3}$? Ans. $154\frac{1}{3}$.

23. A ship worth $860, of which A owned $\frac{1}{8}$, B $\frac{1}{4}$, and C the rest, was lost in a gale; what will each owner lose, the ship having been insured to the amount of $500?

Ans. { A $45. B 90. C 225.

24. If 980 lbs. of tea cost $588, and be sold for $666.40, what is the gain per pound? Ans. 8 cents.

25. If I buy cloth at 1 s. 6 d. per yard, and sell it again at 1 s. 9 d., what do I gain per cent.? Ans. 16 £. 13 s. 4 d.

26. A square piece of land contains 43560000 feet of surface, what is the length of one of its sides? Ans. 6600 ft.

27. A pile of wood in the form of a cube contains 102727 solid feet; what is the length of one of its sides? Ans. 103 ft.

28. If 1000 apples be placed in a right line a yard distant from each other, and the first a yard from a basket; what distance will a man travel, who shall gather them up singly, and return with them one by one to the basket?

29. A and B laid out equal sums of money in trade. A gained a sum equal to $\frac{1}{4}$ of his stock, and B lost 225 dollars; then A's money was double B's; what did each lay out?

30. Divide $25\frac{4}{21}$ by 20. Ans. $1\frac{108}{420}$.

31. Multiply $\frac{2}{8}$ of $\frac{1}{7}$ by $\frac{3}{7}$. Ans. $\frac{45}{392}$.

32. A man travelled 6 days; the first day he went 4 miles, and doubling this distance each day, his last day's ride was 128 miles; how far did he go in all? Ans. 252 miles.

33. If the length of an inclined plane be 12 ft. 9 in., and its perpendicular height be 4 ft. 3 in.; what force will be necessary to support a cylinder on this plane, weighing 66 lbs. 12 oz., making no allowance for friction? Ans. 22 lbs. 4 oz.

34. If the diameter of a wheel be 4 feet $7\frac{1}{2}$ inches, and the diameter of its axle be $9\frac{3}{4}$ inches; what weight attached to the wheel will balance 112 lbs. attached to the axle?

Ans. 19 lbs. 10 oz. $12\frac{108}{111}$ drs.

QUESTIONS.—What are the conditions of Example 28th? What data have you given in sum 32d, from which you are to determine the true answer?

## SECTION V.

1. Required the number, from which, if 7 be subtracted, and the remainder be divided by 8, and the quotient be multiplied by 5, and 4 be added to the product, the square root of the sum subtracted, and $\frac{3}{4}$ of that root cubed, and the cube divided by 9, the last quotient may be 24? Ans. 103.

2. If 1 acre of land produce 88 bushels of corn, how much will 168 acres produce? Ans. 14784 bush.

3. In a certain army there were 16 regiments, each regiment consisted of 12 companies, and each company of 74 men; 1749 of these were slain in battle; how many were left? Ans. 12459.

4. A man bought a farm of 425 acres, for which he gave 6840 dollars; what was that per acre? Ans. $\$16\frac{8}{85}$.

5. Reduce $\frac{40}{425}$ to its lowest terms. Ans. $\frac{8}{85}$.

6. What is the largest number that will divide 132 and 356 separately without a remainder? Ans. 4.

7. What will 349 lbs. of beef come to at 2 d. per lb.? Ans. 2 £. 18 s. 2 d.

8. From $\frac{13}{20}$ take $\frac{4}{10}$. Ans. $\frac{1}{4}$.

9. If a coat and waistcoat can be made from 3 yds. and 3 qrs. of cloth that is $1\frac{1}{2}$ yds. wide, how much will it require to make the same from cloth only 2 qrs. 3 nls. wide? Ans. 9 yds.

10. A owes B $\$400$, to be paid as follows: $\$100$ in 6 months, $\$150$ in 8 months, and the remainder in one year; in what time may the whole be paid at once, without loss to either? Ans. 9 months.

11. Reduce $\frac{6}{10}$. $\frac{4}{8}$, $\frac{1}{9}$, and $\frac{6}{7}$ to equivalent fractions having a common denominator. Ans. $\frac{378}{630}$, $\frac{315}{630}$, $\frac{70}{630}$, $\frac{540}{630}$.

12. What is the interest of 9672 dollars for 2 yrs. 7 mo. and 4 dys. at 8 per cent? Ans. $\$2007.477$.

13. What is the square root of $\frac{27883}{82088}$? Ans. $\frac{7}{12}$.

14. If 40 bushels of rye at 4 s. per bushel, 10 bushels at 6 s. per bushel, 30 at 5 s. per bushel, and 20 at 3 s. per bushel, be mixed together, what will 10 bushels of the mixture be worth? Ans. $\$7.166$.

15. How many cubic feet of wood in a load 7 ft. 10 in. long, 3 ft. 11 in. wide, and 3 ft. 6 in. high? Ans. 107 ft. 4 in. $7''$.

16. If 7 men can reap 84 acres of wheat in 24 days, how many men can reap 100 acres in 10 days? Ans. 20 men.

17. What is the difference between the sum of the squares of 23 and 42, and the cube of 25? Ans. 13332.

18. Reduce $\frac{144}{216}$ to its lowest terms. Ans. $\frac{2}{3}$.

19. If a legacy is left me of 2000 dollars, of which 500 are payable in 6 months, 800 in 1 year, and the remainder at the end of 3 years; how much ready money should I receive for said legacy, allowing 6 per cent. discount?

Ans. $1833.374.

20. In 61 £. 12 s. how many shillings, pence, and farthings? Ans. 1232 shill.; 14784 d.; 59136 qrs.

21. A merchant has 16 hogsheads of tobacco, each weighing 8 cwt. 15 lbs.; what will it amount to if he sell it for $12\frac{1}{2}$ cents per lb.? Ans. $1630.*

22. In 13 bars of gold, each weighing 9 oz. 5 pwts., how many grains? Ans. 57720.

23. What will 4560 yards of cambric cost at 1 s. 6 d. per yard? Ans. 342 £.

24. Reduce 342 £. to dollars. Ans. $1140.

25. Reduce 1 s. 6 d. to Federal money. Ans. 25 cents.

26. If 4560 yards of cloth cost 1140 dollars, what is it per yard? Ans. 25 cents.

27. Reduce 2 roods 16 rods to the decimal of an acre.

Ans. .6.

28. Reduce $\frac{1}{3}$ to a decimal fraction. Ans. .3333.×

29. Reduce .6 of an acre to its proper denominations.

Ans. 2 roods 16 rods.

30. *London, Feb. 13th*, 1832.

For value received, I promise to pay A B or order forty-two dollars and fifty cents, on demand, with interest after four months. W. M.

What is due on this note at the present time? Ans.

31. What is the value of 1 cwt. of sugar at 9 d. per pound? Ans. 3 £. 15 s.

32. Reduce 146 days, 2 hours, 24 minutes, to the decimal of a year? Ans. .4.

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* The answers to question 21st, and all of the kind, hereafter, are found by considering 100 pounds a hundred weight—which is now generally the case in the transactions of business. The answers to all previous questions in which *avoirdupois* weight is concerned, are agreeable to the old method of considering 112 pounds a hundred weight. If question 21st be considered after the latter method, the answer will be $1822.

33. Reduce .625 of a day to its proper denominations.
Ans. 15 hours.

34. A farmer sent 7 loads of wheat to market. Three of which contained 219 bushels, 3 pecks, 7 quarts, and each of the others 69 bushels, 3 pecks, 5 quarts; how much did he send in all? Ans. 499 bush. 2 pks. 3 qts.

35. Divide 3264 by 24. Ans. 136.

36. What is the value of $\frac{7}{16}$ of a dollar? Ans. 0.437.

37. If a silver globe 3 inches in diameter be worth $45; what is the value of one a foot in diameter? Ans. $2880.

38. If wood be 3 feet long, how high must it be piled so that a pile 8 feet long may make a cord? Ans. 5 ft. 4 in.

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## SECTION VI.

1. Divide $3515 among A B C and D, so as to give B three times as much as A, C four times as much as B, and D five times as much as C; what will be the share of each?
Ans. A's $46.25, B's $138.75, C's $555, D's $2775.

2. If $\frac{7}{8}$ of a ton of hay cost $16.24; what is it a ton?
Ans. $18.56.

3. A man sold $\frac{8}{12}$ of a field, which contained 24 acres, at the rate of $\frac{3}{8}$ of a dollar for $\frac{4}{16}$ of an acre; how much money did he receive? Ans. $288.

4. What number is that which being divided by 72, its quotient will be 19? Ans. 1368.

5. At what time between 4 and 5, will the hour and minute hands of a watch be exactly together?
Ans. $21\frac{9}{11}$ min. past 4 o'clk.

6. What is the difference between the interest of 350 £. at 4 per cent. for 8 years, and the discount of the same sum at the same rate and for the same time? Ans. 27 £. $3\frac{1}{3}$ s.

7. A man bought $\frac{8}{12}$ of a ship for $4800, and freighted her with corn at $\frac{4}{5}$ of a dollar per bushel. He paid a sum equal to $\frac{1}{8}$ of $\frac{1}{2}$ of $\frac{2}{3}$ of the value of the whole ship for insurance. The cargo and insurance cost a sum equal to $\frac{4}{5}$ of what he paid for his part of the ship; he sold the corn in England and received a profit after deducting $920. Expen-

What is the first thing to be done in solving example 1st?

ses were equal to $\frac{1}{5}$ of $\frac{1}{12}$ of the whole cost of all the ship, cargo and insurance; how did he sell his corn per bushel.
Ans. 70 cents.

8. What is the cube root of $\frac{12}{324}$? Ans. $\frac{1}{3}$.

9. If 16 s. 10½ d. be divided equally among 9 men; how much will each man receive? Ans. 1 s. 10½ d.

10. Multiply 732 by 99. Ans. 72468.

11. What part of 3 pence is $\frac{1}{8}$ of 2 pence? Ans. $\frac{3}{8}$.

12. Rays of light are said to pass from the sun to the earth, a distance of 95 millions of miles in 8 minutes; what distance do they travel in a second? Ans. 197916$\frac{2}{3}$ miles.

13. B and C can build a house in 18 days; with the aid of A they can do it in 11 days; in what time would A do it alone? Ans. 28$\frac{2}{7}$ days.

14. What is the value of $\frac{2}{5}$ of a guinea? Ans. 4 s. 8 d.

15. What is the difference between 200 yards, 2 feet, 10 inches, 1 bar. and 59 yards, 2 feet, 11 in. 2 bar.?
Ans. 140 yds. 2 ft. 10 in. 2 bar.

16. A goldsmith received 6 parcels of jewelry; the first weighed 48 lbs. 11 oz. 18 pwts. 21 grs; the second, 42 lbs. 10 oz. 14 pwts; the third, 40 lbs. 9 oz. 16 pwts. 20 grs.; the fourth, 36 lbs. 8 oz. 15 pwts. 22 grs.; the fifth, 30 lbs. 10 oz. 10 pwts; the sixth, 53 lbs. 17 pwts. 13 grs.; how much did he receive in all? Ans. 261 lbs. 4 oz. 13 pwts. 4 grs.

17. How many yards of serge 3 qrs. of a yard wide will line 7½ yards of broadcloth, 1½ yards wide? Ans. 15 yards.

18. What is the present worth of 150 £. payable in ¼ of a year, discounting at 5 per cent? Ans. 148 £. 2 s. 11½ d.

19. Multiply 364111 by 56. Ans. 20390216.

20. How many tiles 8 inches square will lay a floor 20 feet long and 16 broad? Ans. 720.

21. If it cost 16 cents per day to keep 1 horse; what will it cost to keep 11 horses a year? Ans. $642.40.

22. A pond 1 mile square froze uniformly to the depth of one foot; if a cubic foot of water weigh 1000 oz.; what was the weight of ice on that pond? Ans. 1742400000 lbs.

23. In 1742400000 lbs. how many tons? Ans. 871200.

24. If a bullet 6 inches in diameter weigh 32 lbs.; what will a bullet of the same metal weigh, whose diameter is 3 inches. Ans. 4 lbs.

25. Three farmers hired a pasture in company for $60.50. A put in 5 oxen for 4½ months, B put in 8 oxen for 5 months, and C. put in 9 oxen for 6½ months; how much of the rent should each pay? Ans. A $11.25—B $20—C $29.25.

26. Divide 1200 dollars among A, B and C, so that B shall have 100 dollars more than A, and C 64 more than B. Ans. A $312—B $412—C $476.

27. A merchant made a composition of 5 lbs. of tea at 7 s. per pound, 9 lbs. at 8 s. 6 d. per pound, and $14\frac{1}{2}$ lbs. at 5 s. 10 d. per pound; what was a pound of the mixture worth? Ans. 6 s. $10\frac{1}{2}$ d.

28. Divide $\frac{3}{8}$ by $\frac{7}{5}$. Ans. $\frac{9}{14}$.

29. Divide 18 by $\frac{6}{5}$. Ans. 15.

30. A guardian paid his ward $3500 for $2500 which he had had in his hands 8 years; what rate of interest did he allow him? Ans. 5 per cent.

31. Reduce $1\frac{2}{4}{}^{3}$ to a whole or mixed number. Ans. $48\frac{1}{4}$.

32. If I lend a friend 100 dollars for three months; how long may I keep 75 dollars of his money to balance the favour? Ans. 4 m.

33. If $\frac{4}{5}$ of a ship cost 12580 dollars; what is the whole ship worth? Ans. 15725 dolls.

34. A man spent $\frac{1}{3}$ of his life in England, $\frac{1}{4}$ of it in France, and the remainder of it, which was 20 years, in the United States; to what age did he live? Ans. 48 years.

35. What will a horse cost, computing his value in a geometrical proportion, by the nails in his shoes, at 1 farthing for the first nail, 3 for the second, and so on in a triple proportion to the last, or 32d nail?

Ans. 965114681693 £. 13 s. 4 d.

36. What is the compound interest of $210.50 for 3 yrs. at 6 per cent.? Ans. $29.487.

37. What is the cube of 45? Ans. 91125.

38. What will 780 dolls. amount to at 6 per cent. in 5 yrs. 7 months and 12 days? Ans. $975.99.

QUESTION. What data have you for obtaining the answer to example 35?

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## SECTION VII.

1. From 800.135 take 16.37. Ans. 783.765.
2. Multiply 516 by 21. Ans. 1083.6.
3. What cost 160 bushels of oats at 50 cents per bushel? Ans. $80.
4. How many solid feet in a load of wood $9\frac{1}{2}$ ft. long, $3\frac{1}{2}$ ft. wide, and 3 ft. 7 in. high? Ans. 113 ft. 5 in. 8″.
5. In 9 miles how many yards? Ans. 15840.

6. What number multiplied by 9 will produce 225?
Ans. 25.

7. Divide 178464 by 16. Ans. 11154.

8. If 145260 dollars be divided equally among 108 men, how much will each receive? Ans. $1345.

9. Divide 1575360 by 144. Ans. 10940.

10. A set out from London for Lincoln precisely at the time B left Lincoln for London, distant 100 miles. After 7 hours they met, and it then appeared that A had travelled $1\frac{1}{2}$ miles an hour more than B; at what rate per hour did each travel? Ans. A $7\frac{2}{28}\frac{4}{5}$; B $6\frac{1}{28}\frac{1}{5}$ miles.

11. It is required to lay out a lot of land in the form of a parallelogram, to be surrounded by 100 rods of fence, and to contain 3 acres 2 roods 29 rods; what must be the length and width of the lot? Ans. Length 31 rods; width 19 rods.

12. The sum of two numbers is 43, and their product is 442; what are those numbers? Ans. 26, and 17.

13. If $\frac{7}{8}$ yds. of cloth cost $\frac{7}{12}$ of a pound, what will $15\frac{3}{4}$ of a yard cost? Ans. 10 £. 9. s. 10 d. 2 qrs.

14. If $\frac{5}{8}$ of a gallon of wine cost $\frac{5}{8}$ of a pound, what will $\frac{5}{8}$ of a ton cost? Ans. 140 £.

15. What is the cube root of $\frac{15}{51}\frac{29}{39}$? Ans. $\frac{2}{3}$.

16. A gentleman purchased a horse, chaise, and harness, for which he paid $60. The harness cost half as much as the horse, and the horse twice as mush as the chaise; what was the cost of each?

Ans. { Harness $15. Horse 30. Chaise 15.

17. What sum of money must be divided among 18 men, so that each may receive $112? Ans. $2016.

18. What is the sum of 66947, 46742, and 132684?
Ans. 246373.

19. What is the difference between 124682 and 113465?
Ans. 11217.

20. Divide 293839455936 by 8405. Ans. 34960078.

21. In 1790 the population of Massachusetts was 378787; in 1830 it was 610014; what was the increase during the period between these dates? Ans. 231227.

22. Divide 4637064283 by 57606. Ans. 80496.

23. Maine contains 31960 square miles; if it were divided into townships, each 6 miles square, how many would there be? Ans. $887\frac{7}{9}$.

24. In $\frac{7}{9}$ of 6 miles, how many acres? Ans. 17928.

25. Pennsylvania is 280 miles long, and 157 broad; how many square miles does it contain? Ans. 43960.

26. In 43960 square miles, how many acres?
Ans. 28134400.

27. A square building covers just 2304 square feet of ground; what is the length of one of its sides? Ans. 48 ft.

28. Connecticut has a population of $62\frac{2}{5}\frac{1}{3}\frac{8}{0}$ to a square mile; how many square miles does the state contain, the population being 297711? Ans. 4770 miles.

29. Suppose a town to contain 145 houses, each house 2 families, and each family to contain 6 persons; how many inhabitants are in the town? Ans. 1740.

30. Bought 150 bales of cotton cloth, each bale containing 49 pieces, and each piece 26 yards; how many yards did I purchase? Ans. 191100 yards.

31. Divide 12098730 by 35. Ans. 345678.

32. I sold a watch for $\frac{4}{5}$ of the sum which it cost me; and it cost me $\frac{5}{4}$ of 20 dollars; what did I receive for it?

33. Multiply 72568 by 54729. Ans. 3938918472.

34. What number multiplied by itself will produce 841?
Ans. 29.

35. Extract the square root of 841. Ans. 29.

36. What number multiplied by itself twice will produce 729? Ans. 9.

37. I have a solid block of wood which measures 9 inches on each side; how many solid inches does it contain?
Ans. 729.

38. What is the cube root of 729? Ans. 9.

39. What is the square root of $27\frac{9}{16}$? Ans. $5\frac{1}{4}$.

40. Multiply $5\frac{1}{4}$ by $5\frac{1}{4}$. Ans. $27\frac{9}{16}$.

41. A gentleman bought 3 townships of land; for the first he gave 3500 dollars, for the second 4440 dollars, for the third 5700 dollars; he paid at the rate of 50 cents per acre; how many acres did he buy? Ans. 27280.

42. There is a house worth 1200 dollars; A owns $\frac{1}{4}$ of it, and B the rest; it is insured to the amount of $\frac{2}{3}$ of its value; if destroyed by fire what will each owner lose?
Ans. A $100, B $300.

43. In 57395520000 seconds, how many years? Ans. 1820.

44. In 1536 pints of wheat, how many bush.? Ans. 24.

In obtaining the answer to the 39th sum, what is the first thing to be done? How may it be proved that $5\frac{1}{4}$ is the correct answer?

## SECTION VIII.

1. Divide 45757390 by 9365. Ans. 4886.
2. Multiply 345678 by 234. Ans. 80888652.
3. Suppose a man to be 32 years of age; how many seconds has he lived, allowing 365 dys. 5 h. 48 min. 48 sec. to a year?
4. In 190080 inches, how many yards? Ans. 5280.
5. If a cubic inch of lead weigh 10 oz., what will be the weight of a plate of this metal 8 ft. long, 4 ft. wide, and $\frac{1}{2}$ in. thick? Ans. 1440 lbs.
6. Reduce 1440 lbs. to the fraction of a ton. Ans. $\frac{18}{25}$.
7. Reduce $\frac{18}{25}$ to a decimal fraction. Ans. .72.
8. If .72 of a ton of lead cost $50, what will 1440 lbs. cost? Ans. $50.
9. Multiply 123456789 by 123456789. Ans. 15241578750190521.
10. If the distance from Boston to London be 3000 miles, how many days will a ship be on its passage, sailing at the rate of 2 leagues per hour? Ans. $20\frac{5}{6}$ days.
11. Reduce $\frac{5}{6}$ of a day to hours. Ans. 20 hours.
12. If the difference between the longitude of Boston and Cincinnati be 13° 30′, what time will it be at Cincinnati when it is noon at Boston? Ans. 6 min. past 11.
13. Reduce 11 h. 6 m. to the fraction of a day. Ans. $\frac{37}{80}$.
14. Reduce $\frac{37}{80}$ to a decimal fraction. Ans. .4625.
15. Reduce .4625 of a day to hours and minutes. Ans. 11 h. 6 m.
16. The sun rises at A 1 h. and 20 min. sooner than at B, which is in longitude 25° 15′ west; what is the longitude of A? Ans. 5° 15″ west.
17. A man died leaving $1000 to be divided between his two sons, one 14, and the other 18 years of age, in such a proportion, that the share of each, being put to interest at 6 per cent., should amount to the same sum when they should arrive at the age of 21 years; what did each receive?

Ans. The younger 453.846+; the elder $546.153+.

QUESTIONS.—What principles are involved in the 11th sum? To what rule does it belong?

What principles are involved in the 17th example? What is the first thing to be done, in order to obtain the answer?

18. A rough stone was put into a vessel whose capacity was 14* wine quarts, and the vessel was then filled with $2\frac{1}{2}$ quarts of water; what was the cubic content of the stone? Ans. $664\frac{1}{8}$ inches.

19. If 800 soldiers were in a garrison, with provisions for 2 months, how many must depart, that the provisions may last 5 months? Ans. 480.

20. Take 480 from 800; how many remain? Ans. 320.

21. What is $\frac{2}{5}$ of 800? Ans. 320.

22. If 320 soldiers have provisions for 5 months, how many soldiers would consume the whole in 2 months? Ans. 800.

23. If 480 men have provisions for 3 months, how many must be added, that it may last but 2 months? Ans. 240.

24. What part of 480 is 240? Ans. $\frac{1}{2}$.

25. What is $\frac{2}{3}$ of 720? Ans. 480.

26. Reduce $\frac{480}{720}$ to its lowest terms. Ans. $\frac{2}{3}$.

27. Reduce $\frac{2}{3}$ to a decimal fraction. Ans. .6666+.

28. A can do a piece of work in 6 days, B can do twice as much in 8 days, and C can do three times as much in 9 days; in what time can they finish it if they work together? Ans. $1\frac{1}{3}$ day.

29. Bought 2000 bushels of wheat, of which I lost 1100 bushels; sold the remainder for $1.30 per bushel, and lost $830; what did I give per bushel? Ans. $1.

30. A man owns 3 lots of woodland; the first is 100 rods long and 80 wide—the second is 84 rods long and 68 wide—the third is 8 rods square; how many acres do all contain? Ans. 125 A. 2 roods 32 rods.

31. How much is $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{5}{8}$? Ans. $\frac{5}{64}$.

32. How much is $20\frac{3}{11}$ times $\frac{1}{2}$ of $\frac{1}{3}$? Ans. $3\frac{25}{66}$.

33. Divide $\frac{4}{15}$ by 6. Ans. $\frac{2}{45}$.

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## SECTION IX.

1. What is the length of one of the sides of a square field, which contains 15 acres and 1 rod? Ans. 29 rods.

What are the principles involved in the 28th example? Describe the process of doing it.

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* A wine pint contains $28\frac{1}{2}$ cubic inches.

2. If from 856 yds., be taken 200 yds. 2 qrs. 1 na. and 1 in., how much will remain? Ans. 655 yds. 1 qr. 2 na. $1\frac{1}{4}$ in.

3. What is the value of 81 barrels of flour, at 3 £. 16 s. 5 d. per barrel? Ans. 309 £. 9 s. 9 d.

4. What will 1210 yards of cloth cost, at 2 s. 5 d. per yard? Ans. 146 £. 4 s. 2 d.

5. If 151 barrels of cider cost 39 £. 12 s. 9d., what is that per barrel? Ans. 5 s. 3 d.

6. If 23 bushels of corn cost 6 £. 6 s. 6 d., what is it per bushel? Ans. 5 s. 6 d.

7. Reduce 5 s. 6 d. to cents. Ans. $91\frac{2}{3}$.

8. What will 23 bushels of corn cost, at $91\frac{2}{3}$ cents per bushel? Ans. $\$21.08\frac{1}{3}$.

9. What is the value of $\$21.08\frac{1}{3}$ in English money? Ans. 6 £. 6 s. 6 d.

10. In 43 pieces of cloth, each piece containing $25\frac{3}{4}$ yards, how many yards? Ans. 1107 yds. 1 qr.

11. In a township containing 1235 lots, and each lot 195 acres, 2 roods, 30 rods, how many acres? Ans. 241674 A. 10 rods.

12. If 6 horses in a given time eat 2 tons 4 cwt. 16 lbs. of hay, what is that to each horse? Ans. 7 cwt. 36 lbs.

13. Reduce 7 cwt. 36 lbs. to lbs. Ans. 736 lbs.

14. In 213210 grains, how many pounds? Ans. 37 lbs. 3 pwts. 18 grs.

15. What is the number of square feet in a marble slab, whose length is 5 ft. 7 in., and breadth 1 ft. 10 in.? Ans. 10 ft. 2 in.

16. From 274 £. 14 s. $2\frac{1}{4}$ d. take 85 £. 15 s. $7\frac{3}{4}$ d. Ans. 188 £. 18 s. $6\frac{1}{2}$ d.

17. Take 34 lbs. 9 oz. 4 dr. 16 gr. from 93 lbs. 10 oz. 5 dr. 19 grs. Ans. 59 lbs. 1 oz. 1 dr. 3 grs.

18. What interest will be due on a note of $\$936.42$, dated January 6th, 1832, if paid July 6th, 1835, interest at 6 per cent.? Ans. $\$196.644$.

19. March 15th, 1825, discharged a note of 6 yrs. $8\frac{1}{2}$ mo. standing; when was the note dated? *Ans. June 30, 1818.

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* In calculating interest, discount, &c. 30 days are reckoned a month, and 12 months a year,—which is less than the true year by about 5 dys. 6 hrs.

The answers to all questions in which years, months and days are considered, in this and the following sections, are agreeable to the above mode of reckoning, except when otherwise specified.

20. The sum for which the above note was given was $437.21, and interest at 6 per cent.; what sum was necessary to discharge it? Ans. $613.187.

21. From 1835 yrs. 6 mo. 6 dys. take 1832 yrs. 6 dys. Ans. 3 yrs. 6 mo.

22. From 1825 yrs. 2 mo. 15 dys. take 1818 yrs. 6 mo. Ans. 6 yrs. 8½ mo.

23. *C——d, June 4th*, 1831.

For value received, I the subscriber, of W——, county of Essex and state of Vermont, promise D. O. to pay him or order nine hundred and thirty-six dollars on demand, and interest at 4 per cent. A. L.

What interest will be due on this note Nov. 4th, 1832? Ans. $53.04.

24. From 1832 yrs. 10 mo. 4 dys., take 1831 yrs. 5 mo. 4 dys. Ans. 1 yr. 5 mo.

25. A commission merchant sold a quantity of goods, amounting to $583.47, for which he received 4 per cent. commission; what was the amount of the commission? Ans. $23.338.

26. What is the interest of $583.47 for 1 year, at 4 per cent? Ans. 23.338.

27. What is the discount of $583.47 for 1 year, at 4 per cent? Ans. $22.441.

28. A gave me his note for $426, payable in 4 yrs. 12 dys. without interest—on condition, however, of an immediate payment I agreed to discount 5 per cent; how much did I receive for the note? Ans. $354.519.

29. What is the discount of $350, payable in 6 months, discounting at the rate of 6 per cent. per annum? Ans. $10.194.

30. What is the present worth of $583.47 due 1 year hence, discounting at 4 per cent.? Ans. $561.028.

31. A owes B $380, of which $100 are to be paid in 6 months, $120 in 7 months, and $160 in 10 months; what is the equated time to pay the whole? Ans. 8 months.

32. A merchant had 1000 yards of canvass, worth 9½ d. per yard, which he bartered for serge worth 10¼ d. per yard; how many yards of serge did he receive? Ans. 926$\frac{34}{41}$.

33. A man bought a farm for $650, and agreed to pay $80 in 4 months, $150 in 6 months, $200 in 8 months, and $220 in 10 months; but at the end of 8 months, having been una-

ble to make any payments, he sold his farm and paid the whole at once; what did his creditor lose?

Ans. 92½ cents.

34. What is the compound interest of $236 for 4 yrs. 7 mo. 6 dys.? Ans. $72.669.

35. If I buy an article at $1.20 per pound, and sell it at 90 cents per pound; what do I lose per cent.? Ans. $0.25.

36. A general disposing his army into a square, found that he had 360 soldiers over and above; but if he increased each side with one soldier, he wanted 41 to fill up the square; how many soldiers had he? Ans. 40360.

37. A man raised 113 bushels, 3 pecks, 4 quarts of corn from 2 acres, 3 roods, and 10 rods; how much was that to the acre? Ans. 40 bu. 1 pk. 7 qrts. $1\frac{3}{4\frac{3}{5}}$ pt.

38. I have a square field which contains 44 acres; how many rails 12 feet long must I procure to surround it with a fence 5 rails high?

39. If 1 acre produce 40 bushels, 1 peck, 7 qts. 1 $\frac{3}{4\frac{3}{5}}$ pts.; what will 2 acres, 3 roods, 10 rods produce?

Ans. 113 bu. 3 pks. 4 qts.

40. A and B barter; A has cloth that cost 28 d., B's cost him 22 d., and he puts it at 25 d.; how high must A put his to obtain 10 per cent. more than B? Ans. 35 d.

41. C and D barter; C of 7 s. makes 6 s. 8 d.; D of 7 s. 6 d. makes 7 s. 3 d.; who loses most and by how much per cent.? Ans. C loses $1\frac{3}{7}$ per cent. more than D.

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## SECTION X.

1. What is the sum of $34.56, $127, $1.72 and $0.75.

Ans. $164.03.

2. A man bought a horse for 120 dollars and sold him for 93 dolls. 74 cts.; what did he lose in the trade?

Ans. 26 dolls. 26 cts.

3. What will 1375 yards of broadcloth cost at 3 dolls. 25 cts. per yard? Ans. $44687.

4. If 16.25 barrels of cider cost $42.05; what is it per barrel? Ans. $2.581.

5. Divide 120 dollars into three shares which shall be to each other as 1.2.3. Ans. $20.40.60.

6. If 100 dollars are paid for corn at 75 cents per bushel, and it be sold for .80 per bushel; what is gained?

Ans. $6.66\frac{2}{3}$.

7. Bought 37 gallons of oil for $1.10 per gallon, and sold the whole for 40 dollars; did I gain or lose and how much per cent? Ans. Lost $1.719 per cent.

8. B and C buy goods in company; B pays 450 dollars and C 600 dollars; they trade and gain 273 dollars; what is each man's share of the gain?

Ans. B's 117 dolls. C's 156 dolls.

9. Sold a watch for fifty guineas, and by so doing lost seventeen per cent., whereas I should have gained in trading twenty per cent.; for how much less than its true value was the watch sold? Ans. 23 £. 8 s. $0\frac{3}{4}$ d.

10. Two men hired a pasture for 27 dollars. A put in a number of horses for 4 months, B put in 5 horses for 3 months and paid $\frac{5}{9}$ of the rent; how many horses did A put in?

Ans. 3.

11. A. and B companied; A put in 120 dollars and took $\frac{5}{8}$ of the gain; what did B put in? Ans. 96 dolls.

12. A, B and C, commenced trade in company; A put in 400 yards of cloth, B put in 300 lbs. of tea, and C 200 dollars in cash; at the end of 8 months B took out the amount of his stock, 2 months after C took out his stock; A continued his stock in trade two months longer, when they found their gain to be 298 dolls., of which C's share is 100 dolls.; what was A's cloth per yard, and B's tea per pound.

Ans. A's cloth 45 cts, B's tea 75 cts.

13. Three men bought a farm for $2400; A paid $\frac{1}{6}$, B $\frac{1}{3}$, and C $\frac{1}{2}$; they sold it for $3000; what did each man receive?

Ans. A $500, B$1000, and C $1500.

14. A and B returned equal sums of money, and closed by joint trade 154 dollars. By agreement A received 8 per cent. because he spent his time in the execution of the business, while B received only 5 per cent.; what was A allowed for his trouble? Ans. $35.53\frac{1}{13}$.

15. A farmer has oats worth 38 cents per bushel, which he wishes to mix with corn worth 75 cents per bushel, so that the mixture may be worth 50 cents per bushel; how many bushels of each must he take?

Ans. 12 of oats, 25 of corn.

16. What will 12 bushels of oats at 38 cents per bushel, and 25 bushels of corn at 75 cents per bushel come to?

Ans. $23.31.

17. A composition being made of 5 lbs. of tea at 7 s. per lb; 9 lbs. at 8 s. 6 d. per lb.; and $14\frac{1}{2}$ lbs. at 5 s. 10 d. per lb.; what is a pound of it worth? Ans. 6 s. $10\frac{1}{2}$ d.

18. If 1 lb. of tea cost 6 s. $10\frac{1}{2}$ d.; what will 12 lbs. cost? Ans. 4 £. 2 s. 6 d.

19. A grocer has currants at 4 d., 6 d., 9 d. and 11 d. per pound, and he would make a mixture of 240 pounds, so that it might be sold at 8 d. per pound; how much of each sort must he take? Ans. 72 lbs. at 4 d.; 24 lbs. at 6 d.; 48 lbs. at 9 d.; and 96 lbs at 11 d.

20. What will 240 lbs. of currants cost at 8 d. per pound? Ans. 8 £.

21. In 8 £. how many dollars? Ans. $26.666+.

22. A cistern containing 60 gallons of water has 3 cocks for discharging it; the first will empty it in one hour; the second in two hours; and the third in three hours; in what time will it be emptied if all run together? Ans. $32\frac{8}{11}$ min.

23. In 35 guineas how many farthings? Ans. 35280.

24. In 6169 pence, how many pounds? Ans. 25 £. 14 s. 1 d.

25. What will 78455 bricks cost at $8.25 per thousand? Ans. $647.253.

26. What will 24570 feet of boards cost at $18.75 per thousand? Ans. $460.687.

27. If 77 yards of calico cost $26.18, what will one yard cost? Ans. 34 cents.

28. A, B and C put 810 £. in trade and gained 420 £., of which as often as A took 1 £., B took 3 £., and C took 5 £; what did each put in, and what did each share of the gain?

Ans. A put in 90 £. and shared $46\frac{2}{3}$ £.
B " 270 £. " 140 £
C " 450 £. " $223\frac{1}{3}$ £.

29. If $\frac{1}{8}$ of a barrel of cider cost $60\frac{1}{2}$ cents, what will a barrel cost? Ans. $4.84.

30. If $\frac{1}{12}$ lb. of myrrh cost $8.70$\frac{1}{2}$, what will a pound cost? Ans. $9.50.

31. What will $24\frac{1}{2}$ yards of broadcloth cost at $3.24 per yard? Ans. $79.38.

32. If 200 lbs. of butter cost $22.40, what is it per pound? Ans. $0.112.

33. Reduce 52 days to the decimal of a year. Ans. .142465753.

34. Reduce 142465753 to a vulgar fraction. Ans. $\frac{142465753}{1000000000}$.

## SECTION XI.

1. Reduce .75 to a vulgar fraction. Ans. $\frac{75}{100}$.

2. What is the greatest number that will divide 75 and 100 without a remainder? Ans. 25.

3. Reduce $\frac{75}{100}$ to its lowest terms. Ans. $\frac{3}{4}$.

4. Reduce $\frac{4}{5}$ to a decimal fraction. Ans. .8.

5. Divide $\frac{4}{5}$ by 7. Ans. $\frac{4}{35}$.

6. Divide $\frac{5}{8}$ by $\frac{3}{4}$. Ans. $\frac{5}{6}$.

7. Multiply $\frac{4}{7}$ by $\frac{7}{8}$. Ans. $\frac{4}{8}$.

8. Two men have the same income; A spends $\frac{4}{5}$ of his; but B by spending 50 £. per annum more than A, at the end of 4 years finds himself 100 £. in debt; what does each receive and spend per annum?

Ans. They receive 125 £. per annum.
A spends 100 £. "
B " 150 £. "

9. A ship has a leak which will sink her in 10 hours; she has also a pump which will clear her in 15 hours; now, if the pump is started when the ship begins to leak in what time will she sink? Ans. 30 hours.

10. There is an island 20 miles in curcumference, and two men start to travel the same way around it; one goes 2 miles per hour, the other 6 miles; how long before they will again be together? Ans. 5 hours.

11. What is the tax upon $1153 at $0.03 on a dollar?

12. A house being let upon a lease of 5 years at $60 per year, and the rent being in arrear for the whole time; what is the sum due at the end of the term, simple interest being allowed? Ans. $336.

13. A hare starts 40 yards before a hound and is not seen by him till she has been running 40 seconds; the hare runs at the rate of 10 miles per hour—the dog pursues at the rate of 18 miles per hour; how long and what distance will the dog run before he overtakes the hare?

Ans. $60\frac{5}{22}$ sec. and 530 yards.

14. In an orchard of fruit trees $\frac{1}{2}$ of them bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plums, 40 peaches, and 10 cherries; how many trees in the orchard? Ans. 600.

15. If 12 barrels of cider cost $36; what will 18 barrels cost? Ans. $54.

16. Reduce $\frac{12}{36}$ and $\frac{18}{54}$ to a common denominator.

Ans. $\frac{648}{1944}$ and $\frac{648}{1944}$.

17. Reduce $\frac{648}{1944}$ to its lowest terms. Ans. $\frac{1}{3}$.

18. What is the ratio of 36 to 12? Ans. $\frac{1}{3}$.

19. What is the ratio of 54 to 18? Ans. $\frac{1}{3}$.

20. If 18 barrels of cider cost $54; what will 12 barrels cost? Ans. 36 dolls.

21. If $54 buy 18 barrels of cider; how many barrels will $36 buy? Ans. 12.

22. Reduce $\frac{5}{18}$ and $\frac{7}{12}$ to a common denominator.
Ans. $\frac{648}{2160}$ and $\frac{648}{2160}$.

23. Reduce $\frac{648}{2160}$ to its equivalent whole number. Ans. 3.

24. If $36 buy 12 barrels of cider; how many barrels will $54 buy. Ans. 18.

25. If 15 horses in 24 days consume 100 bushels of oats; in how many days will 20 horses consume the same quantity?
Ans. 18 days.

26. Reduce $\frac{20}{15}$ and $\frac{24}{18}$ to a common denominator.
Ans. $\frac{360}{270}$ and $\frac{360}{270}$.

27. Reduce $\frac{360}{270}$ to its lowest terms. Ans. $\frac{4}{3}$

28. What is a half year's rent of 547 acres of land at 15 s. 6 d. per acre? Ans. 211 £. 19 s. 3 d.

29. Reduce $\frac{4}{3}$ to its equivalent mixed number. Ans. $1\frac{1}{3}$.

30. If 100 dollars gain 6 dollars interest in one year; how much will 480 dollars gain in the same time. Ans. $28.80

31. Divide 682752 by 24. Ans. 28448.

32. If an ounce of silver be worth $1.10; what is the value of 10 silver spoons, each weighing, 1 oz. 4 pwts.?
Ans. $13.20.

33. If 7 yards of ribbon cost 3 s. 4 d.; what will 126 yds. cost? Ans. 3 £.

34. How long will it take 5 men to do a piece of work, which 37 men can do in 15 days? Ans. 111 days.

35. If 5 men can do a piece of work in 111 days; how many men can do the same in 15 days? Ans. 37 men.

36. If 3 men can dig a trench 48 feet long in a certain time; how many feet will 12 men dig in the same time?
Ans. 192 feet.

37. An usurer put out 75 dollars at interest, and at the end of 8 months received for principal and interest 79 dollars; at what rate per cent did he receive interest?
Ans. 8 per cent.

38. How many square feet in 15 boards, each 12 feet 8 inches long and 13 inches wide? Ans. 205 ft. 10 in.

39. Suppose a tax of 2000 dollars laid on a town, and the

inventory of all the estates in the town amount to 45000 dollars; what must A pay whose estate is valued at 456 dollars? Ans. $20.666.

40. Reduce $\frac{4}{11}$ of a mill to the fraction of a dollar.
Ans. $\frac{1}{2750}$.

41. What is the sum of $\frac{1}{8}$ of 95 and $\frac{7}{8}$ of 14?
Ans. $43\frac{1}{12}$.

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## SECTION XII.

1. There is a cistern having a pipe which will empty it in 25 hours; how many pipes of the same capacity will empty it in 25 minutes? Ans. 60.

2. If 16 horses eat 9 bushels of oats in 6 days; how many horses will eat 168 bushels in 7 days at the same rate?
Ans. 256.

3. I have two fields, one containing 40 acres, the other 50 acres, which I am to exchange for a square field containing the same number of acres as both these; what must be the length of one of the sides of the square field?
Ans. 120 rods.

4. What is the interest of $273.51 at 7 per cent. for one year and ten days? Ans. $19.677.

5. Suppose 450 men are in a garrison, and the provisions on hand will last all only 5 months; how many must leave the garrison that the same provisions may last 9 months?
Ans. 200 men.

6. There is a wheel 4 inches in diameter; what is the diameter of a wheel 5 times as large? Ans. 8.94 in.

7. A man worked for me 15 days, when the price of labour was $1 per day; how long must I work for him to ballance the favour, when the price of labour is 67 cents per day? Ans. $22\frac{26}{67}$ days.

8. At 67 cents per day; what will $22\frac{26}{67}$ days work come to? Ans. $15.

9. If a force equal to 12 lbs. applied to the longer arm of a lever 8 feet long, be required to raise 96 lbs. attached to the shorter arm; what is the length of the shorter arm?
Ans. $10\frac{2}{3}$ inches.

10. Divide $\frac{2}{81}$ by $\frac{3}{41}$. Ans. $\frac{82}{243}$.

11. From .1 of a pound take .0678 of a pound.
Ans. .0322.

12. A and B depart from the same place and travel the same road; A goes 5 days before B, at the rate of 20 miles per day; B follows at the rate of 25 miles per day; in what time, and at what distance, will B overtake A?

Ans. 20 days, and 500 miles.

13. Divide 8 into two numbers, that shall be to each other as 1 is to 8.

Ans. $\begin{cases} .8888+. \\ 7.1111+. \end{cases}$

14. If in 9 days 24 men make 80 rods of road, in how many days will 50 men make $\frac{5}{8}$ of a mile? Ans. $10\frac{4}{5}$.

15. What is the sum of .8888 and 7.1111?

Ans. 7.9999.

16. Reduce $\frac{5}{8}$ of a mile to rods. Ans. 200.

17. What is the difference between 7.99999 and 8?

Ans. .00001.

18. A man gave his note for 22 £. 14 s. 6 d. and interest at the rate of 7 per cent., which note he paid 3 yrs. 8 mo. 12 dys. after date; what was the amount of principal and interest? Ans. 28 £. 12 s. $2\frac{1}{2}$ d.

19. If in 10 dys. 19 hrs. 12 min. 50 men make 200 rods of road, how many rods will 24 men make in 9 days?

Ans. 80 rods.

20. Reduce 19 hrs. 12 min. to the fraction of a day.

Ans. $\frac{4}{5}$ of a day.

21. A goldsmith sold 1 lb. of gold, at 2 cts. for the first oz., 8 cts. for the second oz., 32 cts. for the third oz., and so on in a quadruple proportion; what did the pound come to?

Ans. $111848.10.

22. Bought a piece of cloth containing 85 yds. for $191.25, and sold it at $2.81 per yard; what did I make in the bargain? Ans. 47.60.

23. What will 14 cwt. of beef cost, at 5 cents per pound?

Ans. $70.

24. The district of Columbia is 10 miles square; how many acres does it contain? Ans. 64000 A.

25. Suppose a farmer had three pieces of land, the first containing 27 A. 2 roods 25 rods—the second, 21 A. 3 roods 19 rods—the third, 37 A. 2 roods 29 rods; of which he sold at one time 10 A. 1 rood—at another, 3 A. 19 rods—and at another, 2 A. 1 rood 20 rods; how many acres had he left?

Ans. 71 A. 1 R. 34 rods.

26. What is the cube root of $2\frac{19}{27}$? Ans. $1\frac{1}{3}$.

27. What is the square root of 36? Ans. 6.

28. What is the square of 36? Ans. 1296.

29. How many bricks 8 in. long, 4 in. wide, and 2 in. thick, will build a house 44 ft. long, 40 ft. wide, and 20 ft. high, the walls being 12 in. thick? Ans. 88560.

30. What is the cube root of 216? Ans. 6.

31. What is the cube of 216? Ans. 10077696.

32. How many solid feet in a load of wood 7 ft. 7 in. long, 3 ft. 5 in. wide, and 3 ft. 8 in. high? Ans. $95\frac{1}{4}\frac{1}{3}\frac{1}{2}$.

33. What is the biquadrate root of 6561? Ans. 9.

34. If a ream of paper make a pile 6 in. thick, what is the thickness of each sheet? Ans. $\frac{1}{1}\frac{1}{0}\frac{1}{0}$ of an inch.

35. A man, being asked how many sheep he had, replied, that he had a number, which, if increased by its $\frac{1}{2}$ and $\frac{1}{4}$ and 18 more, would be doubled; how many had he?

Ans. 72.

36. If a man weighing 260 lbs. rest on the end of a lever 12 ft. long, what weight will he balance on the other end of it, if the prop be 1 foot from the weight?

37. If the diameter of a wheel be 48 inches, and that of its axle be 6 inches, what weight applied to the wheel will balance 1268 lbs. at the axle? Ans. $366\frac{5}{8}$ lbs.

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## SECTION XIII.

1. What is the cube root of 10077696? Ans. 216.

2. Change $\frac{1}{4}\frac{5}{2}\frac{8}{5}\frac{1}{6}$ to a decimal fraction. Ans. .3742+.

3. What will 10 doz. of books cost, at $9.50 per dozen?

Ans. $95.

4. If $\frac{1}{2}$ of a ton of hay cost $12, what will $\frac{9}{1}\frac{}{2}$ of a ton cost?

Ans. $18.

5. What cost 59 lbs. of butter, at $12\frac{1}{2}$ cts. per pound?

Ans. $7.37\frac{1}{2}$.

6. If 450 lbs. of beef cost $15, what is it per pound?

Ans. $3\frac{1}{3}$ cents.

7. A privateer took a prize, which was sold for $12465; of which the captain had $\frac{1}{2}$, the other officers $\frac{1}{4}$, and the remainder was divided equally among 125 sailors; what did each sailor receive? Ans. $24.93.

8. What is the interest of sixty-four dollars and eighteen cents for 1 year and 10 months, at 4 per cent?

Ans. $4.706.

9. A vessel lies at anchor 620 yards from the base of a perpendicular rock, which is washed by the sea; the distance from the top of the rock to that part of the vessel level with the water is 846 yards, and the distance from the top of a flag-staff erected on the summit of the rock to the same part of the vessel, is 900 yards; what is the height of the flag-staff?

Ans. 76.77 yds.

10. What is the superficial content of a board $12\frac{1}{2}$ ft. 6 in. long, and 2 ft. 4 in. broad? Ans. 29 ft. 2 in.

11. What is the solid content of a bale of cotton 7 ft. 6 in. long, 3 ft. 3 in. wide, and 1 ft. 10 in. thick?

Ans. 44 ft. $8\frac{1}{4}$ in.

12. What is the cube of 6 ft. 6 inches?

Ans. 274 ft. $7\frac{1}{2}$ in.

13. In a certain house there are 3 tiers of windows: in the first tier are 36 windows, each containing 34 panes of glass; in the second, are 34 windows of the same dimensions; and in the third, are 28 windows, each containing 18 panes; in all the windows the dimensions of the panes are 7 in. by 9 in.; what is the number of square feet of glass in the house? Ans. 955 ft. 72 in.

14. What did the glass of the above house cost at 6 cents per foot? Ans. $57.33.

15. Reduce 72 square inches to the fraction of a square foot. Ans. $\frac{1}{2}$.

16. There is a square field containing 10 acres; what is the distance from the centre to each corner?

Ans. 28.28+ rods.

17. In 20692 square rods, how many acres?

Ans. 129 A. 12 R. 19 rods.

18. Reduce 145 rods and 90 inches to the fraction of a mile. Ans. $\frac{5}{11}$.

19. If the snow be uniformly 6 in. deep, what is the quantity on an acre of land? Ans. 23400 cub. ft.

20. In a pile of wood, each side of which measures 6 feet, how many solid feet? Ans. 216.

21. In 10368 solid inches, how many feet? Ans. 6.

22. What is the square root of 119550669121?

Ans. 345761.

23. What is the cube root of 146708.483. Ans. 52.74.

24. Bought 6 tuns of wine for $500.50; what did 1 pipe cost? Ans. $41.70$\frac{8}{+}$.

25. Two ships sailed from the same port; one sailed due east at the rate of $22\frac{1}{2}$ leagues per day—the other sailed exactly south at the rate of 38 miles per day; how far were they apart at the end of the second day?

Ans. 88.32 leagues.

26. What is the cube root of .0001357? Ans. 00.5138+.

27. Multiply 460321 by 999. Ans. 459860679.

28. A general has an army of 4096 men; how many must he place in rank and file to form them into a square?

Ans. 64.

29. What is the product of 3216 multiplied by itself?

Ans. 10342656.

30. What is the cube of 103? 1092727.

31. What is the number of solid inches in a block of lead 1 foot square? Ans. 1728.

32. If a cube of silver, whose side is 4 inches, be worth 150 dollars, what is the side of a cube of the same silver, worth 450 dollars? Ans. 6.349 in.

33. A ladder 40 feet long placed in the street, reached a window 33 feet from the ground, but when turned over without moving its foot, reaches only 21 feet from the ground, to a window on the opposite side; what is the width of the street? Ans. 56.649 ft.

34. What is the cube root of 275894.451? Ans. 65.1.

35. Multiply 78910 by 1984. Ans. 156557440.

36. In a floor 24 feet long and 16 feet wide, how many feet of plank? Ans. 384.

37. How much per day does a labourer receive, who works 359 days for $314.125? Ans. $0.875.

38. What is the square of 999? Ans. 998001.

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## SECTION XIV.

1. A man has an orchard containing 46 rows, and each row 35 trees; how many trees are there in the orchard?

Ans. 1610.

2. In 324 plates of lead, each plate 12 inches long, 8 inches wide, and $\frac{1}{12}$ of an inch thick, how many solid inches?

Ans. 2592.

3. A kite was intercepted by the top of a steeple when 250 feet of line were expended; the boy who held it then

measured the distance to the base of the steeple, which he found to be 200 feet; what was the height of the steeple?

Ans. 150 feet.

4. A gallon, dry measure, contains $268\frac{4}{5}$ cubic inches; how many cubic inches in a bushel? Ans. $2150\frac{2}{5}$.

5. Wishing to secure grain, I procured 324 sheets of tin, each 1 foot long, 8 inches wide, and $\frac{1}{12}$ of an inch thick, which were just enough to line a cubical box with its cover; after thus lining the box, I filled it accurately with rye, which I sold afterwards for $\$129.27\frac{5}{11}\frac{1}{2}$; what did I receive per bushel? Ans. 75 cents.

6. What is the largest number, that will divide 475200 and 216000 without a remainder? Ans. 43200.

7. If the quantity of air taken into the lungs at once be 16 cubic inches, and 20 inspirations be made in a minute; how long could a person, confined in a close room 10 feet long, 7 feet wide, and 8 feet high, be supplied with air, provided none of it could be inspired more than once?

Ans. 2 d. 2 ho. 5 min. 15 sec.

8. A and B have gained 1260 dollars, of which A is to have 10 per cent. more than B; what is the share of each?

Ans. A's $660; B's $600.

9. Reduce $\frac{5}{7}$ of a solid foot to inches. Ans. $1234\frac{2}{7}$.

10. A is 5 miles before B, and travels at the rate of 5 miles in 12 hours; B follows at the rate of 60 miles in 12 hours; in what time will B overtake A? Ans. 1 ho. 5 min. $27\frac{3}{11}$ sec.

11. What is the most simple fraction equivalent to $\frac{3936}{10752}$?

Ans. $\frac{41}{112}$.

12. A lady being married on the day she was 18 years old, received a note dated on the day she was born, which then amounted to $5200, at 6 per cent. per annum, simple interest; how much was the principal? Ans. $2500.

13. From 373248 take 2592. Ans. 370656.

14. If a man travel 60 miles in 24 hours, how long will it take him to travel $5\frac{5}{11}$ miles? Ans. 2 h. 10 min. $54\frac{6}{11}$ sec.

15. What is the greatest common measure of 10752 and 3936? Ans. 96.

16. If a school room, in which are 70 scholars, be 34 feet long, 30 feet wide, and 10 feet high, how many cubic feet of space are allowed to each scholar? Ans. $145\frac{5}{7}$.

17. Change $\frac{172\frac{41}{112}}{12927\frac{5}{11}\frac{1}{2}}$ to a simple fraction.

Ans. $\frac{19305}{1447875}$.

18. Reduce 7 ft. 6 in. to the fraction of a rod. Ans. $\frac{5}{11}$.

19. Reduce $\frac{4}{7}$ of a guinea to the fraction of a pound.

Ans. $\frac{4}{5}$.

20. What is the value of .003125 £.? Ans. 3 qrs.

21. What is the difference between $\frac{1}{2}$ and $\frac{1}{8}$? Ans. $\frac{1}{8}$.

22. A goose is 5 rods before a fox; they both start at the same moment and travel the same way, the goose at the rate of 5 rods in 12 seconds, and the fox at the rate of 60 rods in 12 seconds; at what distance from the place where the fox started will he overtake the goose? Ans. 5 rods $7\frac{1}{2}$ ft.

23. Multiply 23400 by 1728. Ans. 40435200.

24. Divide 373248 by $2150\frac{2}{5}$. Ans. $173\frac{4}{7}$.

25. Reduce $\frac{6144}{10752}$ to its lowest terms. Ans. $\frac{4}{7}$.

26. What is the cube root of 46656? Ans. 36.

27. Reduce $\frac{2000}{150}$ to an equivalent whole number.

Ans. 14.

28. If a man travel 5 miles in 12 hours, how far will he travel in 1 ho. 5 min. and $27\frac{3}{11}$ sec.? Ans. $\frac{5}{11}$ of a mile.

29. What is the value of $\frac{500}{630}$ of a hogshead?

Ans. 50 galls.

30. A goldsmith bought 1 lb. of gold dust at $14 per oz., and sold it at the rate of 1 cent for the first oz., 4 cents for the second oz., 16 for the third, and so on in quadruple proportion; how much did he gain by the bargain?

Ans. $55756.05.

31. What will be the interest of $2363.635 for 1 year at $2\frac{3}{4}$ per cent? Ans. $65.009.

32. The mean distance of the earth from the sun is 95 millions of miles; what distance does it travel in performing one revolution, allowing the proportion between the diameter and the circumference of a circle to be as 1 to 3?

Ans. 570000000 miles.

33. An insurance company insure property to the amount of 70 per cent. of its real value; a man, whose house is insured by this company for a premium of $2\frac{3}{4}$ per cent. per annum, pays 65 dollars yearly; what is the real value of his house? Ans. $3376.622+.

34. What is the solid content of a wall 53 ft. 6 in. long, 12 ft. 3 in. high, and 2 ft. thick? Ans. 1310 ft. 9 in.

35. Four men rated a house as follows: A at $6700, B at $9000, C at $8750, and D at $7380; what is the average judgment? Ans. 7957.50.

36. Reduce 3 s. 6 d. to the fraction of a pound.

Ans. $\frac{7}{40}$.

37. Add together $\frac{2}{3}$ of a yard, $\frac{3}{4}$ of a foot, and $\frac{3}{8}$ of a mile.
Ans. 660 yds. 2 ft. 9 in.

38. Reduce 3 qrs. to the decimal of a shilling.
Ans. .0625.

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## SECTION XV.

1. Divide 7380964 by 23000. Ans. $320\frac{2964}{23000}$.
2. Divide 83016572 by 240. Ans. $345902\frac{92}{240}$.
3. In 36 guineas, how many farthings? Ans. 36288.
4. In 36288 farthings, how many guineas? Ans. 36.
5. Reduce 12 to a fraction whose denominator shall be 13.
Ans. $\frac{156}{13}$.
6. What is the square root of $\frac{9}{12}$? Ans. 0.866025.
7. What will be the charge of keeping 20 horses for a year, at the rate of $14\frac{1}{2}$ d. per day for each horse?
Ans. 441 £. 0 s. 10 d.
8. Required the product of 6, and $\frac{2}{3}$ of 5. Ans. 20.
9. Divide $\frac{9}{16}$ by 3. Ans. $\frac{3}{16}$.
10. The governor of a besieged place having provisions for 54 days, at the rate of $1\frac{1}{2}$ lb. of bread, is desirous to prolong the siege to 80 days, in hope of succour; what must the ration of bread be to last this time? Ans. $1\frac{1}{80}$ lb.
11. Find the amount of 50 £. in 5 years, at 5 per cent.
Ans. 63 £. 16 s. $3\frac{1}{4}$ d.
12. What is the cube root of 520? Ans. 8.04145+.
13. A, B and C companied; A put in 40 £., B 60 £., and C a sum unknown; they gained 72 £., of which C took 32 £. for his share; what did A and B gain, and C put in?
Ans. A's gain = 16 £.
B's " = 24 £.
C put in 80 £.
14. What number is that, which, being increased by $\frac{2}{5}$, $\frac{3}{8}$ and $\frac{5}{6}$ of itself, the sum will be be $234\frac{3}{4}$? Ans. 90.
15. What will $567\frac{3}{4}$ yards of cloth cost at 2 s. per yard?
Ans. 56 £. 15 s. 6 d.
16. A general was asked by his king what reward he should confer on him for his services; the general only required a penny for every file, of 10 men in a file, which he could make out of a company of 90 men; what did it amount to? Ans. 23836022841 £. 7 s. $11\frac{65}{1154}$ d.

17. What is the value of 87 barrels of flour, at $\$6.37\frac{1}{2}$ per barrel? Ans. $\$554.62\frac{1}{2}$.

18. What is the weight of 6 chests of tea, each weighing 3 cwt. 2 qrs. 9 lbs.? Ans. 1 ton 1 cwt. 2 qrs. 24 lbs.

19. A merchant bought a number of bales of velvet, each containing $129\frac{17}{7}$ yards, at the rate of 7 dollars for 5 yards, and sold them out at the rate of 11 dollars for 7 yards, and gained 200 dollars by the bargain; how many bales were there? Ans. 9.

20. What is the cube of 322? Ans. 33386248.

21. There is a circle whose diameter is 4 inches; what is the diameter of a circle 5 times as large? Ans. 8.94 in.

22. What will the paving of a foot-path cost, at 3 s. 4 d. per yard, the length being 35 ft. 4 in., and the breadth 8 ft. 3 in.? Ans. 5 £. 7 s. $11\frac{1}{4}$ d.

23. What cost the covering and guttering a roof with lead, at 18 s. the cwt., the length of the roof being 43 feet, and breadth agirt over it 32 feet—the guttering 57 feet long and 2 feet wide—the former 9.831 lbs., and the latter 7.373 lbs. to the square foot? Ans. 115 £. 9 s. $1\frac{1}{2}$ d.

24. What cost 24 tons of hay, at 3 £. 7 s. 6 d. per ton? Ans. 81 £.

25. Divide 123 £. 11 s. $2\frac{1}{2}$ d. by 127. Ans. 19 s. $5\frac{1}{2}$ d.

26. Reduce 2, and $\frac{2}{5}$ of $\frac{5}{6}$, to a simple fraction. Ans $\frac{7}{3}$.

27. What will $3\frac{3}{8}$ oz. of silver cost, at 6 s. 4 d. per ounce? Ans. 1 £. 1 s. $4\frac{1}{2}$ d.

28. Reduce $\frac{550}{3842}$ to a decimal. Ans. .143155+.

29. What is the cube root of 2? Ans. 1.25992+.

30. A certain work having been raised in 12 days by working 4 hours per day, how long would it have been in raising by working 6 hours per day? Ans. 8 days.

31. A person looking on his watch, was asked what was the time of the day, who answered, "It is between 5 and 6;" but a more particular answer being required, he said that the hour and minute were then exactly together; what was the time? Ans. $27\frac{3}{11}$ min. past 5.

## SECTION XVI.

1. What is the value of .0125 lb. Troy? Ans. 3 dwts.
2. What is the value of .6875 yd.? Ans. 2 qrs. 3 nls.
3. Reduce 14 min. to the decimal of a day.
Ans. .00972+ of a day.
4. Multiply 12 ft. 5 in. by 6 ft. 8 in. Ans. 82 ft. $9\frac{1}{3}$ in.
5. What principal in 1 yr. 2 mo., at 6 per cent, will amount to 642 dollars? Ans. 600 dolls.
6. A gentleman bought 3 yds. of broadcloth, $1\frac{1}{2}$ yds. wide; how many yards of flannel which is only $\frac{3}{4}$ yd. wide, will line the same? Ans. 6 yds.
7. A captain, 2 lieutenants, and 30 seamen, take a prize worth 7002 dollars, which they divide into 100 shares—of which the captain takes 12, the two lieutenants each 5, and the remainder is to be divided equally among the sailors; how much will each man receive?

Ans. { The captain's share = $840.24.
Each lieutenant's = 350.10.
" seaman = 182.052.

8. Divide 101442075 by 4025. Ans. 25203.
9. Multiply 45678 by 144. Ans. 6577632.
10. Add together $14\frac{3}{4}$ and $15\frac{5}{6}$. Ans. $30\frac{7}{12}$.
11. From $\frac{1}{2}$ of $\frac{6}{10}$ take $\frac{1}{2}$ of $\frac{9}{26}$. Ans. $\frac{66}{520}$.
12. What is the square of 686? Ans. 470596.
13. If 20 men can perform a piece of work in 12 days; how many men will accomplish another thrice as large in $\frac{1}{5}$ of the time? Ans. 300 men.
14. A person being asked the hour of the day, said, the time past noon is equal to $\frac{4}{5}$ of the time till midnight; what was the time? Ans. 20 min. past 5.
15. What is the fifth power of .029?
Ans. .000000020511149.
16. What is the fourth power of $\frac{3}{4}$? Ans. $\frac{81}{256}$.
17. A farmer mixes 10 bushels of wheat at 5 s. per bushel with 18 bushels of rye at 3 s. per bushel, and 20 bushels of barley at 2 s. per bushel; how much is a bushel of the mixture worth? Ans. 3 s.
18. What is the interest of 720 £. for 3 years at 5 per cent? Ans. 108 £.
19. A detachment consisting of 5 companies, being sent

into winter quarters, in which the duty required 76 men a day; what number of men must be furnished by each company in proportion to their strength; the first consisting of 54 men, the second of 51 men, the third of 48 men, the fourth of 39 men, and the fifth of 36 men? Ans. The 1st must furnish 18—the 2nd. 17—the 3rd. 16—the 4th. 13—and the 5th. 12 men.

20. Divide 206 mo. 4 days by 26.

Ans. 7 mo. 3 weeks. 5 days.

21. How many seconds are in a solar year, or 365 days, 5 hours, 48 minutes, 48 seconds? Ans. 31556928.

22. What is the cube of 451? Ans. 91733851.

23. What is the cube root of 451? Ans. 7.66876+.

24. What is the square root of 451? Ans. 21.23676+

25. A man died leaving 3 sons, to whom he bequeathed his estate in the following manner, viz. to the oldest he gave 184 dolls.—to the second 155 dolls.—to the third 96 dolls.; but when his debts were paid, there were but 184 dolls. left; what is each one's proportion of the estate?

Ans. { The 1st. $77.829.
" 2nd. 65.563.
" 3rd. 40.606.

26. A man travelled 180 miles in 6 days; he increased his journey each day by 4 miles; how far did he travel the first day? Ans. 20 miles.

27. If the last term be 262144, the ratio 4, and the number of terms 9; what is the first term? Ans. 4.

28. If 120 lbs. of steel cost 7 £.; how must I sell it per pound to gain 15½ £. Ans. 1 s. 4 d. per lb.

29. Reduce $\frac{1}{2}$, $\frac{2}{3}$ of $\frac{5}{6}$, 7 $\frac{3}{4}$, and $\frac{3}{13}$ to a common denominator. Ans. $\frac{936}{1872}$, $\frac{1040}{1872}$, $\frac{1450\,8}{1872}$, and $\frac{432}{1872}$.

30. Increase the value of $\frac{1}{114}$, nineteen times. Ans. $\frac{1}{6}$.

31. A man distributed 37 £. 10 s. among 4 persons in the following manner, viz. that as often as the first had 20 s. the second should have 15 s. and the third 10 s. and the fourth 5 s.; what did each person receive? Ans. The 1st. 15 £.—the 2nd. 11 £. 5 s.—the 3rd. 7 £. 10 s.—the 4th. 3 £. 15 s.

32. A lady bought a gold ring, at the rate of 20 dollars per ounce; she gave for the ring $1.25; how much did it weigh?

33. What is the amount of $53 at 10 per cent. for 7 months? Ans. $56.091.

34. What is the square of 317? Ans. 100489.

35. What is the interest of 200 guineas for 4 years, 7 mo. and 25 days, at $4\frac{1}{2}$ per cent.? Ans. 43 £. 19 s. $2\frac{1}{4}$ d.

36. If 1000 men, besieged in a town with rations for 5 weeks, allowing 16 ounces per day to each man, be reinforced with 500 men more; and supposing they cannot be relieved till the end of 8 weeks, how many ounces a day must each man have, that the provisions may last that time?

Ans. $6\frac{2}{3}$ ounces.

## SECTION XVII.

1. Bought 16 yards, 2 qrs. 3 nails of broadcloth for $100,-125; what was that per yard? Ans. 6 dolls.

2. From a piece of cloth containing $36\frac{5}{8}$ yards, a merchant sold at one time $7\frac{3}{10}$ yards, and at another time, $12\frac{5}{8}$ yards; how much of the cloth had he left? Ans. 167. yds.

3. A farmer bought 7 yards of broadcloth for $8\frac{6}{13}$ £., a barrel of flour for $2\frac{4}{15}$ £., a cask of lime for $1\frac{3}{8}$ £., and 7 lbs. of rice for $\frac{5}{24}$ £.; he paid 1 ton of hay at $3\frac{7}{16}$ £., 1 cow at $6\frac{2}{3}$ £., and a balance in pork at $\frac{1}{40}$ £., per lb.; how much pork did he sell? Ans. $104\frac{4}{5}$ lbs.

4. What is the greatest common divisor of 35 and 100?

Ans. 5.

5. Multiply 49561776 by 20000. Ans. 991235520000.

6. Divide 49561776 by 5137. Ans. 9648.

7. What is the area of a parallelogram, of which one side is 13 rods, and the perpendicular 9 rods?

Ans. 117 square rods.

8. What is the area of a triangular field, of which one side is 28 rods, and the perpendicular 15 rods?

Ans. 210 square rods, or 1 acre and 50 rods.

9. How many cubic feet of water will a round cistern hold which is 3 feet in diameter at the bottom, 4 feet at the top, and 5 feet high? Ans. 48.433 feet.

10. What is the amount of .5—.05—.005—.555—.18765 and 8567? Ans. 8567.29765.

11. If 6 yards of cloth cost $\$5\frac{3}{5}$; what will $14\frac{3}{4}$ yds. cost?

Ans. $\$13\frac{23}{30}$.

12. What will be the amount of $700 for 5 years, compound interest? Ans. $\$936.757\frac{8}{10}+$.

13. What will be the amount of $700 for 5 years simple interest? Ans. $910.

14. Add $\frac{1}{8}$ £. $\frac{2}{7}$ s. and $\frac{4}{5}$ d. together. Ans. 2 s. $8\frac{64}{105}$ d.

15. A bridge built across a river in 6 months by 45 men,

was washed away by a freshet; how many men will be sufficient to build another of twice the value in 4 months?

Ans. 135 men.

16. What is the square root of 998001? Ans. 999.

17. There are two globes; 1 of them is one foot in diameter—and the other 40 feet in diameter; how many of the smaller globes would it take to make one of the larger?

Ans. 64000.

18. There is a cubic box, one side of which is 3 feet; how many solid feet does it contain? Ans. 27 feet.

19. What is the cube of $\frac{1}{125}$? Ans. $\frac{1}{5}$.

20. In a pile of wood 126 feet in length, 3 feet 9 inches wide, and 4 feet 3 inches high; how many cords?

Ans. 21 cords $7\frac{5}{16}$ of a cord.

21. From 480 take 245.0075. Ans. 234.9925.

22. If a piece of land, 40 rods in length and 4 in breadth, make an acre, how wide must it be when it is but 25 rods long? Ans. $6\frac{2}{5}$ rods.

23. A man bought 25 horses, agreeing to pay for them one cent for every different order in which they could all be placed; how much did the horses cost him?

Ans. $155112100433309859840000.

24. A, B and C, talking of their ages, B said his age was once and a half the age of A, and C said his age was twice and one tenth the age of both, and that the sum of their ages was 93; what was the age of each?

Ans. A 12 yrs.—B 18 yrs.—C 63 yrs. old.

25. In 8012131 grains, how many pounds? &c.

Ans. 1390 lbs. 11 oz. 18 pwts. 19 grs.

26. What cost 6 lbs. of butter at $9\frac{1}{2}$ d. per lb.?

Ans. 4 s. 9 d.

27. Divide 705 lbs. 10 s. 2 d. by 7.

Ans. 100 lbs. 15 s. $8\frac{3}{4}$ d.

28. Reduce .21 pint to the decimal of a peck.

Ans. .013125 peck.

29. Multiply 4 feet, 7 inches, by 9 feet 6 in.

Ans. 43 feet, $6\frac{1}{2}$ inches.

30. If the extremes be 10 and 70, and the number of terms 21; what is the common difference, and the sum of the series? Ans. com. diff. is 3, and the sum 840.

31. What is the cube root of 23887872? Ans. 288.

What is necessary in order to obtain the true answer to the 17th question?

32. In 109 £. 3 s. 8 d. how many dollars and cents?
Ans. $363.94.

33. What is the interest of $1600 for 1 year and 3 mo.?
Ans. $120.

34. What will 126 yards of tape cost at $\frac{1}{2}$ d. per yard?
Ans. 5 s. 3 d.

35. If 120 lbs. of iron cost 7 £; how must I sell it per pound to gain $15\frac{1}{2}$ per cent? Ans. 1 s. 4 d. per lb.

36. A and B cleared in an adventure at sea 45 guineas, which was 35 £. per cent. upon the money advanced, and with which they agreed to buy a horse and carriage, whereof they were to have the use in proportion to the sums adventured, which was found to be 11 to A, as often as 8 to B; what money did each adventure?
Ans. A 104 £. 4 s. $2\frac{10}{19}$ d.—B 75 £. 15 s. $9\frac{9}{19}$ d.

37. Divide 37895429 by 112. Ans. $338352\frac{5}{112}$.

38. Reduce $\frac{3}{4}$ and $\frac{5}{8}$ to the least common denominator.
Ans. $\frac{6}{8}$ and $\frac{5}{8}$.

39. At $\$\frac{1}{16}$ a pound, what will 40 pounds of sugar cost?
Ans. $\$2\frac{1}{2}$.

40. Three men hire a pasture for $48; A puts in 80 cows for 4 months, B 60 cows for two months, and C 72 cows for 5 months; what part of the rent must each man pay?
Ans. A's $19.20. B's 7.20. C's 21.61.

41. Involve $\frac{2}{3}$ to the 2nd power. Ans. $\frac{4}{9}$.

42. What is the square root of $\frac{4}{9}$? Ans. $\frac{2}{3}$.

## SECTION XVIII.

1. How much in length, that is $8\frac{4}{7}$ inches broad, will make a square foot? Ans. $16\frac{32}{61}$ inches.

2. From $\frac{1}{4}$ £. take $\frac{9}{10}$ shill. Ans. 4 s. $1\frac{1}{5}$ d.

3. From $\frac{1}{2}$ of a league take $\frac{3}{5}$ of a mile. Ans. 1 mi. 1 fur.

4. In 12 tons 15 cwt. 1 qr. 19 lbs. 6 oz. 12 dr. how drachms? Ans. 7323500.

5. A and B barter; A has 150 bushels of wheat at $1.25 per bushel, for which B gives 65 bushels of barley, worth $62\frac{1}{2}$ cts. per bushel, and the balance in oats at $37\frac{1}{2}$ cts. per bushel; what quantity of oats must A receive from B?
Ans. $391\frac{2}{3}$ bushels.

6. Multiply $5\frac{1}{2}$ by $\frac{1}{8}$. Ans. $\frac{7}{8}$.

7. If my income be $1750 a year, and I spend 19 s. 7 d. per day, how much shall I have saved at the end of the year?
Ans. 167 £. 12 s. 1 d.

8. Let the extremes be 3 and 39, and the sum 399; what is the common difference? Ans. 2.

9. What will a pension of 75 dollars per annum, payable yearly, amount to in 9 years, at 5 per cent. compound interest? Ans. $826.992\frac{3}{10}$.

10. If the first term be 5, the ratio 3, and the number of terms 7; what is the sum of the series? Ans. 5465.

11. What is the cube of 4.39? Ans. 84.604519.

12. What is the square of 15.3? Ans. 234.09.

13. There are 320 rods in a mile; how many rods from Andover to Boston, considering the distance $20\frac{1}{2}$ miles?
Ans. 6560 rods.

14. How many feet in the above answer?
Ans. 108240 feet.

15. In 108240 feet, how many miles? Ans. $20\frac{1}{2}$ miles.

16. A put in $\frac{1}{2}$ for $\frac{3}{4}$ of a year, B $\frac{2}{5}$ for $\frac{1}{2}$ of a year, and C the rest for one year; their joint stock was 1, and their gain 1; what is each one's share?
Ans. { A's = $\frac{15}{27}$. B's = $\frac{8}{27}$. C's = $\frac{4}{27}$.

17. The number of inhabitants on the globe is computed to be 750000000; suppose they had each counted one for every second from the creation to this time, or 6000 years of 365 days each; how many would they have counted?
Ans. 141912000 billions.

18. Divide 73146085 by 4. Ans. $18286521\frac{1}{4}$.

19. What will 120 days' wages amount to, at 5 s. 9 d. per day? Ans. 34 £ 10 s.

20. Reduce $12\frac{7}{9}$ to an improper fraction. Ans. $\frac{115}{9}$.

21. Reduce $\frac{2}{15}$£. to the fraction of a penny. Ans. $\frac{32}{1}$d.

22. How many yards of paper that is 30 inches wide, will hang a room that is 20 yards in circuit and 9 feet high?
Ans. 72 yds.

23. A debt can be paid in a year, by paying 1 shill. the first week, 3 shill. the next, and so on, always 2 shill. more every week; what is the debt, and what will the last payment be?
Ans. { The debt is 135 £. 4 s. The last payment will be 5 £. 3 s.

24. Four persons, W, X, Y, and Z, spend among them 25 shill. and agree that their shares shall be in proportion as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$; what are their shares?

Ans. W must pay 9 s. 8 d. $3\frac{1}{7}$ q.
X " " 6 ' 5 ' $3\frac{3}{7}$ '
Y " " 4 ' 10 ' $1\frac{5}{7}$ '
Z " " 3 ' 10 ' $3\frac{1}{7}$ '

25. What is the square of 230? Ans. 52900.
26. What is the square root of 230? Ans. 15.16575+.
27. What is the cube of 230? Ans. 12167000.
28. What is the cube root of 230? Ans. 6.1269+.
29. If 27 shill. be the wages of 4 men for 7 days, what will be the wages of 14 men for 10 days? Ans. 6 £. 15 s.
30. Reduce $_1\frac{6}{12}$ to a decimal fraction. Ans. .031350.
31. Divide $\frac{2}{7}$ by $\frac{3}{5}$. Ans. $\frac{10}{21}$.
32. Multiply $\frac{2}{7}$ by $\frac{5}{8}$. Ans. $\frac{5}{28}$.
33. What is the sum of $\frac{1}{4}$ of a pound, $\frac{2}{9}$ of a shill. and $\frac{5}{12}$ of a penny? Ans. $\frac{31}{160}\frac{3}{8}$ shill.
34. Reduce $\frac{31}{160}\frac{3}{8}$ shill. to the fraction of shillings, pence, &c. Ans. 3 s. 1 d. $1\frac{1}{2}\frac{0}{1}$ q.
35. Having melted together 7 oz. of gold of 22 carats fine, $12\frac{1}{2}$ oz. of 21 carats fine, and 17 oz. of 19 carats fine, I would know the fineness of the composition?
Ans. $20\frac{2}{3}$ carats fine.
36. Find two arithmetical means between 2 and 8.
Ans. 4 and 6.

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## SECTION XIX.

1. What is the interest of 715 £. 12 s. 6 d. for a year, at $4\frac{1}{2}$ per cent. per annum? Ans. 32 £. 4 shill.
2. How many days can 7 persons be placed in a different position at dinner? Ans. 5040.
3. A son asking his father how old he was, received this answer: "Your age is now one third of mine; but five years ago, your age was only one fourth of mine;" what were the two ages? Ans. 15 and 45.
4. Multiply 683456000 by $\frac{1}{2}$. Ans. 341728000.
5. Multiply 168416000 by $\frac{1}{4}$. Ans. 42104000.
6. Divide 10369 by $\frac{1}{2}$. Ans. 20738.
7. Divide 1468325000 by $\frac{1}{5}$. Ans. 4404975000.

8. What is the difference between $\frac{4}{5}$ of a pound, and $\frac{2}{3}$ of $\frac{3}{4}$ of a shilling? Ans. 10 s. 7 d. $1\frac{1}{3}$ q.

9. Reduce 15 s. 6 d. to the decimal of a pound.
Ans. .775 £.

10. What is the third power of $\frac{4}{5}$? Ans. $\frac{64}{125}$.

11. A man having 50 shill. to pay among his labourers for a day's work, would give to every boy 6 d.—to every woman 8 d.—and to every man 16 d.; the number of the men, women and boys, was the same; I demand the number of each.
Ans. 20.

12. If a field will feed 7 horses 8 weeks, how long will it feed 28 horses? Ans. 2 weeks.

13. A bushel measure is 18.5 inches in diameter and 8 inches deep; how many cubic inches does it contain?
Ans. 2150.4+.

14. How many square inches of leather will cover a ball $3\frac{1}{2}$ inches in diameter? Ans. $38\frac{1}{2}$ sq. in.

15. How many solid inches in a ball 7 inches in diameter?
Ans. $179\frac{2}{3}$.

16. A person who was possessed of $\frac{2}{5}$ of a vessel, sold $\frac{4}{5}$ of his share for 375 £.; what was the vessel worth?
Ans. 1500 £.

17. If the planet Saturn is 1000 times as large as the earth, and the earth is 7900 miles in diameter, what is the diameter of Saturn? Ans. 79000 miles.

18. Bought a load of hay for $9.17, paying at the rate of $16 per ton; what was the weight of the hay?
Ans. 11 cwt. 1 qr. 23 lbs.

19. From thirty-five thousand take thirty-five thousandths.
Ans. 34999.965.

20. How much hay in 7 loads, each containing 23.571 cwt.? Ans. 164.997 cwt.

21. What is the least number that 3, 5, 8 and 10 will measure? Ans. 120.

22. A man purchased 30 sheep for $82\frac{1}{2}$; what was the price of each sheep? Ans. $2\frac{3}{4}$.

23. One Sessa, an Indian, having invented the game of chess, showed it to his prince, who was so delighted with it, that he promised him any reward he should ask; on which Sessa requested that he might be allowed one grain of wheat for the first square on the chess board, 2 for the second, 4 for the third, and so on, doubling continually to 64, the whole number of squares. Now supposing a pint to contain 7680

of these grains, and one quarter, or 8 bushels, to be worth 27 s. 6 d., it is required to compute the value of the corn.

Ans. 6450468216285 £. 17 s. 3 d. $\frac{3}{32}\frac{7}{76}\frac{1}{8}$ q.

24. There is a number, which if multiplied by $\frac{2}{3}$ of $\frac{4}{5}$ of $1\frac{1}{2}$, will produce 1; what is the square of that number?

Ans. $1\frac{9}{16}$.

25. How many combinations can be made of 2 letters out of the 24 letters of the alphabet? Ans. 276.

26. A vintner has wine at 4 s., at 5 s., 5 s. 6 d., and at 6 s. a gallon; he would make a mixture of 18 galls. so that it may be afforded at 5 s. 4 d. per gallon; how much of each sort must he take?

Ans. { 3 gallons at 4 shillings.
3 " " 5 "
6 " " 5 " 6d.
6 " " 6 "

27. Of what fineness is that composition, which is made by mixing 3 lbs. of silver of 9 oz. fine, with 5 lbs. 8 oz. of 10 oz. fine, and 1 lb. 10 oz. of alloy? Ans. $7\frac{8}{8}\frac{1}{3}$ oz. fine.

28. A and B companied; A put in 50 £. for 4 months—B put in 60 £. for 5 months, at the end of which time they had gained 24 £.; what is each one's share of the gain?

Ans. { A's share = 9 £. 12 s.
B's " = 14 8.

29. What is the square of 930? Ans. 864900.

30. What is the square root of 930? Ans. 30.49590+.

31. What is the square root of 864900? Ans. 930.

32. What is the cube root of 930? Ans. 9.7610+.

33. What is the difference between 1.9185 and 2.73?

Ans. 0.8115.

34. If $\frac{3}{1}\frac{}{8}$ of a ship be worth 273 £. 2 s. 6 d., what are $\frac{6}{3}\frac{}{2}$ of her worth? Ans. 227 £. 12 s. 1 d.

35. Divide $\frac{14}{8}$ by $\frac{7}{6}$. Ans. $1\frac{1}{3}$.

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## SECTION XX.

1. In 7 £, 14 s. 6 d. 1 qr. how many farthings?

Ans. 7417 qrs.

2. In 18 £. 4 s. 11 d. 1 qr., 26 £. 15 s. 3 d., and 8 £. 1 s. 7 d. 3 qrs., how many pounds, shillings, pence and farthings?

Ans. 63 £. 1 s. 10 d. 0 qr.

3. Multiply 29831 by 952. Ans. 28399112.

4. What number is that to which if you add 789 it will become 6350? Ans. 5561.

5. Suppose an estate of 36582 dollars to be divided among 13 sons; how much would each one receive? Ans. $2814.

6. If flour be $6.75 per barrel; how much would 9 barrels cost? Ans. $60.75.

7. If sugar be 12½ cents per pound; how much would 93½ lbs. cost? Ans. $11.68¾.

8. If 263 bushels of wheat cost 86 £. 7 s. 10 d. what is it per bushel? Ans. 6 s. 6¾ d.

9. A began trade June 1st. with 40 dollars, and took B in as a partner, Sept. 8th following, with 120 dollars; December 24th A put in 190 dolls. more, and continued the whole in trade till May 5th following, when their whole gain was found to be 82 dollars; what is each partner's share?

Ans. { A's share $47.065+.
B's " 34.934+.

10. If a weight of 1440 lbs. be placed 1 foot from the prop, at what distance from the prop must a power of 160 lbs. be applied to balance it? Ans. 9 feet.

11. What is the square of 562? Ans. 315844.

12. A merchant has canisters, some holding 5 lbs. some 7 lbs. and some 12 lbs.; how many of each an equal number, can be filled out of 12 cwt. 3 qrs. 12 lbs. of tea?

Ans. 60.

13. There was a farm, of which A owned $\frac{2}{7}$, and B $\frac{5}{12}$; the farm was sold for $1764; what was each one's share of the money? Ans. A's $504, and B's $1260.

14. Suppose a man had put out one cent at compound interest in 1620; what would have been the amount in 1824, allowing it to double once in 12 years? Ans. $1310.72.

15. If the extremes be 4 and 131072, and the ratio 8, what is the whole amount of the series? Ans. 149796.

16. There is a square field containing 90 acres; how many rods in length is each side of the field? and how many rods apart are the opposite corners?

Ans. 120 rods, and 169.7+ rods.

17. There is a circle whose diameter is 4 inches; what is the diameter of a circle 9 times as large? Ans. 12 inch.

18. There is a room plastered, the compass of which is 47 feet, 3 inches, and the height 7 feet, 6 inches; what is the content? Ans. 39 yds. 3 feet, 4 inches, 6″.

19. A bankrupt is indebted to A $780, to B $460, and to C $760; his estate is worth only $600; how must it be divided? Ans. A $234, B $138, and C $228.

20. How much land at $2.50 per acre, must be given in exchange for 360 acres, at $3.75 per acre? Ans. 540.

21. If $400 gain $18 in 9 months, what is the rate per cent. per annum? Ans. 6 per cent.

22. If 7 lbs. of sugar cost $\frac{3}{4}$ of a dollar; what cost 12 lbs.? Ans. $1\frac{2}{7}$.

23. If $\frac{3}{5}$ yards cost $\frac{7}{8}$, what will $40\frac{1}{2}$ yards cost? Ans. $59.062.

24. If a field will feed 6 cows 91 days, how long will it feed 21 cows? Ans. 26 days.

25. Suppose 2000 soldiers had been supplied with bread sufficient to last them 12 weeks, allowing each man 14 oz. per day; but on examination they find 105 barrels, containing 200 lbs. each, wholly spoiled; what must the allowance be to each man, that the remainder may last them the same time? Ans. 12 oz. per day.

26. If the moon move 13° 10′ 35″ in a day, in what time does it perform one revolution? Ans. 27 days, 7 hours, 43 minutes.

27. The interest on a certain note, for 1 year, 9 months, was $49.875; what was the principal? Ans. $475.

28. Bought hats at 4 s. apiece and sold them at 4 s. 9 d. what is the profit in laying out 100 £? Ans. 18 £. 15 s.

29. What is the interest of $486 for 1 year, 3 months, 19 days, at 8 per cent.? Ans. $50.652.

30. A merchant purchased goods for 250 dollars ready money, and sold them again for 300 dollars, payable in 9 months; what did he gain discounting at 6 per cent.? Ans. $37.081.

31. Multiply 1000000 by $\frac{5}{6}$. Ans. $555555\frac{5}{6}$.

32. Multiply $\frac{7}{8}$ by $2\frac{2}{3}$. Ans. $2\frac{1}{3}$.

33. At $\frac{5}{6}$ of a dollar per yard, what cost $6\frac{5}{8}$ yards? Ans. $6\frac{11}{24}$.

34. What is the continual product of 7, $\frac{1}{2}$, $\frac{5}{7}$ of $\frac{3}{8}$ and $3\frac{1}{5}$? Ans. $2\frac{11}{12}$.

35. How many times is $8\frac{5}{9}$ contained in 53? Ans. $6\frac{17}{17}$.

## SECTION XXI.

1. In 622080 cubic inches, how many tons of round timber? Ans. 8.

2. There is a house the roof of which is $44\frac{1}{2}$ feet in length and 20 feet in width, on each of the two sides; if 3 shingles in width cover one foot in length, how many shingles will it take to lay one course on this roof? if 3 courses make one foot, how many courses will there be on one side of the roof? and how many shingles will it take to cover both sides of the roof? First Ans. $133\frac{1}{2}$; 2d Ans. 60; 3d Ans. 16020.

3. There is a room 18 feet in length, 16 feet in width, and 8 feet in height; how many rolls of paper 2 feet wide, and containing 11 yards in each roll, will it take to paper the room? Ans. $8\frac{16}{66}$.

4. A merchant sold goods to the amount of 136 £. 7 s. $6\frac{1}{2}$ d., and received in payment 50 £. 10 s. $4\frac{3}{4}$ d.; how much remained due? Ans. 85 £. 17 s. $1\frac{3}{4}$ d.

5. A note bearing date of Sept. 29, 1816, was paid April 2, 1819; how long was the note at interest?
Ans. 2 years, 6 months, 3 days.

6. Massachusetts being about 72°, and the Sandwich Islands about 155° West longitude, when it is 28 minutes past 6 o'clock, A. M. at the Sandwich Islands; what will be the hour in Massachusetts? Ans. 12 o'clock at noon.

7. From $1\frac{4}{16}$ take $\frac{5}{9}$. Ans. $1\frac{59}{144}$.

8. From $\frac{15}{16}$ take $\frac{4}{7}$. Ans. $\frac{41}{112}$.

9. From $\frac{40}{41}$ take $\frac{5}{6}$. Ans. $\frac{35}{246}$.

10. Involve 8 to the fifth power. Ans. 32768.

11. Involve $\frac{1}{3}$ to the fourth power. Ans. $\frac{1}{81}$.

12. How many barley-corns will reach across the Atlantic ocean, supposing it to be 3000 miles? Ans. 570240000.

13. How many times will a clock tick in 20 years, if it tick at the usual rate of 60 times in a minute?
Ans. 631152000.

14. A father left legacies to his children as follows; to Thomas 75 £. 14 s. 6 d., to William 3 times as much as to Thomas, to Mary $\frac{1}{6}$ as much as to Thomas, and to Sarah as much as to all the others lacking 20 £. 13 s. 8 d.; how much did each receive?

Ans. { William 227 £. 3 s. 6 d.
Mary 12 £. 12 s. 5 d.
Sarah 294 £. 16 s. 9 d.

15. How much wine can be bought for $322.56, at 4 cents a gill? Ans. 1 tun.

16. A farmer sold a grocer 20 bushels of rye, at $0.75 per bushel; 200 lbs. of cheese, at 10 cents per pound; in exchange for which he received 20 gallons of molasses, at 22 cents per gallon, and the balance in money; how much did he receive? Ans. $30.60.

17. Tubes may be made of gold weighing not more than at the rate of $\frac{1}{1625}$ of a grain per foot; what would be the weight of such a tube, which would extend across the Atlantic, considering the distance to be 3000 miles?

Ans. 2 oz. 6 dwt. $3\frac{69}{100}$ gr.

18. What is the cube root of 167284151? Ans. 551.

19. A cannon ball as it leaves the gun, flies about a mile in eight seconds; at this rate, how long would a ball be on its passage from the Earth to the Sun, considering the distance to be 95173000 miles?

Ans. 24 yrs. 46 dys. 7 h. 33 min. 20 sec.

20. A certain usurer lent 90 £. for 12 months, and received for principal and interest 95 £. 8 s.; at what rate per cent. did he receive interest? Ans. 6 per cent.

21. What is the length of a piece of land, which being 33 feet wide, contains an acre? Ans. 80 rods.

22. If a quarter of wheat affords 60 ten-penny loaves, how many eight-penny loaves may be made from it? Ans. 75.

23. Reduce 14 guineas and 75 £. 13 s. $6\frac{3}{4}$ d. to Federal money. Ans. $317.593.

24. A owed B $317.19, for which he gave his note on interest bearing date July 12th, 1797. On the back of the note are the following endorsements, viz.

October 17th, 1797, received in cash $61.10.

March 20th, 1798, received 17 cwt. beef, at $4.33 per cwt.

January 1st, 1800, received in cash $84.

What was due from A to B of principal and interest, September 18th, 1801? Ans. $144.363.

25. Reduce $653\frac{3}{19}$ to an improper fraction. Ans. $\frac{12410}{19}$.

26. Reduce $\frac{12410}{19}$ to a mixed number. Ans. $653\frac{3}{19}$.

27. What is the least number that 9, 8, 15 and 16 will measure? Ans. 720.

28. Suppose a light-house built on the top of a rock—the distance between the place of observation and that part of the rock level with the eye 620 yards—the distance from the top of the rock to the place of observation 846 yards, and

from the top of the light-house 900 yards; the height of the light-house is required. Ans. 76.77 yards.

29. Reduce $\frac{5}{6}$, $2\frac{3}{5}$, and 4, to a common denominator.
Ans. $\frac{25}{30}$, $\frac{78}{30}$, $\frac{120}{30}$.

30. There is an island 50 miles in circumference, and 3 men start together to travel round it; A goes 7 miles per day, B 8, and C 9: when will they come together again, and how far will each travel?
Ans. { They will meet again in 50 days.
A goes 350 miles—B 400—and C 450 miles.

31. Sound uninterrupted, moves about 1142 feet in a second; how long, then, after firing a cannon at Boston, before it will be heard at Quincy, estimating the distance at 10 miles? Ans. $46\frac{134}{171}$ seconds.

32. In a thunder storm, I observed by my watch that it was 6 seconds between the flash and the report; at what distance from me was the explosion? Ans. $1\frac{321}{440}$ mile.

33. The difference of two numbers is 20—and the difference of their squares is 2000; what are the numbers?
Ans. 60 and 40.

34. If 24 yards at Boston make 16 ells at Paris, how many ells at Paris will make 128 yards at Boston? Ans. $85\frac{1}{3}$.

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## SECTION XXII.

1. What is the value of .009943 miles?
Ans. 17 yds. 1 ft. 5.98848 in.

2. Reduce 24 yards to the decimal of a mile.
Ans. .013636+ of a mile.

3. Multiply 79.347 by 23.15. Ans. 1836.88305.

4. A regiment of soldiers, consisting of 976 men, are to be new clothed, each coat to contain $2\frac{1}{2}$ yards of cloth that is $1\frac{5}{8}$ yards wide, and lined with shalloon $\frac{7}{8}$ yard wide; how many yards of shalloon will line them?
Ans. 4531 yds. 1 qr. $2\frac{6}{7}$ nails.

5. Divide $\frac{4}{6}$ by $\frac{15}{7}$. Ans. $\frac{7}{15}$.

6. Multiply $\frac{2}{3}$ of $\frac{3}{5}$ and $\frac{5}{8}$ of $3\frac{3}{7}$ together. Ans. $\frac{23}{84}$.

7. If a footman travel 130 miles in 3 days, when the days are 12 hours long, in how many days, of 10 hours each, may he travel 360 miles? Ans. $9\frac{3}{5}$.

8. The circumference of the earth is about 24877 miles; at what rate per hour is a person at the middle of its surface carried round, one whole revolution being made in 23 hours and 56 minutes? Ans. $1039\frac{154}{359}$ miles.

9. If a gentleman's estate be worth 384 £. 16 s. a year, and the land tax be assessed at 2 s. $9\frac{1}{2}$ d. per pound, what is his net annual income? Ans. 331 £. 1 s. $9\frac{1}{5}$ d.

10. Wanting just an acre of land cut off from a piece which is $13\frac{1}{2}$ poles in breadth, of what length must the piece be? Ans. 11 pls. 4 yds. 2 ft. $0\frac{8}{17}$ in.

11. If an ounce of gold cost 4 guineas, what is the value of a grain? Ans. $2\frac{1}{10}$ d.

12. How many strokes do the clocks of Venice strike in the space of 2 days, which go continually from 1 to 24 o'clock? Ans. 600.

13. Three merchants, A, B and C, made a stock of 700 £., of which A contributed 123 £.—B 358 £.—and C the rest; by trading they gain 125 £. 10 s.; what must each have of it?

Ans. A's share = 22 £. 1 s. 0 d. $2\frac{2}{35}$ qr.
B's = 64 ' 3 ' 8 ' $0\frac{32}{35}$ '
C's = 39 ' 5 ' 3 ' $1\frac{1}{35}$ '

14. The slow or parade step being 70 paces per minute, at 28 inches at each pace, it is required to determine at what rate per hour that movement is. Ans. $1\frac{113}{132}$ miles.

15. What is the ratio of the velocity of light to that of a cannon ball, which issues from the gun with a velocity of 1500 feet per second—light passing from the Sun to the Earth in $7\frac{1}{2}$ minutes? Ans. The ratio of $782222\frac{2}{3}$ to 1.

16. At $3\frac{5}{8}$ £. per cwt., what will $9\frac{2}{3}$ lbs. cost? Ans. 6 s. $3\frac{5}{56}$ d.

17. If 2 oz. of silver cost \$2.24, what cost $\frac{3}{4}$ oz.? Ans. \$0.84.

18. What is the amount of $29\frac{3}{10}$, $374\frac{9}{1000}$, $97\frac{253}{1000}$, $315\frac{4}{1000}$, 27, and $100\frac{4}{10}$? Ans. 942.957009.

19. Divide .012 by .005. Ans. 2.4.

20. What is the interest of \$1000 for 120 years? Ans. \$7200.

21. There is a pole, $\frac{1}{4}$ of which stands in the mud, $\frac{1}{3}$ of it in the water, and the rest of it out of the water; required the part out of the water? Ans. $\frac{5}{12}$.

22. What is the interest of \$1000 for 5 days? Ans. \$0.83+.

23. Supposing two boats start on a river at the same time,

from places 300 miles apart; the one proceeding up stream is retarded by the current 2 miles per hour, while that moving down stream is accelerated the same; if both be propelled by a steam engine which would move them 8 miles per hour in still water, how far from each starting place will the boats meet? Ans. 112½ miles from the lower place, and 187½ from the upper.

24. What is the area of a square field, of which the opposite angles are 70.71 rods apart? and what is the length of each side? Ans. to the last. 50 rods, nearly.

25. If the floor of a square room contain 36 square yards, how many feet does it measure on each side? Ans. 18.

26. What is the area of a triangle, of which the base is 30 rods, and the perpendicular 10 rods? Ans. 150 rods.

27. If the area be 150 rods, and the base 30 rods, what is the perpendicular? Ans. 10 rods.

28. If the perpendicular be 10 rods, and the area 150 rods, what is the base? Ans. 30 rods.

29. What is the area of a circle, whose circumference is 176 rods? Ans. 2464 rods.

30. What is the diameter of a circle, whose circumference is 462 feet? Ans. 147 feet.

31. What is the square root of 964.5192360241? Ans. 31.05671.

32. I would set out, at equal distances, 784 apple trees, so that my orchard may be 4 times as long as it is broad; how many rows of trees must I have, and how many trees in each row? Ans. 14 rows, and 56 trees in each row.

33. There are two circular ponds; the diameter of the less is 100 feet—and the greater is three times as large; what is its diameter? Ans. 173.2+ feet.

34. There is a building 30 feet in length and 22 feet in width, and the eaves project beyond the wall 1 foot on every side; the roof terminates in a point at the centre of the building, and is there supported by a post, the top of which is 10 feet above the beams on which the rafters rest; what is the distance from the foot of the post to the corners of the eaves?—and what is the length of a rafter reaching to the

QUESTIONS.—What is first necessary in obtaining the answer to the 23d example? Will you explain the whole process?

How is the answer to the 34th example obtained? What principles are necessarily understood in order to obtain it?

middle of one side?—a rafter reaching to the middle of one end?—and a rafter reaching to the corners of the eaves?

Answers in order, 20 ft.; 15.62+ ft.;—18.86+ ft.;—and 22.36+ ft.

35. There is a field 800 rods long and 600 rods wide; what is the distance between two opposite corners, or of its diagonal? Ans. 1000 rods.

36. What is the cube of 276? Ans. 21024576.

37. What is the cube root of $\frac{8}{27}$? Ans. $\frac{2}{3}$.

38. How many times does a clock strike in 12 hours? Ans. 78.

39. If the first term be 50, the last term 107, and the number of terms 20, what is the sum of the series? Ans. 1570.

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## SECTION XXIII.

1. There are two farms; one is valued at 3750, and the other at 1500 dollars; what is the difference in the value of the two farms? Ans. 2250 dolls.

2. A man, owing 379 dollars, paid at one time 47 dollars, at another time 84 dollars, at another time 23 dollars, and at another time 143 dollars; how much did he then owe? Ans. 82 dolls.

3. How many inches in 253 feet? Ans. 3036 in.

4. If it is 436 miles from Boston to Washington, how many rods is it? Ans. 139520.

5. What number is that, the factors of which are 4, 7, 6, and 20? Ans. 3360.

6. $\frac{24}{9}\frac{6}{4}$ equal how many? Ans. $384\frac{8}{8}$.

7. How many dollars in 6487 mills? Ans. $6\frac{487}{1000}$.

8. Let 60 cents be divided among three boys, in such a manner, that as often as the first has 3 cents, the second shall have 5 cents, and the third 7 cents; how many cents will each receive? Ans. 12, 20, and 28 cents.

9. Four men traded together on a capital of $3000, of which A put in $\frac{1}{2}$, B $\frac{1}{4}$, C $\frac{1}{6}$, and D $\frac{1}{12}$; at the end of 3 years they had gained $2364; what was each one's share of the gain?

Ans. A's $1182.
B's 591.
C's 394.
D's 197.

10. How many wine gallons in a cask whose bung diameter is 36 inches, head diameter 27 inches, and length 45 inches? Ans. 166.617.

11. If 7412 eggs be put up in 34 baskets, how many in a basket? Ans. 218.

12. A merchant bought a piece of shalloon, containing 34 yards, at 3 s. 4 d. per yard, and sold it at $12\frac{1}{2}$ per cent. loss; how much did he sell it for per yard? Ans. 2 s. 11 d.

13. A is owing B 120 £., whereof one half is to be paid in 3 months, one quarter in 6 months, and the remainder in 9 months; what is the equated time for the payment of the whole? Ans. 5 mo. $7\frac{1}{2}$ dys.

14. How much is the discount of 308 £. 15 s. due in 18 months, at 8 per cent. per annum? Ans. 33 £. 1 s. $3\frac{3}{7}$ d.

15. What is the interest of 57 dolls. 78 cts. for 1 year, 4 months and 17 days, at 4 per cent. per annum?
Ans. 3 dolls. 19 cts.

16. Suppose 120 seamen are provided with 7200 gallons of water for a cruise of 4 months, each month 30 days; how much is each man's share per day? Ans. 2 quarts.

17. If a chest of tea, weighing 79 lbs. neat, cost 32 £. 11 s. 9 d., what is it per pound? Ans. 8 s. 3 d.

18. What is the square root of $51\frac{3}{4}\frac{1}{6}$? Ans. $7\frac{1}{2}$.

19. What is the square root of $6\frac{2}{3}$? Ans. 2.5298+.

20. What is the cube root of $12\frac{1}{2}\frac{8}{7}$? Ans. $2\frac{1}{3}$.

21. There is a cellar that is 12 feet every way, in length, breadth, and thickness or depth; how many solid feet of earth were taken out of it? Ans. 1728.

22. If the solid content of a globe is 10648, what is the side of a cube of equal solidity? Ans. 22.

23. If 673 bushels of rye cost $769.239, what is it per bushel? Ans. $1.143.

24. Suppose I import from France, 12 bales of cloth, containing 10 pieces each, which with the charges there amounted to $360; I pay duty here 92 cents per piece, for freight $12, and portage $1.25; what does it stand me in per piece, and how must I sell it per piece to gain $10 per cent. Ans. $4.433 the price at which it must be sold per piece.

25. What will wages of 25 dolls. per month amount to in a year, at $\frac{1}{2}$ per cent. per month, compound interest?
Ans. $308.389.

26. A wine merchant mixes 12 gallons of wine, at 75 cts.

per gallon, with 24 gallons at 90 cts. and 16 gallons at 10 dolls.; what is a gallon of this mixture worth?

Ans. 92 cents 6 mills.

27. A person lent his friend a sum of money unknown, to receive interest for the sum at 6 per cent. per annum, simple interest, at the end of 12 years he received for principal and interest 860 dollars; what was the sum lent?

Ans. 500 dolls.

28. A certain sum of money is to be divided between 5 men, in such a manner as that A shall have $\frac{1}{4}$, B $\frac{1}{5}$, C $\frac{1}{10}$, D $\frac{1}{20}$, and E the remainder, which is 40 £.; what is the sum?

Ans. 100 £.

29. Christ church, in Boston, has 8 bells; how many changes may be rung on them? Ans. 40320.

30. How many variations will the alphabet admit of?

Ans. 620448501733239439360000.

31. How far may a mountain be seen on level ground, which is a mile high, supposing the eye of the observer elevated 5 feet above the surface? Ans. 91.999 miles.

32. What number is that from which $\frac{3}{7}$ being taken, the remainder will be $\frac{1}{5}$? Ans. $\frac{22}{35}$.

33. What number is that of which $19\frac{3}{13}$ is $\frac{4}{7}$?

Ans. $26\frac{12}{13}$.

34. A line 35 yards long will exactly reach from the top of a fort, standing on the brink of a river, known to be 27 yards broad to the opposite bank; what is the height of the wall? Ans. 22 yards $9\frac{3}{4}$ inches.

35. The river Po is 1000 feet broad, and 10 feet deep, and runs at the rate of 4 miles an hour. In what time will it discharge a cubic mile of water, (reckoning 5000 feet to the mile,) into the sea? Ans. 26 days 1 hour.

36. If a man have an estate of 1000 £. per annum, how much may he spend per day to lay up 60 guineas at the end of the year? Ans. 2 £. 10 s. 2 d. $1\frac{4}{73}$ qrs.

37. If 80 dollars worth of provisions will serve 20 men 25 days; what number of men will the same provisions serve 5 days. Ans. 100.

38. A wall was to be built 700 yards long in 29 days. Now after 12 men had been employed on it for 11 days, it was found that they had completed only 220 yards of the wall. It is required to determine how many men must be

QUESTION. How do you answer the 38th example?

added to the former, that the whole number of them may just finish the wall in the time proposed, at the same rate of working? Ans. 4 men must be added.

39. A younger son received 8400 £. which was $\frac{7}{8}$ of an elder son's fortune; what was the father worth at his death? Ans. 19200 £.

40. Reduce $\frac{2}{5}$ qrs. to the fraction of a pouud. Ans. $\frac{1}{2400}$.

41. What is the value of $\frac{3}{8}$ of a pound sterling? Ans. 7 s. 6 d.

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## SECTION XXIV.

1. If $2\frac{1}{4}$ cwt. of cotton wool cost 11 £. 17 s. 6 d.; what is it per pound? Ans. $11\frac{1}{4}$ d.

2. How many revolutions does the moon perform in 144 years, 2 days, 10 hours, one revolution being in 27 days, 7 hours, 43 minutes? Ans. 1925.

3. A man spends 23 dollars, 69 cents, 5 mills in a year; what is that per day? Ans. 0.064.

4. Bought a hogshead of molasses, containing 60.72 gallons, how much can I sell from it, and save 19.999 gallons for my own use? Ans. 40.721.

5. What number multiplied by $\frac{2}{7}$ will make $\frac{8}{13}$. Ans. $2\frac{1}{26}$.

6. Find the greatest common divisor of 144 and 132. Ans. 12.

7. How much is 12 times $\frac{8}{15}$? Ans. $6\frac{2}{5}$.

8. Four men A, B, C and D, found a purse of money, containing 12 dollars; they agree that A shall have $\frac{1}{3}$, B $\frac{1}{4}$, C $\frac{1}{8}$ and D $\frac{1}{8}$ of it; what must each man have according to this agreement?

Ans. A's share $4.571\frac{3}{7}$.
B's " 3.428$\frac{4}{7}$.
C's " 2.285$\frac{5}{7}$.
D's " 1.714$\frac{4}{7}$.

9. Two men depart both from one place, the one goes North, and the other South; the one goes 7 miles a day, the other 11 miles a day; how far are they distant the 12th day after their departure? Ans. 216 miles.

10. What is the square of 887? Ans. 786769.

11. What is the cube of 887? Ans. 698764103.

12. What is the square root of 887? Ans. 29.78254+.

13. What is the cube root of 887? Ans. 9.60818+.

14. Divide $5205\frac{1}{5}$ by $\frac{4}{5}$ of 91. Ans. $71\frac{1}{2}$.

15. A man spending $\frac{1}{6}$ of a dollar a day in 83 days would spend $\frac{83}{6}$ of a dollar; how many dollars would that be?
Ans. $\$13\frac{5}{6}$.

16. In $\frac{7}{12}\frac{8}{3}$ of a shilling, how many shillings?
Ans. $730\frac{3}{12}$.

17. There are three pieces of cloth, one containing $7\frac{3}{4}$ yards, another $13\frac{5}{8}$ yards, and the other $15\frac{7}{8}$ yards; how many yards in the 3 pieces? Ans. $37\frac{1}{4}\frac{1}{4}$.

18. If the diameter of a circular pond be 147 feet, what is its circumference? Ans. 462 feet.

19. If the diameter of the earth be 7911 miles, how many miles around it? Ans. 24853 miles nearly.

20. Bought 45 barrels of beef, at $3.50 per barrel, among which are 16 barrels, whereof 4 are worth no more than 3 of the others; how much must I pay? Ans. $143.50.

21. What is the value of a plank, at $1\frac{1}{2}$ d. per foot, whose length is 12 feet, 6 inches, and mean breadth 11 inches?
Ans. 1 s. 5 d.

22. How much will a sheet of lead weigh, which is 39 feet, 6 inches long, and 3 feet, 3 inches broad, at $8\frac{1}{2}$ lbs. to the square foot? Ans. $1091\frac{3}{16}$ lbs.

23. How many square feet in a window, 4.25 feet long, and 2.75 broad? Ans. $11\frac{2}{3}$.

24. What is the area of a triangle, whose base is 625, and perpendicular 520 links? Ans. 162500 square links.

25. If a barrel of potash cost $15, how must it be sold to lose 10 per cent.? Ans. $13.50.

26. How many square yards in a triangle, whose sides are 30, 40, 50 feet? Ans. $66\frac{2}{3}$.

27. How much gold of 15, 17, and 22 carats fine, must be mixed with 5 oz. of 18 carats fine, so that the composition may be 20 carats fine? Ans. 5 oz. of 15 car. fine,—5 of 17,—and 25 of 22 car. fine.

28. How many square feet are contained in the plank, whose length is 12 ft. 6 in.—the breadth at the greater end 15 inches, and at the smaller 11 inches? Ans. $13\frac{1}{2}\frac{3}{4}$ ft.

29. A plain of a certain extent having supplied a body of 3000 horse with forage for 18 days, then how many days would the same plain have supplied a body of 2000 horse?
Ans. 27 days.

30. What will 72 yards of cloth cost, at the rate of 9 yards for 5 £. 12 shill.? Ans. 44 £. 16 shill.

31. A person's annual income being 146 £., how much is that per day? Ans. 8 shill.

32. What quantity of shalloon that is $\frac{3}{4}$ yard wide, will line $9\frac{1}{2}$ yards of cloth that is $2\frac{1}{2}$ yards wide? Ans. $31\frac{2}{3}$ yds.

33. Divide $\frac{2}{3}$ of $\frac{1}{3}$ by $\frac{5}{7}$ of $7\frac{7}{5}$. Ans. $\frac{7}{11}$.

34. Multiply 6, and $\frac{2}{3}$ of 5. Ans. 20.

35. If 3 cwt. of tea cost 40 £. 12 s., at how much a pound must it be retailed, to gain 10 £. by the whole?

Ans. $3\frac{4}{336}$ shill.

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## SECTION XXV.

1. Multiply 9876543 by 246. Ans. 2429629578.

2. Four men enter into partnership—each man puts in $4060; what is the whole amount of stock? Ans. $16240.

3. Reduce $\frac{2}{10}$, $\frac{3}{5}$, $\frac{4}{7}$, and $\frac{5}{9}$ to a common denominator.

Ans. $\frac{630}{3150}$, $\frac{1890}{3150}$, $\frac{1800}{3150}$, $\frac{1750}{3150}$.

4. Add 0.00004, 2, 14.6 and 12.6666 together.

Ans. 29.26664.

5. Bought 30 hogsheads of molasses at $600; paid in duties $20.66,—for freight $40.78,—for porterage $6.5,—and for insurance $30.84; if I sell it at $26 per hogshead, how much shall I gain per cent.? Ans. $11.695.

6. What ready money will discharge a debt of $1595, due 5 months and 20 days hence, at 6 per cent.?

Ans. $1541.326.

7. If the area of a triangle be 160, what is the side of a square equal in area thereto? Ans. 12.64+.

8. If the diameter of a circle be 20 feet, what is the diameter of a circle one third as large? Ans. 11.54 feet.

9. How much barley at 40 cents, rye at 60 cents, and wheat at 80 cents per bushel, must be mixed together, that the compound may be worth $62\frac{1}{2}$ cents per bushel?

Ans. $\begin{cases} 17\frac{1}{2} \text{ barley.} \\ 17\frac{1}{2} \text{ rye.} \\ 25 \text{ wheat.} \end{cases}$

10. How much corn at $1 per bushel, must I give for 8 cwt. of sugar, at $9.45 per cwt.? Ans. 75 bu. 2 pks. $3\frac{1}{5}$ qts.

11. How many gallons, wine measure, will a cask hold, whose bung diameter is 20 inches, head diameter 17 inches, and length 24 inches? Ans. 29 galls. $1\frac{252}{294}$ qt.

12. What number, added to the 31st part of 3813, will make 200? Ans. 77.

13. What is the difference between twice 5 and 20, and twice 25? Ans. 20.

14. If the earth be 7911 miles in diameter, and the moon 2180 miles, how many moons would it take to make one earth? Ans. 47.788.

15. Sound, uninterrupted, moves at the rate of 1142 feet per second; if the time between the lightning and thunder be one minute, at what distance was the explosion?

Ans. 12.977+ miles.

16. Add $\frac{1}{4}$ of a week, $\frac{1}{8}$ of a day, $\frac{1}{2}$ of an hour, and $\frac{3}{4}$ of a minute together. Ans. 2 dys. 2 h. 30 min. 45 sec.

17. From 5 weeks take $19\frac{4}{5}$ days.

Ans. 15 dys. 4 h. 48 min.

18. Multiply $9\frac{3}{4}$, $\frac{1}{2}$ of $\frac{2}{5}$, and $12\frac{4}{7}$ continually together.

Ans. $24\frac{18}{35}$.

19. Divide $4204\frac{1}{6}$ by $\frac{7}{8}$ of 112. Ans. $42\frac{529}{588}$.

20. If a man count 100 cents in a minute for 10 hours in a day, in how many days will he count a million of cents?

Ans. $16\frac{2}{3}$.

21. In a certain school $\frac{1}{20}$ of the scholars study Greek, $\frac{1}{10}$ Latin, $\frac{2}{5}$ arithmetic, $\frac{1}{4}$ read and write, and 20 attend to other things; how many in the school? Ans. 100.

22. How much in length, that is $13\frac{7}{8}$ poles in breadth, will make a square acre? Ans. $11\frac{59}{111}$ poles.

23. If 950 soldiers consume 350 quarters of wheat in 7 months, how many soldiers will consume 1464 quarters in 1 month? Ans. 27816.

24. A frigate pursues a ship at 8 leagues distance, and sails twice as fast as the ship; how far must the frigate sail, before she comes up with her? Ans. 16 leagues.

25. A debt of $252 was paid in geometrical progression; the first payment was $4, and the last $128; in what ratio did the payments exceed each other?

Ans. 2, viz. a double ratio.

26. The present worth of a lease of a house is 431 £. 15 s. 7 d. 2.7819 qrs. taken in reversion for 20 years, but not to commence till the end of 8 years, allowing 6 £. per cent. to the purchaser; what is the yearly rent? Ans. 60 £.

27. A person having about him a certain number of dollars, said that $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of them would make 57; how many dollars had he? Ans. 60.

28. It is required to mix several sorts of wine, at 60 cents, 90 cents, and $1.15 cents per gallon, with water, that the mixture may be worth 75 cents per gallon; of how much of each sort must the composition consist?

Ans. { 40 galls. of water.
15 " of wine at 60 cts. per gall.
15 " " at 90 cts. "
75 " " at $1.15 "

29. How many changes are there in throwing 4 dice?
Ans. 1296.

30. A can do a piece of work alone in 13 days, and A and B together in 8 days; in what time can B do it alone?
Ans. 20⅘ days.

31. What number is that, which multiplied by ⅔, produces ¼? Ans. ⅜.

32. What number is that, of which 9 is the ⅔ part?
Ans. 13½.

33. If a town raise a tax of $1850, and the whole town be valued at $37000, what will that be on $1? Ans. $0.05.

34. A usurer put out $75 at interest, and at the end of 8 months received for principal and interest $79; at what rate per cent. did he receive interest? Ans. 8 per cent.

35. What is the cube root of 170953875? Ans. 555.

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## SECTION XXVI.

1. At 5 dolls. 50 cts. per thousand, what will 37 thousand of boards cost? Ans. $203.50.

2. A man failing in business, finds his debts amount to 2119 £. 17 s. 6 d., and he is worth but 1384 £. 18 s. 5¼ d.; if he delivers this to his creditors, how much do they receive on the pound? Ans. 12 s. 6 d.

3. How many bushels of wheat, at $1.12½ per bushel, can I buy for $81.76? Ans. 73.

4. A ship's company of 16 men is on an allowance of 6 oz. of bread per day, when meeting with a vessel from which they are supplied with 2 cwt. of bread; what addition will this make to their daily rations, if their voyage last 28 days?
Ans. 8 oz.

5. From ¾ £. take ¾ of a shilling. Ans. 14 s. 3 d.

6. How many crowns, of $1.10 each, will pay a debt of 82 £. 16 s. 7½ d.? Ans. 251.

7. There is a stone of a cubic form, which contains 389017 solid feet; what is the superficial content of one of its sides? Ans. 5329.

8. Reduce $\frac{5}{8}$ of a penny to the fraction of a guinea.

Ans. $\frac{5}{2088}$.

9. If I give 80 bushels of potatoes at 21 cents per bushel, and 240 lbs. of flax at 15 cents per pound, for 64 bushels of salt, what is the salt per bushel? Ans. $0.825.

10. What is the present worth of 482 dolls. payable 4 yrs. hence, discounting at the rate of 6 per cent.?

Ans. $388.709.

11. A bankrupt, whose effects are $3948, can pay his creditors but 28 cts. 5 mills on the dollar; what does he owe? Ans. $13852.631.

12. A person increased his estate annually by 100 £. more than the $\frac{1}{4}$ part of it, and at the end of 4 years found that his estate amounted to 10342 £. 3 s. 9 d.; what had he at first?

Ans. 4000 £.

13. Divide 1200 acres of land among A, B, and C, so that B may have 100 more than A, and C 64 more than B.

Ans. A 312, B 412, C 476.

14. What number is that, from which if there be taken $\frac{2}{7}$ of $\frac{3}{8}$, and to the remainder be added $\frac{3}{15}$ of $\frac{5}{16}$, the sum will be 10? Ans. $9\frac{73}{84}$.

15. How many variations can be made from the letters in the word Bacchanalia? Ans. 831600.

16. How many changes may be rung on 12 bells?—and what time would it require, supposing 10 changes to be rung in 1 minute, and the year to consist of 365 dys. 5 h. 49 min.?

Ans. { 479001600 changes.
{ 91 yrs. 26 dys. 22 hrs. 41 min.

17. How much wine at 5 s., at 5 s. 6 d., and 6 s. per gallon, must be mixed with 3 gallons at 4 s. per gallon, so that the mixture may be worth 5 s. 4 d. per gallon?

Ans. 3 gall. at 5 s.—6 at 5 s. 6 d.—and 6 at 6 s. per gall.

18. What is the interest of 170 £. for 1½ year, at 5 per cent. per annum? Ans. 12 £. 5 s.

19. C and D hold a piece of ground in common, for which they are to pay 54 £. C put in 28 horses for 27 days, and D 21 horses for 39 days; how much ought each man to pay of the rent?

Ans. { C must pay 23 £. 5 s. 9 d.
{ D " " 30 £. 14 s. 3 d.

20. A man, having $10000, gave away 9 dollars; how many had he left? Ans. $9991.

21. A ship worth 900 £. being entirely lost, of which $\frac{1}{8}$ belonged to S, $\frac{1}{4}$ to T, and the rest to V; what loss will each sustain, supposing 540 £. of her were ensured?

Ans. S will lose 45 £., T 90 £., and V 225 £.

22. What is the square of 878? Ans. 770884.

23. What is the square root of 770884? Ans. 878.

24. What is the square root of $17\frac{3}{8}$? Ans. 4.168333.

25. Reduce $\frac{1}{2}$ pint of wine to the decimal of a hogshead.

Ans. .00238+.

26. What is the value of .4694 lbs. Troy?

Ans. 5 oz. 12 dwts. 15.74 grs.

27. If 180 men in 6 days of 10 hours each, can dig a trench 200 yards long, 3 wide, and 2 deep; in how many days of 8 hours long, will 100 men dig a trench of 360 yds. long, 4 wide, and 3 deep? Ans. 15 days.

28. How many yards of canvass that is an ell wide, will line 50 yards of Say that is 3 quarters wide? Ans. 30 yds.

29. A wall that is to be built to the height of 36 feet, was raised 9 feet high by 16 men in 6 days; how many men must be employed to finish the wall in 4 days, at the same rate of working? Ans. 72 men.

30. A person's annual income being $146; how much is that per day? Ans. $0.40.

31. If a carriage wheel turns round 32870 times in running the distance of 95 miles; how many times does it turn round in running one mile? Ans. 346.

32. How many square feet of boards will it take for the floor of a room 16 feet long, and 15 feet wide, if we allow 12 square feet for waste? Ans. 250 feet.

33. If 500 men consume $102\frac{5}{7}\frac{4}{3}$ barrels of provisions in 9 months, how much will 365 men consume in the same time? Ans. 75 barrels.

34. There is a cistern having a pipe which will empty it in 10 hours; how many pipes of the same capacity will empty it in 24 minutes? Ans. 25 pipes.

35. Add $\frac{3}{4}$ of a yard, $\frac{1}{3}$ of a foot, and $\frac{5}{8}$ of a mile together. Ans. 1100 yards, 2 feet, 7 inches.

36. Ruduce $\frac{3}{5000}$ of an eagle to the fraction of a mill.

Ans. $\frac{3}{5}$.

37. If a staff 5 feet, 8 inches in length, cast a shadow of 6 feet, how high is that steeple whose shadow measures 153 feet? Ans. $144\frac{1}{2}$ feet.

## SECTION XXVII.

1. What is the sum of 27.148,—918.73,—14016,—294-304,—.7138,—and 221.7? Ans. 309488.2918.

2. What is the difference between 16.37 and 800.135? Ans. 783.765.

3. Multiply 8634.875 by 843.7527, retaining only the integers in the product. Ans. 7285699.

4. Reduce 10 weeks, 2 days to the decimal of a year. Ans. .1972602+.

5. What is the value of .42857 of a month? Ans. 1 w. 4 d. 23 h. 59′ 56″.

6. The extremes of a geometrical progression are 1 and 65536, and the ratio 4; what is the sum of the series?

7. At $1\frac{1}{2}$ £. per cwt. what does $3\frac{1}{3}$ lbs. come to? Ans. $10\frac{5}{7}$ d.

8. Suppose a gentleman's income to be $1750 a year, and he spends 19 s. 7 d. per day, one day with another; how much will he save during the year? Ans. 167 £. 12 s. 1 d.

9. A young man received $210, which was $\frac{2}{5}$ of his elder brother's portion, and three times the elder brother's portion was half of the father's estate; what was the value of the estate? Ans. 1890 £.

10. There are two numbers, the one 25, the other the square of 25; what is the square root of the sum of their squares? Ans. 625.499+.

11. What number is that which being multiplied by $\frac{3}{4}$, the product will be $15\frac{3}{4}$? Ans. 21.

12. What is the amount of $1000 for $5\frac{1}{2}$ years, at $4\frac{1}{2}$ per cent. per annum? Ans. $1247.50.

13. A certain town is taxed 39 £. for the building a bridge, the town rent is 900 £. per annum; what must a man of that town pay towards it, whose income is 100 £. per annum? Ans. 4 £. 6 s. 8 d.

14. My correspondent writes me from Manchester in England, that he has paid for goods on my order to the amount of 738 £. 17 s. 6 d. sterling; how much Federal money must I remit to him allowing him $2\frac{1}{2}$ per cent. commission? Ans. $3365.98$\frac{1}{2}$.

15. At 40 cents per yard, how much must be paid for painting the floor of a hall, which is 51 feet long, and 21 ft. wide? Ans. $47.60.

16. A merchant shipped 400 cloths at 12 £. each, for Spain, to have returns one half in wines at 30 £. per tun, and the other half in rice at 28 s. per cwt.; how much of each had he returned? Ans. { 80 tuns of wine. 1714 cwt. 1 qr. 4 lbs. of rice.

17. What is the decimal of $\frac{3}{8}$? Ans. .375.

18. Jacob, by contract, was to serve Laban for his two daughters 14 years; when he had accomplished 10 years, 10 months, 10 weeks, 10 days, 10 hours, 10 minutes, how many minutes had he then to serve? Ans. 1416350 mi.

19. How many minutes from the commencement of the Christian era, to the end of the year 1827?

Ans. 960928920.

20. Washington was born A. D. 1732; how many years old would he have been, had he lived until the end of the year 1827? and how many seconds old, allowing the year to contain $365\frac{1}{4}$ days. Ans. 95 years, 2997972000 seconds.

21. What number is that which being multiplied by $\frac{4}{5}$, the product will be $3\frac{1}{8}$? Ans. $3\frac{23}{32}$.

22. What is the number of square miles on the surface of the earth, supposing its diameter 7911 miles?

Ans. 196612083.

23. What is the number of cubic miles in the earth, supposing its diameter as above? Ans. 259233031435.

24. A man put 20 apples into a wine gallon measure, which he afterwards filled up, by pouring into it 1 quart of water; required the contents of the apples in cubic inches?

Ans. $173\frac{1}{4}$ inches.

25. Reduce $\frac{1}{15}$, $\frac{3}{4}$ of $2\frac{1}{2}$, $\frac{7}{12}$, and $\frac{5}{6}$, to a common denominator. Ans. $\frac{8448}{11520}$, $\frac{21600}{11520}$, $\frac{6720}{11520}$, $\frac{7200}{11520}$.

26. Reduce $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{3}{4}$ of $12\frac{1}{2}$ to a simple fraction.

Ans. $1\frac{1}{64}$.

27. Increase the value of $\frac{3}{7}$ seven fold. Ans. 3.

28. How many cubic feet of wood in a load 6 feet, 7 inches long, 3 feet, 5 inches high, and 3 feet, 8 inches wide?

Ans. 82 feet, 5′. 8″. 4‴.

29. If the diameter of the planet Jupiter is 12 times as much as the diameter of the earth; how many globes of the size of the earth, would it take to make one as large as Jupiter? Ans. 1728.

30. If the sun is 1000000 times as large as the earth, and the earth is 8000 miles in diameter, what is the diameter of the sun? Ans. 800000 miles.

31. A sum of money is to be divided omong 10 persons; the first is to have $10, the second $30, and so on in a threefold proportion; what will the last have? Ans. $196830.

33. There is a certain elm 20 feet in diameter, growing in the centre of a circular island; the distance from the top of the tree to the water in a direct line is 120 feet, and the distance from the foot 90 feet; what is the height of the tree?

Ans. 66.332 feet.

33. What is the solid content of a pyramid whose base is 4 feet square, and the perpendicular height 9 feet?

Ans. 48 feet.

34. There is a cone, whose height is 27 feet, and whose base is 7 feet in diameter; what is its content?

Ans. 346½ feet.

35. How many square yards are there in a circle whose diameter is 3½ feet? Ans. 1.069.

36. How many gallons of water will a cistern contain, its length being 3 feet, 2 inches, breadth 2 feet, 8 inches, and thickness 2 feet, 6 inches, allowing 282 cubic inches to each gallon? Ans. 129$\frac{17}{47}$.

37. What is the area of a triangle, whose base is 18 feet, 4 inches, and height 11 feet, 10 inches? Ans. 108 ft. 5$\frac{2}{3}$ in.

38. What is the area of a square, whose side is 35.25 chains? Ans. 124 acres, 1 rood, 1 perch.

## SECTION XXVIII.

1. A gentleman left his son a fortune, $\frac{4}{15}$ of which he spent in 3 months, $\frac{3}{4}$ of $\frac{5}{6}$ of the remainder lasted him 9 months longer, when he had only 537 £. left; what did his father bequeath him? Ans. 2082 £. 18 s. 2$\frac{2}{11}$ d.

2. What number is that, to which if $\frac{3}{7}$ of $\frac{1}{5}$ of $\frac{1}{3}\frac{2}{1}\frac{9}{3}$ be added the total will be 1? Ans. $\frac{6}{10}\frac{3}{9}\frac{11}{5}\frac{}{5}$.

3. Suppose the battering ram of Vespasian weighed 6000 lbs.; that it was moved at the rate of 24 feet per second, and that this was sufficient to demolish the walls of Jerusalem; with what velocity must a cannon ball, which weighs 42 lbs. be moved, to do the same execution?

Ans. 34285$\frac{5}{7}$ feet per second.

4. If $\frac{7}{15}$ of a ship cost $1163, what is the whole worth?

Ans. $2658.285.

5. A farm containing 125 acres, 3 roods, 27 poles, is rented at $11.50 per acre; what is the yearly rent of the farm?
Ans. $1447.065$\frac{5}{8}$.

6. In a distance of 31 miles, how many times will a wheel whose circumference is 15$\frac{1}{2}$ feet, turn round?
Ans. 10560.

7. A owes B 3475 £., but B compounds with him for 13 s. 4 d. on the pound; what must he receive for his debt?
Ans. 2316 £. 13 s. 4 d.

8. If 19 yards of cloth cost $25.75; what will 435.5 yds. come to? Ans. $590.217$\frac{2}{19}$.

9. Suppose I lend a friend 350 £. for 5 months; he promises the like kindness, but when requested so to do, can spare only 125 £.; how long may I keep it to balance the favour?
Ans. 14 months.

10. If a suit of clothes can be made of 4$\frac{1}{8}$ yards of cloth, 1$\frac{3}{8}$ yards wide; how many yards of coating $\frac{7}{8}$ of a yard wide, will it require for the same person?
Ans. 6 yards, 1 qr. 3$\frac{1}{7}$ nails.

11. Multiply 8496427 by 874359. Ans. 7428927415293.

12. If 95 lbs. Flemish, make 100 lbs. American, how many American lbs. are equal to 550 lbs. Flemish?
Ans. 578$\frac{1}{19}$ lbs.

13. A, B and C put in $720, and gained $540, of which so often as A took up $3, B took $5, and C $7; what did each put in and gain?
Ans. A put in $144 and gained $108.
B " $240 " $180.
C " $336 " $252.

14. A and B companied; A put in the 1st of January 150 £.; but B could not put in until the first of May; what did he then put in, to have an equal share with A at the year's end? Ans. 225 £.

15. If a 15 inch cable require an anchor 35.15625 cwt; what must the circumference of a cable be, for an anchor of 18 cwt.? Ans. 12 inches.

16. If the first term be 3, the common difference 2, and the number of terms 19; what is the sum of the series?
Ans. 399.

17. If the first term be 3, the number of terms 19, and the sum 399; what is the last term? Ans. 39.

18. If the first term be 2, and the ratio 2; what is the 13th term? Ans. 8192.

19. Suppose a ball to be put in motion by a force which

impels it 10 rods the first minute, 8 the second, and so on decreasing by a ratio of 1.25 each minute to infinity; what space would it move through? Ans. 50 rods.

20. A owes B the following sums, with interest at 6 per cent. per annum: $60 for 7 months, $150 for 9 months, $75,50 for 3 months, $365.25 for 8 months, and $510.20 for 5 months; required the amount. Ans. $1198.297.

21. At what rate per cent. will 391 £. 17 s. amount to 449 £. 3 s. $1\frac{1}{2}$ d. in $3\frac{1}{4}$ years?

22. What is the cube root of 742? Ans. 9.05318+.

23. The greatest velocity of a cannon ball, is about 2000 feet in a second. In what time, at this rate, would a ball be on its way from the earth to the sun, admitting the distance to be 100 millions of miles, and the year to contain 365 days 6 hours? Ans. $8\frac{4808}{131149}$ years.

24. How far would 500 millions of guineas reach, when laid down in a straight line touching one another; supposing each guinea to be one inch in diameter?

Ans. 7891 miles, 728 yds. 2 ft. 8 in.

25. What is the value of 6 bushels of coal, at the rate of 1 £. 14 s. 16 d. the chaldron? Ans. 5 s. 9 d.

26. What will be the tax upon 763 £. 15 s. at the rate of 3 s. 6 d. per pound sterling? Ans. 133 £. 13 s. $1\frac{1}{2}$ d.

27. In 86 £. 6 s. $5\frac{1}{4}$ d. how many dollars, cents and mills?

Ans. $230.191.

28. Change $6\frac{3}{4}$ d. to Federal money. Ans. 9 cts. 3 mills.

29. Change 71 cents to shillings, pence, &c.

Ans. 4 s. 3 d. N. E. Currency.

30. A bankrupt owes in all 972 dolls., and he is worth only $607.50; what will a creditor receive on $11.333?

Ans. $7.083.

31. If a family of 9 persons expend 450 dolls. in 5 months; how much would be sufficient to maintain them 8 months, if 5 persons more were added to the family? Ans. $1120.

32. Suppose the Legislature of this State should grant a tax of 7 cents 3 mills on a dollar; what will a man's tax be, who is $142.40 on the list? Ans. $10.395.

33. A gentleman had 7 £. 17 s. 6 d. to pay among his labourers; to every boy he gave 6 d., to every woman 8 d., and to every man 16 d.; and there were for every boy 3 women, and for every woman 2 men; how many of each were there?

Ans. 15 boys, 45 women, and 90 men.

## SECTION XXIX.

1. If a circle be 14 feet in diameter, what is its circumference? Ans. 44 feet.

2. How many square feet in a board 20 feet long, and 2 feet wide at one end, and 1 foot at the other? Ans. 30 feet.

3. What commission must I receive for selling $478 worth of books, at 8 per cent.? Ans. $38.24.

4. A had 41 cwt. of hops, at 30 s. per cwt., for which B gave him 20 £. in money, and the rest in prunes, at 5 d. per lb.; how many prunes did A receive?

Ans. 17 cwt. 3 qrs. 4 lbs.

5. If 9 bushels of wheat cost $13.50, what is it per bushel?

Ans. 1.50.

6. If 3 bushels of oats cost 7 s. 6 d., how much are they per bushel? Ans. 2 s. 6 d.

7. Change 2 s. 6 d. to dollars and cents.

Ans. $0.41$\frac{2}{3}$.

8. A merchant bought a quantity of goods for $734, and sold them so as to gain 21 per cent.; for how much did he sell his goods? Ans. $888.14.

9. A bankrupt is indebted to A $277.33, to B $305.17, to C $152, and to D $105. His estate is worth only $677.50; how must it be divided?

Ans. { A $223.81$\frac{2580}{8395}$. B $246.28$\frac{6154}{8395}$.
{ C 122.66$\frac{6930}{8395}$. D 84.73$\frac{6661}{8395}$.

10. What principal will gain 262 £. 10 s. in 7 years, at 5 £. per cent. per annum? Ans. 750 £.

11. From 476.32 take 84.7697. Ans. 391.5524.

12. A vintner mixed 5 gallons of Canary, at $1.30 per gallon, with 6 gallons of Malaga, at $1.20 per gallon, and 4 gallons of white wine, at $1 per gallon; what is a gallon of this mixture worth? Ans. $1.18.

13. How much wheat at 1 dollar, rye at 66 cents, and barley at 50 cents per bushel, must be mixed to make the mixture worth 75 cents per bushel?

Ans. { 34 bush. of wheat.
{ 25 " of rye.
{ 25 " of barley.

14. A mason has paved a part of a street, which measures in length 236 ft. 8 in., and in breadth 37 ft. 8 in.; how many square yards does it contain? Ans. 990 yds. 4 ft. 5 in.

15. In 36 pieces of linen, measuring equally, there are 927 yards; how many yards in each piece?
Ans. 25 yds. 3 qr.

16. Reduce $\frac{182}{196}$ to its lowest terms. Ans. $\frac{13}{14}$.

17. Reduce $\frac{4}{5}$, $\frac{1}{2}$, $\frac{5}{6}$ and $\frac{7}{8}$ to a common denominator.
Ans. $\frac{384}{480}$, $\frac{240}{480}$, $\frac{400}{480}$, $\frac{420}{480}$.

18. Reduce $19\frac{2}{3}$ to an improper fraction. Ans. $\frac{59}{3}$.

19. Divide 5673.21 by 23.0. Ans. 246.660+.

20. If a man travel 240 miles in 10 days, when the days are 12 hours long, how far will he travel in 12 days, when they are 16 hours long? Ans. 384 miles.

21. Reduce $\frac{18}{112}\frac{17}{}$ to its proper terms. Ans. $16\frac{25}{112}$.

22. How many yards of cloth may be bought for $195.75, of which $9\frac{1}{4}$ yds. cost $11.02? Ans. 168 yds. 3 qrs.

23. Reduce $\frac{1}{4}\frac{2}{}$ of $\frac{5}{6}$ of $\frac{1}{2}$ to a single fraction. Ans. $\frac{15}{48}$.

24. What will $37\frac{1}{2}$ gross of buttons cost, at 13 cents per dozen? Ans. $58.50.

25. Reduce $\frac{6}{7}$ of a pound avoirdupois, to the fraction of a hundred weight. Ans. $\frac{3}{392}$.

26. What is the present worth of 426 dollars, payable in 4 years and 12 days, at 5 per cent.? Ans. $354.519.

27. Reduce $\frac{1}{96}$ of a shilling to the fraction of a farthing.
Ans. $\frac{5}{2}$.

28. A labourer was hired for 60 days upon this condition, that for every day he worked, he should receive 75 cents, and for every day he was idle he should forfeit $37\frac{1}{2}$ cents; at the expiration of the time he received 18 dollars; how many days did he work, and how many was he idle?
Ans. He worked 36 days, and was idle 24.

29. Reduce $\frac{75}{100}$ to a fraction of the same value whose denominator shall be 4. Ans. $\frac{3}{4}$.

30. Reduce $\frac{3}{4\frac{1}{2}}$ to a single fraction.

31. What is the square root of 964.5192360241?
Ans. 31.05671.

32. The eaves of a house are 18 feet high; there is a wharfing in front of the house 12 feet wide; what is the length of a ladder that will just reach from the edge of the wharfing to the eaves of the house? Ans. 21.63 feet.

33. What is the proper quantity of $\frac{3}{16}$ of a barrel?
Ans. 6 galls.

34. Add $6\frac{7}{8}$ of $\frac{9}{10}$, $\frac{4}{7}$ of $\frac{1}{2}$, and $7\frac{1}{2}$ together.
Ans. $14\frac{1284}{2240}$.

35. Add $\frac{3}{4}$ of a mile to $\frac{7}{10}$ of a furlong.

Ans. 6 fur. 28 poles.

36. From $\frac{2}{3}$ of a league take $\frac{7}{10}$ of a mile.

Ans. 1 mi. 2 fur. 16 poles.

37. If $\frac{3}{4}$ of a hogshead of wine cost $30, what will $\frac{1}{8}$ cost?

Ans. $5.

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## SECTION XXX.

1. A person who was possessed of a $\frac{2}{5}$ share of a copper mine, sold $\frac{3}{4}$ of his interest in it for 1800 £.; what was the reputed value of the whole at the same rate? Ans. 4000 £.

2. In the latitude of London, the distance round the earth, measured on the parallel of latitude, is about 15550 miles; as the earth turns round in 23 h. 56 min., at what rate is the city of London carried by this motion from west to east?

Ans. $649\frac{2}{3}\frac{1}{5}\frac{3}{9}$ miles per hour.

3. The quick time or step, in marching, being 2 paces per second, at 28 inches each, then at what rate per hour does a troop march on a route, and how long will they be in arriving at a garrison 20 miles distant, allowing a halt of one hour by the way to refresh?

Ans. { The rate is $3\frac{2}{11}$ miles an hour,
The time $7\frac{7}{9}$ hrs. or 7 hrs. $17\frac{1}{7}$ min.

4. A person, after spending 20 £. more than $\frac{1}{4}$ of his yearly income, had then remaining 30 £. more than the half of it; what was his income? Ans. 200 £.

5. Suppose that I have $\frac{3}{16}$ of a ship worth 1200 £.; what part of her have I left after selling $\frac{2}{5}$ of $\frac{1}{3}$ of my share, and what is it worth?

Ans. { I have left $\frac{37}{240}$.
It is worth 185 £.

6. How many words can be made with 5 letters of the alphabet, supposing 24 letters in all, and that a number of consonants alone will make a word? Ans. 5100480.

7. Two persons, A and B, have both the same income; A saves $\frac{1}{5}$ of his; but B, by spending 50 £. per annum more than A, at the end of 4 years finds himself 100 £. in debt; what does each receive and spend per annum?

Ans. { They receive 125 £. per annum.
A spends 100 £. " "
B " 150 £. " "

8. What number is that, which, being increased by $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of itself, the sum shall be 75? Ans. 36.

9. A grocer would mix teas at 12 s. 10 s. and 6 s. per lb. with 20 lbs. at 4 s. per lb.; how much of each sort must he take to make the composition worth 8 s. per pound?

Ans. { 20 lbs. at 4 s. 10 lbs. at 10 s.
{ 10 lbs. ' 6 s. 20 lbs. ' 12 s.

10. A goldsmith has gold of 16, of 18, of 23, and of 24 carats fine; how much must he take of each, to make it 21 carats fine?

Ans. { 3 of 16, 3 of 23,
{ 2 ' 18, 5 ' 24.

11. What is the amount, at compound interest, of 217£, forborne for $2\frac{1}{4}$ yrs. at 5 per cent. per annum, the interest payable quarterly? Ans. 242£. 13s. $4\frac{1}{2}$d.

12. What is the simple interest of 355£. 15s. for 4 years, at 4 per cent. per annum? Ans. 56£. 18 s. $4\frac{3}{4}$ d.

13. A ship's company take a prize of 1000£. which they agree to divide among them according to their pay and the time they have been on board; now the officers and midshipmen have been on board 6 months, and the sailors 3 months; the officers have 40 s. a month, the midshipmen 30 s. and the sailors 22 s. a month; moreover there are 4 officers, 12 midshipmen, and 110 sailors; what will each one's share be?

Ans. { Each officer must have 23£. 2s. 5d. $0\frac{8\,2}{173}$ qr.
{ Each midship. " 17£. 6s. 9d. $3\frac{6\,9}{173}$ qr.
{ Each seaman " 6£. 7s. 2d. $0\frac{8}{173}$ qr.

14. Two merchants, C and D, make a stock of 120£. of which C put in 75£. and D the rest; by trade they gained 30£.; what must each have of it?

Ans. { C must have 18£. 15s.
{ D " 11£. 5s.

15. If the greatest term be 70, the common difference 3, and the number of terms 21; what is the least term and the sum of the series?

Ans. The least term is 10, and the sum of the series 840.

16. What is the cube root of 1628.1582?

17. What is the square root of $\frac{27}{147}$? Ans. $\frac{3}{7}$.

18. What is the third power of 3.5? Ans. 42.875.

19. What is the value of .3375 acre?

Ans. 1 rood, 14 poles.

20. If $\frac{1}{8}$ of a house be worth 74£. 1s. 3d.; what part of it is worth 250£. 10s.? Ans. $\frac{1}{3}$.

21. What is the difference between $\frac{2}{7}$ of $5\frac{1}{8}$ of a pound, and $\frac{2}{5}$ of a shilling? Ans. $\frac{30}{21}\frac{3}{0}\frac{7}{0}$£. or 1 £. 8 s. $11\frac{3}{35}$d.

22. What is the sum of $\frac{3}{5}$ of a shilling and $\frac{4}{15}$ of a penny? Ans. $\frac{11}{15}^{2}$d. or 7 d. $1\frac{3}{15}$qr.

23. A garrison of 536 men have provisions for 12 months; how long will those provisions last, if the garrison be increased to 1124 men? Ans. $174\frac{64}{1124}$ dys.

24. It is supposed that the wars of Bonaparte, in 20 years, were the cause of 2,000,000 deaths; how many was this per hour, allowing the year to contain 365 dys. 6 hours?

Ans. $11\frac{7}{17}\frac{14}{53}\frac{8}{20}\frac{0}{0}$.

25. How many seconds is it from the birth of our Saviour to Christmas, 1828, allowing the year to contain $365\frac{1}{4}$ dys.?

Ans. 57687292800.

26. A man brings to market 3 loads of wood; the first containing 1 crd. 64 ft. 864 in.; the second, 2 crds. 63 ft. 64 in.; and the third, 1 crd. 60 ft. 931 in.; how much did he bring in all? Ans. 5 crds. 60 ft. 131 in.

27. What will 600 bushels of oats cost, at $\frac{3}{16}$ of a dollar a bushel? Ans. $\$112\frac{1}{2}$.

28. What will 2700 yards of tape cost, at $\frac{1}{8}$ of a dollar per yard? Ans. $\$337\frac{1}{2}$.

29. A merchant bought sugar in a hogshead, both of which weighed 8 cwt. 15 lbs.; the hogshead alone weighed 1 cwt. 1 qr.; what was the cost of the sugar, at $11\frac{1}{2}$ cts. per pound? Ans. $\$86.73\frac{3}{4}$.

30. A wealthy gentleman, at his death, left an estate of $30,000, to be divided among his 5 children, in such a manner that their shares should be to each other as their ages, which are 7, 10, 12, 15, 16 years; what was the share each?

Ans. $3500, $5000, $6000, $7500, $8000.

31. On the first of January, A began trade with $760, and, on the first of February following, he took in B with $540; on the first of June following, he took in C with $800; at the end of the year they found they had gained $872; what was each man's share of the gain?

Ans. A's share, $384.929; B's, $250.71; C's, $236.36.

32. Boston is situated about 6° 40′ E. longitude from the city of Washington; when it is 2 o'clock at Washington, what o'clock is it at Boston?

Ans. 26 min. 40 sec. past 2 o'clock.

33. What number is that which being divided by 15, will make $\frac{1}{20}$? Ans. $\frac{3}{4}$.

34. What is the difference between the compound interest of $500 for 4 years, and the discount of the same sum for the same time? Ans. $34.463.

35. If 120 gallons of water, in one hour, fall into a cistern containing 600 gallons, and by one pipe from the cistern 35 gallons run out, and by another pipe 65 gallons run out, in what time will the cistern be filled? Ans. 30 hours.

36. My correspondent in London informs me that he has paid out on my account 2576 £. sterling; how much in Federal money must I pay for him in New-York, to requite his kindness? Ans. $11448.88$\frac{8}{9}$.

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## SECTION XXXI.

1. Divide 13268.4590 by 29.37.

2. Multiply $9\frac{3}{8}$ of $\frac{16}{20}$ by $\frac{3}{6}$ of $\frac{4}{5}$.

3. How many seconds from the commencement of the Christian era to the 16th of May, 1832?

4. What is the amount of $1763.40, at simple interest, for 3 yrs. 6 months, and 12 dys. at $7\frac{1}{2}$ per cent. per annum?

5. How many solid feet in a pile of wood, 10 ft. 6 in. long, 3 ft. 9 in. wide, and 4 ft. 2 in. high? And how much would it cost at $5,87$\frac{1}{2}$ per cord?

6. What is the interest of $406.927 for 7 days?

7. A merchant owes me $400, to be paid in 46 days, $160 to be paid 140 days, $600 to be paid in 160 days; what is the equated time for the payment of the whole?

8. The whole circumference of the heavenly space (called the zodiac) is divided by astronomers into twelve equal parts, called signs; how many degrees are in one sign?

9. How much is the weight of 12 silver spoons, each weighing 2oz. 15 dwt. 11 grs.

10. Multiply 434687932 by .000426.

11. Divide $\frac{5}{8}$ of $\frac{3}{8}$ of $6\frac{1}{6}$ by $3\frac{3}{8}$ of $\frac{4}{7}$.

12. What is the square root of 949862087?

13. If 7 cwt. of sugar cost 29 £. 8 s.; how much is it per pound?

14. There are three numbers, 17, 19, and 48; I demand the difference between the sum of the squares of the first two and the cube of the third?

15. A merchant sold goods to the amount of $2625, at 25 per cent loss; how much did they cost him? And how much did he lose in all?

16. What is the value of 9736 £. sterling in the New-England currency?—what in the New-York currency?

17. What sum will amount to $6728.26, in $4\frac{3}{4}$ years, at 6 per cent. per annum.

18. Two men depart from the same place, and travel in opposite directions, one at the rate of 27 miles a day, the other 31 miles a day; how far apart would they be at the end of 6 six days? How far at the end of 9 days? How far at the end of 3 days?

19. In one hogshead are 63 gallons; how many gallons in 8 hogsheads? How many quarts? How many gills?

20. A certain town contains 3840 square acres; a tax of 1200 dollars being raised, required the tax upon each acre? Upon 100 acres?

21. What cost 8456 lbs. of tea, at 7 s. 6 d. per lb? At 15 s. 3 d. per lb.? At 18 s. 9 d. per lb.?

22. If 600 cwt. of cocoa cost 19 £. 13 s. 4 d. 3 qr.; what is that per hundred weight?—what per pound?

23. A man divided 13 dollars among a number of poor people, giving $\frac{5}{8}$ of a dollar to each; how many people were there?

24. Multiply .0340067 by $\frac{4}{7}$ of $\frac{5}{8}$.

25. What is the square root of the cube of 456?

26. What is the difference between the square and the cube of 684? Between the square and the cube root of the same number?

27. A merchant bought a piece of cloth, containing 18 yds. 3 qrs.; and sold 6 yds. 2 qrs. 3 n.; how much had he left?

28. A man sold a box of butter for 19 s. 6 d. 3 qr. and in pay received 7 lbs. of sugar, worth 10 d. 3 qr. per lb. and the rest in money; how much did he receive?

29. 3001049068 is how many times 36847?

30. How many lbs. of coffee, at 22 cts. per pound, can I have for 97 lbs. of tea, at 9 s. 3 d. per pound?

31. Bought 130 hhds. of salt, at $\$5.37\frac{1}{2}$ per hhd.; how much did it come to? How many quintals of fish, at $\$2.87\frac{1}{2}$ per quintal, will it take to pay for it?

## SECTION XXXII.

1. The war between England and the United States commenced June 18th 1812, and continued 2 years, 8 months, and 18 days; when was peace concluded?

2. If $\frac{1}{57}$ of a barrel of flour will serve 1 man a day, how many men will $7\frac{3}{57}$ barrels serve? How many men will $43\frac{3}{57}$ barrels serve?

3. At $2\frac{84}{87}$ £. per barrel, what cost 16 barrels of flour? what cost 8 barrels?—what cost $5\frac{1}{2}$ barrels?

4. How much is 34 times $\frac{3}{16}\frac{14}{93}$?

5. Multiply 436 by $\frac{17}{18}\frac{1}{12}$.

6. If $\frac{4}{5}$ of a piece of linen cost $19.50; what will $\frac{1}{3}$ of a piece cost? What will $\frac{1}{8}$ cost? What will $6\frac{3}{8}$ pieces cost?

7. At $\frac{32}{100}$ of a dollar a gallon, what cost 3 hhds. of molasses?

8. A gentleman bought stock in a bank to the amount of 9740 dollars, which was $\frac{1}{30}$ of the whole stock in the bank; what was the whole stock?

9. A man travelled 14 miles in $\frac{4}{5}$ of a day; how far did he travel in $\frac{1}{5}$ of a day? How far would he travel in a day at that rate? How far in $3\frac{1}{2}$ days?

10. Bought goods in England to the amount of 147 £. 19 s. 8 d.; the expenses for getting them on board 4 £. 6 s. 9 d.; for freight $13.40; duties in Boston, 17 per cwt. on the invoice; and other expenses in Boston $23.37$\frac{1}{2}$. How many dollars did the goods cost? How much must they be sold for to gain 15 per cent. on the cost?

11. What is the tax upon 1463 dollars, at 3 cents on a dollar?

12. A merchant bought a hogshead of molasses for 22 dollars, and sold it for 25 dollars; how much per cent. did he gain?

13. What is the present worth of 4362 dollars, payable half in 4 months, and the rest in 8 months, allowing discount at $4\frac{1}{2}$ per cent. per annum?

14. There are two numbers, 73 and 41; what is the difference of their sum and product? What is the product of their squares?

15. A and B traded in company and gained 475 dollars. A put in 930 dollars, B put in so much that he must receive 315 dollars of the gain. I demand how much B put in?

16. There are 1200 dollars to be divided among three men in such proportion that as often as A has 4 dolls. B shall have 7 and C 9; how much must each have?

17. What is the interest of $748.63 for 1 yr. 4 mo. 18 dys. at 5 per cent. per annum?

18. What is the amount of $84.40 for 5 years, 8 days, at 6 per cent. per annum?

19. Divide 3468910687 by $\frac{1}{5}$.

20. Divide $240 between A and B, so that B's share shall be to that of A as 3 to 7.

21. I have expended for a merchant $1039, for which he allows me $3\frac{1}{2}$ per cent. commission. I owed him $345.50; how much is the balance in my favour?

22. How many yards of stuff that is 4 quarters wide, will line a cloak, in which are 4 yards of cloth 7 quarters wide?

23. How much is $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of a dollar?

24. What number is that to which if $\frac{1}{4}$ of itself be added, and from the sum $\frac{1}{5}$ of itself be subtracted, 600 will remain?

25. There is a stone of a cubic form, which in solid feet is 3.375; what is the length of each side?

26. What is the cost of 8 cords of wood, at $6.625 per cord?

27. How many hills of corn can be planted on an acre of ground, and how many rows, allowing 4 feet width for each row, and 4 feet distance between the hills in the rows?

28. What part of 140 is $\frac{6}{8}$ of a unit?

29. What is the amount of $640 for 1 yr. 3 mo. 4 days, at 6 per cent.?—what for 2 years?—what for 10 years?—what for 1 month?

30. If 5 a. 1 rood produce 26 qrs. 2 bush. of wheat, how many acres will produce 47 qrs. 4 bushels?

31. Bought 12 chests of tea, each weighing 4 cwt. 3 qrs. 16 lbs., at 6 £. 3 s. 9 d. per cwt.; what did they cost?

32. From $4\frac{9}{12}$ take $3\frac{8}{17}$.

33. What is the amount of 429, $23\frac{40}{100}$, $382\frac{4}{1000}$, $10\frac{8}{10}$, and $2\frac{7}{10}$?

34. Multiply .87 by .32469.

35. What is $\frac{3}{10}$ of 116?

## SECTION XXXIII.

1. Multiply 2760325 by 37072.

2. A body of troops at the distance of 432 miles from head quarters, received orders to be there in 21 days; what number of miles did they march per day to obey their orders?

3. A lends to B 142 £.; how much is B in debt after A has taken goods of him to the amount of 94 £. 14 s. 7¾ d.?

4. What cost 12 cwt. of sugar, at 4 £. 8 s. 4 d. per cwt.?

5. What cost 20 cwt. of hops, at 3 £. 19 s. 10½ d. per cwt.?

6. If 6352 stones of 3 feet long complete a certain quantity of wall, how many stones of 1 ft. 6 in. long will raise a like quantity?

7. How much a year will 173 acres, 2 roods, 14 poles of land give, at the rate of 1 £. 10 s. 1½ d. per acre?

8. How much will 42 chaldrons, 7 bush. of coal come to, at 3 £. 4 s. 3 d. per chaldron?

9. If a family of 8 persons expend 300 £. in 10 months, how much will serve 15 persons for 12 months?

10. What is the interest of 234 £. 6 s. 8 d., at 3¼ per cent., for 4 yrs. 6 months?

11. Divide $\frac{2}{3}$ of $\frac{1}{3}$ by $\frac{3}{5}$ of $8\frac{1}{3}$.

12. A composition being made of 5 lbs. of tea at 7 s. per pound, 9 lbs. at 8 s. 6 d. per pound, and 14½ lbs. at 5 s. 10 d. per pound; what is a pound of the mixture worth?

13. What is the fourth power of 205?

14. What is the fifth power of 84?

15. What number is that which being multiplied by 7, and the product divided by 6, the quotient may be 21?

16. One being asked his age, said, "If $\frac{3}{5}$ of the years I have lived be multiplied by 7, and $\frac{2}{3}$ of them be added to the product, the sum will be 219;" what was his age?

17. How many different numbers can be made from the following figures: 1220005555?

18. In 38560 six-pences, how many pounds?—how many pence?—how many farthings?

19. Delivered to a purchaser at different times, the following quantities of wheat, at $1.25 per bushel, viz: 46¾ bushels, 72¼, 69¼, 56¾, 73¾ bushels; what is the value of the whole?

20. What cost 95¾ acres of land, at $7.36 per acre?

21. What is the value of 24⅝ bushels of wheat, at $1.37½ per bushel?

22. What is the value of 3 tons of hay, at 75 cts. per cwt.?

23. What cost $86\frac{1}{4}$ bushels of salt, at 46 cents per bushel?

24. There is an oblong field, 20 rods wide, and the distance of the opposite corners is $33\frac{1}{3}$ rods; what is the length of the field?—what is its area?

25. If the floor of a square room contain 44 square yards, how many feet does it measure on each side?

26. What is the area of a circle whose diameter is 18 rods?

27. If the diameter of a circular garden be 240 feet, what is its circumference?—what its area?

28. Within a square, containing 2 square rods, a circle is drawn; what is its circumference?—what its area?

29. A farmer bought a sheep, a cow, and a yoke of oxen, for $94.40; for the cow he gave seven times as much as for the sheep—and for the oxen 4 times as much as for the cow; how much did he give for each?

30. A gentleman divided his fortune among his sons, giving A 9 £. as often as B 5 £., and C 3 £. as often as B 7 £.; C's share was $1537\frac{5}{8}$ £.; what did the whole estate amount to?

31. What is the interest of $4.50 for 4 months?

32. A man having $150, would lay it out in sheep, at $1.62\frac{1}{2}$ apiece; how many can he buy?

33. If $4\frac{5}{9}$ of a bushel of corn cost $\frac{19}{40}$ £., what is it per bushel?

34. What will be the insurance, per annum, of a house and furniture, valued at $10435.75, at $2\frac{1}{2}$ per cent.?

35. Bought a quantity of books for $132, but for cash a discount of 12 per cent. was made; what did the books cost?

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## SECTION XXXIV.

1. A tax on a certain town is $2340.20, on which the collector is to receive $3\frac{1}{4}$ per cent. for collecting; what will he receive for collecting the whole tax at that rate?

2. A merchant bought a quantity of goods for $894.42; how much must he sell them for to gain 12 per cent.?

3. The amount of donations given to the "American Colonization Society," from 1821 to 1828, inclusive, was $83,000; in 1829, $20,295.61; in 1830, $27,209.39; in 1831, $32,000; how much in all has been given?—and what is the rate of increase from 1829 to 1831?

4. What is the amount of $1.50, at compound interest, for 50 years, at 6 per cent. per annum?

5. According to the census of 1820, there were, in the United States, 1,538,064 slaves—the census of 1830 estimates their number to be 2,010,572; what was the increase for 10 years?—and supposing their number is doubled every 20 years, how long before the slave population of our country shall equal the present population of the United States?

6. What will be the premium for insuring a ship and cargo from New-York to Canton, valued at $75643, at $3\frac{1}{2}$ per cent.?

7. E's note of $340.41, was given May 18th, 1810, on interest after 6 months; February 14th, 1812, he paid $75.50; what was due June 18th, 1815?

8. What will be the annual insurance, at $\frac{5}{8}$ per cent., on a store and stock valued at $4560?

9. Bought cloth at $5.33\frac{1}{2}$ per yard; how must I sell it to gain $14\frac{1}{3}$ per cent.?

10. What cost $8\frac{1}{2}$ barrels of flour, at $6.42 per barrel?—Suppose I am to pay for them in work, at 3 s. 6 d. per day, how many days shall I have to work to pay the debt?

11. What is the area of a circle, whose diameter is 104 ft.?

12. What is the circumference of a circle, whose diameter is $\frac{1}{20}$ of a mile?

13. The area of a given triangle is 40 rods, the perpendicular 12 rods, 10 feet; what is its base?—what its hypothenuse?

14. A ship from New-York, bound to the Sandwich Islands, was on her passage 103 days—on account of head winds, she was blown back during 12 days, 720 leagues; estimating the distance to be 11,640 miles, how many leagues per day did she make, considering the day 24 hours?

15. What is the fifth power of $\frac{1}{4}$?

16. Reduce $\frac{1}{2}$, $\frac{3}{5}$, and $\frac{5}{6}$ to fractions having the least common denominator, and add them together.

17. What is the amount of $\frac{1}{3}$ of $\frac{4}{5}$ of a yard, $\frac{5}{6}$ of a yard, and $\frac{4}{7}$ of 3 yards?

18. What is the difference between $\frac{1}{2}$ and $\frac{1}{3}$?—$\frac{3}{5}$ and $\frac{1}{4}$?—$\frac{7}{8}$ and $\frac{4}{5}$?—$\frac{5}{6}$ and $\frac{4}{5}$ of $\frac{3}{4}$?—$\frac{8}{11}$ and $3\frac{4}{5}$?

19. Reduce $\frac{2}{17}$ of a pound Troy, to the fraction of an ounce.

20. Reduce $\frac{4}{5}$ to the fraction of a pound Troy.

21. What is the value of $\frac{4}{15}$ of a ton?

22. Reduce 5 cwt. 3 qrs. 14 lbs. 10 oz. $13\frac{4}{15}$dr. to the fraction of a ton.

23. If a family of 8 persons use 4 bushels of meal in one month, how many bushels will serve them the same time, when there are 14 in the family?

24. If a man perform a journey in 20 days, when the days are 12 hours long, in how many will he perform the same when the days are but 9 hours long?

25. If 1333 £. 6 s. 8 d. Massachusetts be equal to 1000 £. sterling, how much sterling is equal to 6 £. Massachusetts?

26. If 1000 £. sterling is equal to 1333 £. 6 s. 8 d., how much Massachusetts is equal to 5 £. sterling?

27. If a staff 9 ft. 4 in. in length cast a shadow 10 feet, how high is a steeple whose shadow measures 175 ft. 3 in.?

28. What is the cube root of 122023936?

29. There was a certain building raised in 7 months by 150 men, but the same being demolished, it is required to be built in 1 month; how many men will it require to effect it?

30. What is the interest of 8 cents, for 99 years, 11 months and 29 days?

31. A's note of $95.44 was given June 6, 1828, on interest after 30 days; what was due February 10, 1829?

32. How many inches will reach round the earth, supposing it, according to the best calculations, to be 24877 miles? —how many round the sun, considering it to be 800,000 times as large as the earth?

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## SECTION XXXV.

1. What is the area of a parallelogram, of which one side is 15 feet, and the perpendicular 9 feet?

2. The table upon which I am writing is 4 ft. long, and 3 ft. 9 in. in width; how many square inches of surface in the table?

3. How many solid inches in a round stick of timber 10 inches in diameter, and 14 feet long?

4. How many degrees does the earth turn in one hour?

5. What is the value of $167.85 in sterling money?

6. What is the value of 250 ducats of Naples, at $\$7.77\frac{7}{9}$ each, in Federal money?

7. A man when he married was three times as old as his

wife; 15 years after he was but twice as old; what was the age of each when they were married?

8. Divide $\frac{1}{3}$ of 3480 by $\frac{1}{5}$ of 236.

9. A person drew two prizes; $\frac{1}{4}$ of the first, and $\frac{1}{3}$ of the second, was $120, and the sum of the two was $400; what was each prize?

10. How much wood in a pile 4 ft. wide, 3 ft. 8 in. high, and 23 ft. 7 in. long?

11. If $\frac{1}{7}$ of a ship is worth 943 £. 7 s. 8 d., what is the whole ship worth?

12. If $9\frac{2}{5}$ barrels of flour cost 21 £. 3 s. 8 d., what cost $23\frac{5}{8}$ barrels?

13. How much water must be added to a pipe of wine, worth $1.50 per gallon, in order to reduce the price to $1.30 per gallon?

14. What is the compound interest of $9650 for 19 yrs. 7 mo. at 5 per cent.?

15. A farmer mixed 15 bushels of rye, at 64 cents per bushel; 18 bushels of corn, at 55 cents per bushel; and 21 bushels of oats, at 28 cents per bushel: how many bushels were there of the mixture?—what was the whole worth?—what was it worth per bushel?

16. Two men, A and B, traded in company; A put in $460 for 8 months, and B $550 for 5 months; they gained $1750: what was the share of each?

17. Reduce 16 s. 6 d. $4\frac{2}{5}$ q. to the fraction of a £.

18. What is the sum of $\frac{1}{2}$, and $\frac{4}{16}$ and $\frac{4}{8}$.

19. In how long time will 1 dollar gain as much interest as 8 dollars will gain in 3 months?

20. In how long time will 28 dollars gain as much interest as 1 dollar will gain in 157 months?

21. A borrowed of B 17 dollars for 11 months, promising him a like kindness; afterwards B lent A 25 dollars; how long ought he to keep it?

22. How many minutes of a degree does the earth turn in one minute of time?

23. How many square inches in a circle, the diameter 10 inches?

24. The latitude of Turk's Island is 21° 30′ N. and its longitude about the same as that of Boston. The latitude of Boston is 42° 23′ N. How many miles apart are they?

25. What is the sum of $\frac{2}{3}$ of $\frac{3}{8}$ of a pound, and $\frac{1}{5}$ of a shilling?

26. What is the difference between $\frac{2}{3}$ of $\frac{3}{5}$ of a pound, and $\frac{1}{5}$ of an ounce?

27. What is the hour of the day when the time past from noon is equal to $\frac{1}{11}$ of the time to midnight?

28. What is the circumference of a wheel, whose diameter is 4 feet and 6 inches? How many times will it turn round in going 48 miles?

29. A man divided his estate among his children as follows; to the first he gave twice as much as to the third, and to the second, two thirds as much as to the first; the portion of the second and third together was 1500 dollars. What was the share of each?

30. Reduce $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{4}{5}$ of 6 to a simple fraction.

31. Divide $1\frac{1}{3}$ by $\frac{7}{6}$.

32. Two persons talking of their ages, one says $\frac{2}{3}$ of mine is equal to $\frac{3}{4}$ of yours, and the difference of our ages is 10 years; what were their ages?

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## SECTION XXXVI.

1. In 75 equal lots of land there are 10345 acres, 3 roods, and 6 perches; how much is in each lot?

2. If 800 lbs. of sugar cost $279.09, what is it per pound?

3. What is the difference of longitude between Boston in longitude 71° 03′ W. and London, 0° 05′ W.?

4. How many English statute miles is half of the circumference of the earth, each degree containing $66\frac{1}{2}$ statute miles?

5. A jeweller bought an amount of jewels for 130 pounds, and sold them again for 340 pounds, payable at the end of 6 months; what is the gain worth in ready money, discounting 6 per cent. per annum, for the present payment?

6. If sound move 1142 feet in a second, and the space between the flash and the report of a gun be 12 seconds; at what distance was the explosion?

7. Two ships sailed from the same port at the same time; one sailed due east 4 leagues an hour, the other due south 6 leagues an hour; how far from each other were they at the end of one hour? At the end of 2 hours? At the end of 10 hours?

29. Reduce $\frac{1}{126}$ of a cwt. to the fraction of a pound, Avoirdupois.

30. What is the value $\frac{1}{7}\frac{3}{7}$ of a Julian year?

31. Reduce 257 dys. 19 h. 45 m. $52\frac{1}{7}\frac{4}{7}$ sec. to the fraction of a Julian year.

32. What is the sum of $\frac{2}{5}$ of 17 £. $9\frac{5}{8}$£. and $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{1}{4}$£.

33. If 20 lbs. at Boston make 23 lbs. at Antwerp, and 155 lbs. at Antwerp make 180 lbs. at Leghorn; how many at Leghorn are equal to 144 lbs. at Boston?

34. A, B and C freighted a ship with 68900 feet of boards; A put in 16520 ft.; B put in 28750 ft.; C put in the rest; in a storm the captain was obliged to throw overboard 26450 ft.; how much must each sustain of the loss?

35. What is the 6th power of 8? what is the 5th power of 7? the 4th of 6? the 3d of 5? the 2d of 4?

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## SECTION XXXVIII.

1. There is a square field containing 10 acres; what distance is the centre from each angle?

2. What principal, at 10 per cent. compound interest, will amount in 4 years to $8.7846?

3. At 5 per cent. compound interest, in what time will $40, amount to 68.40?

4. A merchant bought a cask of sugar, containing 650 pounds for $48.30; how must he sell it per pound to gain 25 per cent.?

5. What sum put to interest at 6 per cent. will in 10 yrs. 6 months, 20 dys. amount to $340?

6. There is a certain island 1400 miles in circumference, and two men start together to travel around it, in opposite directions; A travels $3\frac{1}{4}$ miles per hour, and B $5\frac{1}{2}$ miles per hour; how long before they will meet? and how far will each have travelled?

7. From $\frac{7}{8}$ of $\frac{4}{5}$ of $5. take $\frac{5}{8}$ of 90 cents, added to $\frac{1}{3}$ of $1\frac{1}{8}$ of a dollar.

8. Multiply $\frac{3}{7}$ of $\frac{5}{8}$ by $\frac{4}{5}$ of $\frac{1}{5}$ of $11\frac{3}{7}$.

9. Divide $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{2}{3}$ by $\frac{1}{8}$ of $\frac{3}{4}$.

10. What multiplier is that, by which multiplying 5357, shall give a product equal to the quotient of the same number divided by .004?

8. In 10425 nails, how many yards? Ells Flemish? Ells English? Ells French?

9. In a square mile, how many square poles? How many square yards? Square feet?

10. In 12096 pints of wine, how many quarts? Gallons? Barrels? Hogsheads? Pipes?

11. What is the greatest common measure of 1224 and 1080?

12. What is the least common multiple of 6 and 8?

13. Reduce $\frac{14}{28}\frac{28}{56}$, to its lowest terms.

14. Diminish the value of $\frac{7}{2}$ four times.

15. How many shingles will it take to cover the roof of a house 40 feet long, allowing the length of the rafters to be 16 feet, 6 inches, and 6 shingles to cover one square foot? What will they cost at $\$1.37\frac{1}{2}$ per thousand?

16. What will a square mile of land cost at $\$30.33\frac{1}{3}$ per acre?

17. A man failing in trade, was able to pay his creditors only $\$0.33\frac{1}{3}$ on the dollar; how much will he pay on $\$3.$? On $\$4.$? On 10.? On $\$1000.$? On $\$4350.33\frac{1}{3}$?

18. There are two places, the one situated in 12° E. longitude, the other 4° E. longitude; what is the difference of time between these two places? When it is 30 minutes past 6 o'clock in the former, what hour is it in the latter?

19. Reduce $645\frac{1}{18}$ to its equivalent improper fraction?

20. Reduce 15 to a fraction, whose denominator shall be 12.

21. Reduce $\frac{2163}{17}$ to its equivalent whole, or mixed number.

22. A grocer mixed 6 lbs. of sugar, worth 10 cents per lb., 9 lbs. worth 14 cents, 18 lbs. worth 18 cents; what is a pound of the mixture worth?

23. Divide $\$950$ among 3 persons, so that their shares may be to each other as 1, 3, 5, respectively?

24. If 248 men, in 5 days of 11 hours each, can dig a trench 230 yards long, 3 wide, and two deep, in how many days, of 8 hours each, will 48 men dig a trench 420 yards long, 5 wide, and 3 deep?

25. Reduce $\frac{3}{12}$ of $\frac{13}{15}$ of $\frac{8}{17}$ of 42360 to a simple fraction.

26. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to fractions having a common denominator.

27. Reduce $6\frac{1}{4}$ furlongs to the fraction of a mile.

28. What is the difference between the compound interest of $\$1850$, for 6 years, and the simple interest of the same sum for the same time?

26. At what distance from the earth would a balloon be suspended between the earth and moon?

27. If a ball strike the ground with a velocity of 56 feet per second; from what height did it fall?

28. If a weight of 1440 lbs. were to be raised with a lever 10 feet long, and the prop fixed one foot from the weight, what power or weight applied to the other end of the lever would balance it?

29. If the least term be 2, the greatest term 4374 and the ratio 3; what is the number of terms?

30. A merchant owed to several persons 1080 dollars; to the greatest creditor he paid 142 dollars, to the next greatest 132 dollars, and so on in Arithmetical Progression; what was the number of creditors, and what did the least creditor receive?

31. If a ship cost 21000, what is $\frac{4}{8}$ of her worth?

32. Reduce 9744 pence to guineas.

33. How many square feet in 4 acres?

34. At 2 s. 6 d. per yard, a trough cost 10 s. 5 d. for painting the inside, which was 1 foot wide, 1 foot, 3 in. deep; what was the length of the trough inside?

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## SECTION XXXVII.

1. What number multiplied by 9, will produce 675?

2. America was discovered by Columbus in 1492, and its Independence declared 1776; how many years elapsed between these two periods? How many hours? How many seconds?

3. What is the difference between 1405 £. 8 s. 6 d. and the amount of 440 £. 14 s. 2 d. and 562 £. 12 s. 8 d.?

4. A merchant began trade with $45683; for six years he gained $3604.37$\frac{1}{2}$ per annum; the next five years he made $1462.87$\frac{1}{2}$ per annum; but the last five years he lost $2560.42 per annum; what was he worth at the 16 years' end?

5. If 42000 men march in a column of 7000 deep, how many march abreast?

6. In 432 £. 14 s. 9 d. 3 qrs. how many farthings?

7. In 274800 farthings, how many pence? Three-pences? Six-pences? Nine-pences? Shillings? Pounds?

8. Add $\frac{3}{4}$ of 15 £. $9\frac{1}{2}$ and $\frac{4}{7}$ of $\frac{3}{8}$£. together.

9. From 4 weeks take $7\frac{3}{4}$ days?

10. How much shalloon, that is $\frac{5}{8}$ yds. wide, will line $6\frac{3}{4}$ yds. of cloth that is $1\frac{2}{3}$ yd. wide?

11. A man owning $\frac{3}{4}$ of a house, sold $\frac{2}{3}$ of his share for 400 dollars; what is the whole house valued at?

12. A, B and C occupy a pasture in common, for which they pay 75 dollars per annum. In this pasture A had 95 oxen for 56 days; B had 30 oxen for 42 days, and C had 60 oxen for 70 days. How much must each pay of the $75?

13. What is the cube root of 84.604519?

14. There is a cubical vessel whose side is 18 inches; what will be the side of a vessel which shall contain 5 times as much?

15. A general has an army of 6724 men; how many must he place in a rank and file to form them into a square?

16. What is the square of 82?

17. A note of $475.62, dated July 4th, 1802, has on its back the following endorsements, viz; Sept. 14, 1804, $342.04; March 12, 1810, $24.41; what is there due Jan. 2, 1812?

18. What is the cube of 82?

19. What is the interest of 1800 dollars, for 1 year, 3 months, and 10 days?

20. A man performed a journey in 6 days, and each day travelled 4 miles farther than on the preceding day, till his last day's travel was 40 miles; how far did he travel in the whole?

21. Two merchants barter; A has 30 cwt. of cheese, at 23 s. 6 d. per cwt. and B 9 pieces of broadcloth, at 3 £ 15 s. per piece; which must receive money, and how much?

22. If a bag of cotton, weighing 8 cwt. 2 qrs. 20 lbs. cost $60.62\frac{1}{2}$; how must it be sold per cwt. to gain $8 per cwt.?

23. What ready money will discharge a debt of 2000 dollars, due 2 years hence, at 4 dollars per cent. per annum, compound interest?

24. A body weighing 30 lbs. is impelled by such a force as to send it 20 rods in a second; with what velocity would a body weighing 12 lbs. move, if it were impelled by the same force?

25. How many inches in 4 miles?

11. Reduce $\frac{8}{1125}$ and $\frac{5}{1875}$ to decimals.

12. Two men talking of their ages, one says $\frac{2}{3}$ of my age is equal to $\frac{3}{5}$ of yours, and the sum of our ages 95; what were their ages?

13. Three men enter into partnership and trade as follows; A put in 150 £. and at the end of 7 months took out 50 £.; five months after he put in 170 £. B put in 205 £. and at the end of 5 months, 110 £. more, but took out 150 £. 4 months after. C put in 300 guineas, at 28 s. each, and when 8 months had elapsed, he drew out 150 £., but 5 months after he put in 500 £. At the expiration of 18 months they had gained 459 £.; what is each one's share of the gain?

14. Reduce 55 min. 37 sec. to the decimal of a day.

15. What is the value of .713 of a day?

16. Bought $7\frac{1}{16}$ yds. of cloth at 34 cents per yard; what did I pay for the whole?

17. If 132 bushels of corn cost $133.33, what is that per bushel?

18. What do I pay a pound for cotton, when 99 lbs. cost 6 £. 12 s. 8 d.?

19. A ship has a leak which will fill it so as to sink her in 12 hours; it has a pump also which will clear her of water in $17\frac{1}{2}$ hours; now if they begin to pump when it begins to leak, in what time will it sink?

20. A cistern is supplied by a pipe which will fill it in 30 minutes; how many pipes of the same size will fill it 6 min.?

21. What is the interest of $3.25 for 16 days?

22. The earth being 360 degrees in circumference, turns round on its axis in 24 hours; how far does it turn in one minute, in the 43 parallel of latitude; the degree of longitude, in this latitude, being about 51 statute miles?

23. How many changes are there on throwing 5 dice?

24. Paid 1012 £. 10 s. for a principal of 750 £. taken in 7 years before; at what rate per cent. per annum did I pay interest?

25. What is the square of 501?

26. A can do a piece of work in 12 dys. and B alone in 19; in what time will both together perform the same?

27. What is the cube of 501?

28. What is the square root of 501?

29. What is the cube root of 501?

30. What debt can be dischared in a year, by weekly

payments in arithmetical progression, the first payment being 1 s., and the last or 52nd payment 5 £. 3 s.?

31. Divide 4109.2351 by 230.409, so that the quotient may contain only four decimals.

32. A person failing in trade, owes in all 977 £.; at which time he has in money, goods, and recoverable debts, 420 £. 6 s. 3½d. Now supposing these things delivered to his creditors, how much will they get per pound?

33. What is the difference between $5\frac{3}{8}$ and $\frac{2}{7}$ of $4\frac{1}{6}$?

34. Add $\frac{4}{9}$ and $\frac{3}{5}$ and $2\frac{1}{6}$.

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## SECTION XXXIX.

1. Multiply 3467.8900 by 4593.

2. What is the cost of 40 hogsheads of molasses, at $33\frac{1}{3}$ cents per gallon?

3. What will $1350 amount to in 3 years, at 5 per cent. per annum, compound interest?

4. A has linen cloth worth 40 cents ready money, but in barter he will have 48 cents per yard; B has broadcloth worth $2. per yard in ready money, which he is to barter with A for his linen; what ought the broadcloth to be per yard?

5. Change 600 £. to dollars.

6. In 936 £. 7 s. 6 d. how many dollars?

7. How much sugar at $9 per cwt. at 7, and at $10 per cwt. must be mixed together that the mixture may be worth $9 per cwt.?

8. If one degree of a great circle of the earth be $69\frac{1}{2}$ English statute miles, how many statute miles is the circumference of the earth, it being 360 degrees?

9. At 18 d. a square foot, what is the value of a plank which is $5\frac{1}{4}$ ft. long, $2\frac{1}{2}$ ft. wide at one end, and $1\frac{1}{2}$ ft. at the other?

10. How many tiles of 9 inches square, will lay a floor 50 feet by 20?

11. In 17 £. 6 s. 8 d. how many eight-pences?

12. If 5 barrels of sugar, containing, each 2 cwt. 1 qr. 15 lbs. cost $93.45, how much is that per pound?

13. If 37 yards of cloth cost $314.87, how much is it per yard?

14. Suppose A pays 5 s. 10 d. for an income of 53 £. per annum; what must B pay, whose income is 132 £. 8 s. 9 d. per annum?

15. Bought 127 pieces of cloth, for which I gave 3589 ells of Holland, at 7 s. 11 d. per ell; what cost a piece of the cloth?

16. A grocer laid out for spices, 560 £. at the following prices, viz. cloves at 4 s. per lb., mace at 7 s., cinnamon at 3 s. nutmegs at 12 s., and pepper at 2 s. per lb.; he had of each an equal quantity; how many pound of each had he?

17. What is the cube root of .000000343?

18. Add $12\frac{1}{2}$, $3\frac{3}{8}$, and $4\frac{3}{4}$ together.

19. Reduce 1 rood, and 30 rods of land to the fraction of an acre.

20. Suppose the distance from Boston to New-York to be 240 miles; A sets out from Boston for N. York and travels 20 miles a day; after A had been gone 2 days, B starts for the same place and travels 40 miles a day. In what time will each arrive at N. York? How far from Boston will B overtake A? How far will A be behind B when the latter reaches N. York?

21. What is the area of a triangle, whose base is $10\frac{1}{2}$ rods and its perpendicular $6\frac{3}{4}$ rods?

22. What is the area of a parallelogram, whose length and breadth are as the base and perpendicular of the above triangle?

23. What is the interest of $4.636 mills, at $4\frac{1}{2}$ per cent. for 90 days?

24. How many stones of 10 inches long, 9 inches broad, and 4 inches thick, would be wanted to build a wall 80 ft. long, 20 ft. high, and $2\frac{1}{4}$ ft. thick?

25. What is the difference between the sum of the squares of 86 and 24, and the square of their sum?

26. A bale of merchandise weighed 300 lbs.; its first cost was 15 £. 4 s. 9 d., duties 2 d. per lb., freight 25 s., and porterage 1 s. 6 d.; how much per pound must I sell it for to gain $4\frac{3}{4}$d. per pound?

27. If 30 yds. of cloth cost 15 £., how much will a Dutch ell come to, if 3 yds. are equal to 4 ells?

28. If I am taxed $10 for 50 acres of land, how much is the tax of the whole town, estimating the land taxed to be 23040 acres?

29. At what rate per cent. per annum, will 620 £. amount to 793 £. 12 s. in 7 years?

30. What principal being put to interest for 5 years, at 5 per cent. per annum, will amount to 1150 £. 14 s. 7½d.?

31. What is the interest of $256 from June 1st to Sept. 10th at 6 per cent. per annum?

32. If I sell coffee at 2 s. 3 d. per lb. and gain 35 per cent. what did I give per pound?

33. In $32422.30, how many pounds, &c.?

34. A man bought a quantity of goods, to pay $\frac{1}{5}$ at the end of every three months till the whole was paid; but it is agreed to make one payment of the whole; what is the equated time?

35. What is the square root of $\frac{12}{10}\frac{2}{5}\frac{5}{5}$?

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## SECTION XL.

1. At $\frac{1}{8}$ of a dollar per lb. how many pounds of figs may be bought for $\frac{2}{3}$ of a dollar?—how many pounds for $1\frac{1}{2}$ dollar?—how many for $3\frac{1}{4}$ dollars?

2. If it take $1\frac{1}{2}$ bushels of oats to sow an acre, how many acres will 18 bushels sow?

3. At $\frac{1}{8}$ of a dollar per bushel, how many bushels of apples may be bought for $\frac{1}{5}$ of a dollar?—how many at $\frac{2}{8}$ of a dollar per bushel?

4. A gentleman lost at sea $4843.67, which was $\frac{1}{9}\frac{1}{8}$ of his whole estate; how much was his whole property worth?

5. A man being asked his age answered that he was 24 years old when he was married, and that he had lived with his wife $\frac{5}{8}$ of his whole life; what part of his whole age is 24 years?—what was his age?

6. A ship of war being taken a prize, the captain received $\frac{1}{57}$ of the prize money. His share amounted to $3487$\frac{13}{105}$. What was the whole prize worth?

7. 37 is $\frac{12}{15}\frac{15}{8}$ of what number?

8. 384 is $\frac{11}{13}\frac{28}{9}\frac{86}{46}$ of what number?

9. There is a pole standing, so that $\frac{2}{7}$ of it is in the water, $\frac{2}{5}$ as much in the mud as in the water, and $9\frac{3}{8}$ ft. of it is above the water; what is the length of the pole?

10. Divide $42\frac{3}{8}$ by $2\frac{1}{7}$

11. A merchant sold a quantity of goods for $983.00 by which he lost 12 per cent. How much did the goods cost him?—and how much did he lose?

12. A man, being in want of money, sells a note of $100, payable in 8 months without interest. How much ready money ought he to receive, when the yearly interest of money is 6 per cent.?

13. If the whole surface of the globe is estimated at 198,000,000 square miles, how many acres on the surface of the globe?—and estimating each acre to be worth $20$\frac{5}{8}$, how much is the whole globe worth?

14. How many acres in a piece of land 183 rods long and 97 rods wide?

15. How many solid feet of wood in a pile 5 ft. 4 in. wide, 3 ft. high, and 20 ft. 10 in. long?—how many cords?

16. Add together the following numbers: $38\frac{14}{75}$, $1386\frac{3}{19}$, 7006, $\frac{97}{250}$, $\frac{406}{2875}$, 8, and $460\frac{43}{85}$.

17. From a piece of cloth containing $47\frac{3}{8}$ yards, a merchant sold $29\frac{9}{25}$; how much had he left?

18. What part of 1 yd. is 3 qrs. and 2 nails?

19. What cost 2 qrs. 3 nls. of broadcloth, at $6.42 per yard?

20. Suppose a certain town is to pay a tax of $6145.88, and the whole property of the town is valued at $153647; what is that on a dollar?—how much must a man pay whose property is valued at $23475.67?

21. A gentleman divided $75 between two men, A and B. A's share was $\frac{4}{9}$ of B's; what was the share of each?

22. If 4 lbs. of gold of 23 carats fine be melted with 2 lbs. of 17 carats fine, what will be the fineness of this mixture?

23. What is the square root of $\frac{1}{2}$ of $\frac{1}{3}$?

24. If $1\frac{1}{4}$ yd. in breadth require $20\frac{1}{2}$ yds. in length to make a cloak, what in length that is $\frac{3}{4}$ yd. wide will be required to make the same?

25. A merchant, owning $\frac{4}{9}$ of a ship, sold $\frac{2}{3}$ of his share for $1040; what was the vessel worth?

26. If $1\frac{1}{3}$ lb. of sugar cost $\frac{7}{15}$ of a shilling, what will $\frac{32}{43}$ of a lb. cost?

27. If $\frac{4}{7}$ oz. cost $\$1\frac{1}{2}$, what costs 1 oz.?

28. Supposing a note of $317.92, dated July 5, 1797, on which were endorsed the following payments, viz. Sept. 13, 1799, $208.04; March 10, 1800, $76; what was due Jan. 1, 1801?

29. A man having compromised with his creditors at $62\frac{1}{2}$ cents on a dollar, what must he pay on a debt of $137.46?

30. If a globe of silver, 1 inch in diameter, be worth $6, what is the value of a globe 1 foot in diameter?

31. There are two planets of equal density; the diameter of the less is to that of the greater as 2 to 9; what is the ratio of their solidities?

32. What is the sum of the first 100 numbers in their natural order, that is, 1, 2, 3, 4, 5, &c.?

33. What is the sum of the arithmetical series 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, &c., to the 50th term?

34. A house being let upon a lease of 10 years, at $75 per annum, and the rent being in arrears for the whole time, what is the sum due at the end of the term, simple interest being allowed at 6 per cent.?

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## SECTION XLI.

1. A man is desirous of having a tract of land, containing 140 acres, 2 roods and 20 rods, in a square; what will be the length of each side?

2. What will be the length of one side of a cubical block, whose content shall be equal to another block 32 feet long, 16 feet wide, and 8 feet thick?

3. If the extremes be 3 and 23, and the number of terms 11, what is the common difference?

4. If the amount of $1 for 20 years, at simple interest, be $2.20, what is the rate per cent.?

5. If the extremes be 3 and 4, and the common difference 6, what is the number of terms?

6. What is the present worth of an annual pension of $120, which is to continue 3 years?

7. What number is that, which, being multiplied by 7, and the product divided by 6, the quotient will be 14?

8. Divide $18\frac{5}{9}\frac{3}{7}$ by $\frac{4}{8}\frac{7}{5}$.

9. $427\frac{3}{8}$ is $\frac{5}{7}$ of what number?

10. If 5 men can mow $72\frac{3}{7}$ acres in $11\frac{2}{3}$ days, how many acres will they mow in $8\frac{4}{5}$ days?

11. If $\frac{3}{4}$ of a yard of cloth cost 6 dollars, what costs $\frac{1}{4}$?—what will a yard cost?

12. $\frac{1}{1}\frac{6}{7}$ is $\frac{3}{4}\frac{8}{2}\frac{7}{5}$ of what number?

13. A has tea which he sells B for 10 d. per pound more than it cost him, and in return B sells A cambrick which cost him 10 s. per yard, for 12 s. 6 d. per yard. The gain on each was in the same proportion; what did A's tea cost him?

14. What sum of money, put at interest at 6 per cent., will gain $45 in 1 year and 6 months?

15. A merchant sold 3 hogsheads of oil for $248.37, by which he lost 25 per cent.; how much did it cost him, and how much did he lose?

16. What is the tax upon 434 £. 12 s. 3 d., at 3 s. 2 d. on the pound?

17. How many yards of carpeting that is $1\frac{3}{4}$ yd. wide, will cover a floor 34 ft. 8 in. long, and 22 ft. 9 in. wide?

18. Of how many variations will 20 letters of the alphabet admit?

19. What time is it in 15° west longitude, when it is 6 o'clock in 15° east longitude?

20. A merchant sold 8 bales of linen, 6 of which contained 15 pieces each, and in each piece were 40 yards; the other 2 bales contained 12 pieces each, and in each piece were 27 yards; what did the whole cost, at $1\frac{1}{4}$ per yard?

21. A gentleman increased his estate annually by 120 £. more than $\frac{1}{8}$ part of it, and at the end of 4 years found that his estate amounted to 12436 £. 4 s. 8 d.; what had he at first?

22. What is the square of 643?

23. What is the cube of 643?

24. What is the square root of 413449?

25. What is the cube root of 265847707?

26. Three merchants, A, B, and C, freight a ship with 340 tuns of wine; A owns 110 tuns, B 97, and C the rest. In a storm the seamen were obliged to throw 85 tuns overboard; how much must each sustain of the loss?

27. Three merchants, A, B, and C, freighted a vessel with $65439.40, bound to Canton to buy teas; A put in $15640.35, B put in $24360, and C the rest. After the vessel had been out 4 months, meeting with a storm, the seamen were obliged to throw overboard $39645; how much did each lose, both in specie and interest?

28. What is the sixth power of 5.03?

29. The extremes of a geometrical series are 1024 and 59049, and the ratio is $1\frac{1}{2}$; what is the sum of the series?

30. What must I allow my correspondent for disbursing on my account $9359.33$\frac{1}{3}$, at $2\frac{1}{2}$ per cent.?

31. What part of 3 d. is the third part of 2 d.?

32. A gamester commenced playing with a certain sum of money; at the first game he doubled his money, at the sec-

ond he lost 10 s., at the next game he doubled what he then had, and at the fourth game he lost 20 s.; twice the sum he then had was as much less than 200 s., as three times the sum would be greater than 200 s.; what sum did he commence playing with?

33. What is the circumference of a wheel, of which the diameter is 9 feet?

34. What is the diameter of a circle of which the circumference is 20.75 feet?

35. What is the area of a parallelogram, of which one side is $14\frac{1}{2}$ feet, the other $5\frac{1}{4}$ rods?—how much would it cost, at 42 cents per square foot?

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## SECTION XLII.

1. Reduce 8 cwt. 3 qrs. 14 lbs. 12 oz. 12 dr. to the decimal of a ton.

2. The massacre at Boston, by the British troops, happened March 5, 1770—and the battle of Lexington, April 19, 1775; how long between?

3. What sum of money must be divided among 14 men, so that each may receive $340?

4. There is a certain number, which, being divided by 7, the quotient resulting multiplied by 3, that product divided by 5, from the quotient 20 being subtracted, and 30 added to the remainder, the half sum shall make 35; can you tell me the number?

5. In 40 £. 14 s. 10 d. 3 qrs., how many farthings?

6. In 345358 half-pence, how many pounds, shillings, &c.?

7. How often will a wheel of 4 ft. 3 in. in diameter turn round in the distance from Boston to Worcester, it being 45 miles?

8. What is the interest of $140.34 for 6 yrs. 4 mo. 8 dys., at $4\frac{1}{2}$ per cent. per annum?

9. How many minutes since the commencement of the present century?

10. A gentleman has an annuity of 2000 £. per annum; how much may he spend daily, that, at the year's end, he may lay up 90 guineas, and give 25 cents a day to the poor of his own town?

11. What is the diameter of a circle, whose area is 314.1592 feet?

12. What is the length of one side of a cubical block, which contains 9261 solid feet?

13. A man is to travel from Boston to a certain place in 9 days, and to go but 5 miles the first day, increasing every day by an equal excess, so that the last day's journey may be 37 miles; what is the daily increase?

14. A owes B a certain sum, to be discharged in a year, by paying 6 d. the first week, 18 d. the second, and thus to increase every weekly payment by a shilling, till the last payment should be 2 £. 11 s. 6 d.; what is the debt?

15. $384\frac{4}{13}$ is $\frac{1}{3}$ of what number?

16. How many times is $2\frac{2}{7}$ contained in $13\frac{2}{5}$?

17. What is the interest of $1147 for 8 hours, at 6 per cent.?—for 8 days?—for 8 months?—for 8 years?

18. If a man perform a journey in 35.3 days, when the days are 11.374 hours long, in how many days will he perform it, when the days are 9.13 hours long?

19. Goliah, the Philistine, is said to have been $6\frac{1}{2}$ cubits high, each cubit being 1 ft. 7.16 English inches; what was his height in English feet?

20. Sixteen persons gave in charity to a poor man in such a manner as to form an arithmetical series; the last gave 65 cents, and the whole sum was $5.60; what did each give less than the other, from the last down to the first?

21. If the first term be 5, in geometrical progression, and the ratio 3, what is the seventh term?

22. If the first term be 2, the ratio 3, and the sum of the series 6560, what is the last term?

23. What is the square root of $30\frac{1}{4}$?

24. The ratio being 4, the number of terms 6, and the sum of the series 4095, what is the greatest term?

25. In what time will 391 £. 3 s. $1\frac{1}{4}$ d. be doubled, at $4\frac{1}{2}$ £. per cent. per annum?

26. A man exchanges 760 gallons of molasses, at $37\frac{1}{2}$ cts. per gallon, for $66\frac{2}{3}$ cwt. of cheese, at $4 per cwt.; how much will be the balance in his favour?

27. What is the decimal equivalent to $\frac{1}{3\frac{1}{5}}$?

28. What number is that, which being divided by $\frac{1}{5}$, the quotient will be $30\frac{1}{6}$?

29. If 22 yards of broadcloth cost 21 £. 9 s., what is the price per yard?

30. What premium must I pay for the insurance of my house against loss by fire, at the rate of $\frac{1}{2}$ per cent., if my house be valued at $2475?

31. A merchant bought a quantity of goods at Boston for $750, and paid $75 for their transportation; he sold them so as to gain 24 per cent. on the whole cost; for what did he sell them?

32. Sixteen times six, are how many times eleven and one half?

33. If light passes from the sun to the earth in 8.2 minutes, in what time would it pass from the sun to the Georgium Sidus, it being 1803930416.66 English miles?

34. What is the cube root of 1803930416.66?

35. $\frac{2}{3}$ of 15 is $\frac{2}{3}$ of what number?

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## SECTION XLIII.

1. 20 men built a certain bridge in 60 days, but it being carried away in a freshet, how many men can rebuild it in 8 days?

2. A merchant bought cloth at $3.50 per yard, and sold it at $4.25 per yard; how much did he gain per cent.?

3. Bought 126 gallons of wine for $110; how much water must be added to reduce the first cost to 75 cents per gallon?

4. What number is that, to which if its $\frac{1}{2}$ and $\frac{1}{4}$ be added, the sum will be 84?

5. Suppose one of those meteors, called fire-balls, to move parallel to the earth's surface, and 50 miles from it, at the rate of 20 miles per second; in what time would it move round the earth?

6. If sound, uninterrupted, moves 1142 feet in a second, in how long time would a cannon ball, moving at the same rate, be in passing from the Sun to Saturn, its distance being 903957657.5 English miles?—how long to Jupiter, its distance being 492912533.33?

7. What is the length of a parallelogram, whose area is 12 acres, and the breadth 40 rods?

8. What is the breadth of a parallelogram, whose area is 15 acres, and the length 100 rods?

9. What is the area of a parallelogram, whose length is 80 rods and breadth 40 rods?

10. How many square inches in a board 12 ft. 8 in. long, and 1 ft. 9 in. in width?

11. A man wishes to put down a circular cistern which shall contain 2000 gallons; the place where he wishes to put it, will allow it to be only 4 feet in diameter; how long must the cistern be in order to hold the above named number of gallons?

12. What is the area of a triangle, whose base is 14 rods, and its perpendicular 35 rods?

13. What is the base of a triangle, whose area is 14 acres, and perpendicular 40 rods?

14. What is the perpendicular of a triangle, whose area is 17 acres, and base 15 rods?

15. What is the hypothenuse of a triangle, whose base is 10 rods and perpendicular 40 rods?

16. What is the solid content of a sphere, whose diameter is 4 inches?

17. What is the solid content of a sphere, whose circumference is $12\frac{3}{4}$ inches?

18. What is the sum of 12 terms of the series 1 $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, &c.?

19. The first term of a decreasing arithmetical series is 10; the common difference $\frac{1}{3}$, and the number of terms 21; what is the sum of the series?

20. A triangular battalion, consisting of thirty ranks, in which the first rank is formed of one man only, the second of 3, the third of 5, and so on; what is the strength, or number of men, in such a battalion?

21. What is the sum of 99 terms of the odd numbers, 1, 3, 5, 7, 9, &c.?

22. A person making his will, gave to one child $\frac{13}{20}$ of his estate, and the rest to another. When these legacies came to be paid, the one turned out to be 1200 £. more than the other; what estate did the testator leave?

23. A man after spending 20 £. more than $\frac{1}{4}$ of his yearly income, had then remaining 30 £. more than the $\frac{1}{2}$ of it; what was his income?

24. What is the compound interest of $500.50, for 3 yrs. 6 mo. at $5\frac{1}{4}$ per cent. per annum?

25. What is the simple interest of 32 £. 5 s. 8 d. for 7 years, at $3\frac{1}{4}$ per cent. per annum?

26. What is the cube root of .75?

27. What is the cube root of .001?

28. What is the 8th power of 4?

29. Multiply 480.14936 by 2.72416, retaining only four decimals in the product?

30. If 3 cwt. of tea cost 40 £. 12 s., at how much a pound must it be retailed to gain 10 £. on the whole?

31. What is the amount of $298.59, from the 19th of May 1829, to the 11 of August 1831, at 8 per cent. per annum?

32. If $3\frac{1}{4}$ yds. of cloth that is $1\frac{1}{5}$ yd. wide be sufficient to make a cloak, how much must I have which is $\frac{4}{5}$ yd. wide to make another of the same size?

33. If $25\frac{7}{8}$ s. will transport an cwt. $145\frac{1}{4}$ miles, how far may $6\frac{1}{2}$ cwt. be carried for the same money?

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## SECTION XLIV.

1. If $29\frac{3}{4}$ buy $59\frac{1}{2}$ yards of cloth, what will $60 buy?

2. If $\frac{4}{5}$ yd. cost $\frac{3}{4}$ of $\frac{5}{8}$ of a dollar, what will 50 yds. cost?

3. An usurer put out $175 at interest; at the expiration of 7 months, he received for principal and interest $182; at what rate per cent. per annum did he receive interest?

4. A certain town is taxed $3150; the whole property of the town is estimated at $600000; there are 200 polls, which are taxed $70 each. A's property is valued at 1400, and he pays for 2 polls; what will be A's tax?

5. How many times is $\frac{5}{16}$ contained in $\frac{23}{31}$?

6. What is the interest of $936 for 11 months, at 4 per cent?

7. A and B bartered; A had 12 cwt. of sugar worth 8 cents per pound, for which B gave him $1\frac{3}{4}$ cwt. of cinnamon; what did B value his cinnamon at per lb.?

8. In $1196.565, how many pounds, &c. sterling?

9. If $\frac{3}{4}$ of a hogshead of rice cost $25, what will $\frac{1}{8}$ cost?

10. I would insure an adventure which cost me $9570; how much must I insure so as to cover both principal and premium, when I pay for insurance 3 per cent.?

11. Bought 127 pieces of cloth, for which I gave 3587 ells of Holland, at 7 s. 11 d. per ell; what cost a piece of that cloth?

12. Change 536 £. sterling to the currency of N. England.

13. What number is that to which one fourth of itself being added, and from the sum one tenth of itself be subtracted, 603 will remain?

14. There are two numbers, their product is 1058, and their multiplicand 46; what is the multiplier, the sum of the factors, and the difference between the sum of the cubes of the factors and the square of the product?

15. What is the decimal of $\frac{3}{152}$?

16. Divide 12.169825 by 3.14159, so that the quotient may contain 5 decimals?

17. If $\frac{3}{5}$ of a yard cost $\frac{7}{12}$£., what will $\frac{6}{15}$ of an English ell cost?

18. Shipped for Barbadoes 500 prs. of stockings at 3 s. 6 d. per pair, and 1650 yds. of baize at 1 s. 3 d. per yard, and have received in return 348 gallons of molasses at 6 s. 8 d. per gallon, and 750 lbs. of indigo at 1 s. 4 d. per lb.; what remains due upon my adventure?

19. How many yds. of cloth 3 yrs. wide are equal in measure to 30 yards, 5 qrs. wide?

20. Suppose 840 men are in a garrison, and have provisions sufficient to last them 3 months; how many men must leave the garrison in order to have the provisions last those who remain 7 months?

21. How many times is $27\frac{3}{7}$ contained in $1605\frac{5}{8}$?

22. What part of $3.840\frac{3}{10}$ is $\frac{4}{37}$?

23. How many times is $42\frac{25}{107}$ contained in 1677?

24. If $\frac{3}{5}$ yd of broadcloth cost $\$6.87\frac{1}{2}$; what will $\frac{3}{15}$ cost?

25. Add together $112\frac{1}{2}$, $311\frac{2}{3}$, and $1000\frac{3}{4}$.

26. A man bought 100 yards of cloth in arithmetical progression; for the first yard he gave 4 cents, and for the last 301 cents; what was the common increase of the price on each succeeding yard?

27. A debt is to be discharged at 11 payments in arithmetical series, the first to be $5, and the last $75; what is the whole debt?—the common difference between the payments?

28. A man having $\frac{3}{4}$ of a dollar, expended $\frac{2}{3}$ of it; how much had he left?

29. At $\frac{4}{5}$ of a dollar for building 1 rod of stone wall, how many rods may be built for 34?

30. There are 3 pieces of cloth, one containing $10\frac{4}{5}$ yds. another $14\frac{3}{4}$ yds., and the other $22\frac{1}{6}$ yds.; how many yards in the three pieces?—and how much would all come to at $\$3.87\frac{1}{2}$ per yard?

31. What will be the premium for insuring a ship and cargo from New-York to Canton, valued at $46560, at $3\frac{1}{2}$ per cent.

32. How much ready money must be paid for a note of $48, due 18 months hence, discounting at the rate of 5 per cent?

33. There is a circle whose area, or superficial content is 8592 feet; what will be the length of the side of a square of equal area?—what will be the length of the base of a triangle, whose perpendicular is 62 ft., which shall be of the same area as the square above?

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## SECTION XLV.

1. Two men, A and B, traded in company: A furnished $\frac{5}{8}$ of the stock, and B $\frac{3}{8}$; they gained $944; what is each one's share of the gain?

2. If $\frac{3}{8}$ of a barrel of flour were to be divided equally among 4 men, how much would each have?

3. If 3 yards of cloth cost $9\frac{4}{5}$ dollars, what will 7 yards cost at that rate?

4. What is $\frac{137}{2865}$ of 378.648?

5. What is $\frac{533}{1869}$ of 3?

6. What sum of money, put at interest at 9 per cent., will amount to $152 in 3 years and 4 months?

7. Find the value of .468 of a square foot in square inches.

8. How many minutes of a degree does the earth turn in one minute of time?

9. If the diameter of the sun is 112 times as much as the diameter of the earth, how many globes like the earth would it take to make one as large as the sun?

10. At $3\frac{5}{8}$ s. per cwt. what will $9\frac{3}{8}$ lbs. cost?

11. A rough stone was put into a vessel, whose capacity was $19\frac{1}{2}$ quarts, which was then filled with $4\frac{1}{3}$ quarts of water; what was the cubic content of the stone?

12. There is a screw, whose threads are $\frac{1}{3}$ of an inch apart; if it be turned by a lever 10 feet long, what weight will be balanced by 130 pounds power?

13. What is the solid content of a round stick, 25 feet long and 9 inches through?

14. If a cubic foot of water weigh 1000 oz. avoirdupois, and the weight of mercury be $13\frac{1}{2}$ times greater than of water, and the height of the mercury in the barometer (the weight of which is equal to the weight of a column of air on the same base, extending to the top of the atmosphere) be 30 inches; what will be the weight of the air upon a square foot?—a square mile?—upon the whole earth?

15. Being about to plant 5292 trees equally distant in rows, the length of the grove is to be 3 times the breadth; how many of the shorter rows will there be?

16. What is the sum of $\frac{2}{5}$ of 17 £., $9\frac{5}{8}$ £., and $\frac{3}{8}$ of $\frac{1}{2}$ of $\frac{1}{4}$ £.?

17. From $\frac{1}{5}$ of $2\frac{1}{3}$ $. take $3\frac{1}{3}$ cents.

18. In 28 lbs. avoirdupois, how many pounds Troy?

19. What is the content of a grindstone, whose diameter is 32 inches, and its thickness 3 inches?

20. If 1464 quarters of wheat be used by 27816 soldiers in a month, in what time will 950 soldiers consume 350 qrs.?

21. What is the cube of 542?

22. What is the cube root of the 2d power of 542?

23. What is the present worth of $56.20, payable in 1 yr. 8 months, discounting at 6 per cent.?—at $4\frac{1}{2}$ per cent.?—at 5 per cent.?

24. A man paid $4.25 interest, at the rate of 6 per cent., at the end of 1 yr. 4 months; what was the principal?

25. At $34.40 for the transportation of 62 cwt. 34 miles, what is that per ton?

26. If $\frac{5}{8}$ of a ton of potashes cost $604.5, what is that per ton?

27. What is the value of the infinite series $\frac{2}{100} + \frac{2}{10000}$, &c., descending by the ratio 100, or, which is the same, the repeating decimal .020202 &c.?

28. A man agrees to serve a farmer 20 years without any other reward than 1 kernel of corn for the first year, 10 for the second year, and so on in ten-fold ratio, till the end of the time; what will be the amount of his wages, allowing 1000 kernels to a pint, and supposing he sells his corn at 50 cents per bushel?

23. If a man earn $75 in 5 months, how long must he work to earn $460?

30. A goldsmith melted together 3 oz. of gold 20 carats fine, and 5 oz. 22 carats fine; what is the fineness of the mixture?

31. What the proportions of sugar, at 8, 10, and 14 cents per pound, will compose a mixture worth 12 cents per pound?
32. What is the square of 432?
33. What is the cube of 432?
34. What is the square root of 186624?
35. What is the cube root of 80621568?

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## SECTION XLVI.

1. Boston is situated 6° 40′ east longitude from the city of Washington; when it is 4 o'clock at Washington, what is the hour at Boston?
2. A man received, for interest on a certain note, at the end of 2 yrs. 4 months, $45; what was the principal, allowing the rate to have been 4½ per cent.?
3. A owes B $450, to be paid as follows: ⅓ in 4 months, ¼ in 5 months, ⅛ in 5 months, and the rest in 6 months; what is the equated time?
4. If a man travel 340 miles in 14 days, travelling only 8 hours in a day, how many miles will he travel in 12 days, if he travel 11 hours in a day?
5. A man puts out $2 at 6 per cent. simple interest, which in 1 year amounts to $2.12, in 2 years to $2.24, and so on in arithmetical progression, with a common difference of $0.12; what would be the amount in 99 years?
6. What ready money will purchase the reversion of a lease of $60 per annum, to continue 6 years, but not to commence till the end of 3 years, allowing 6 per cent. compound interest to the purchaser?
7. If a house measures within the walls 52 ft. 8 in. in length, and 30 ft. 6 in. in breadth, and the rafters ¾ of the breadth of the building, what will its roofing come to at 10 s. 6 d. per square foot?
8. What is the government tonnage of a double decked vessel, of the following dimensions: length 82 ft. 3 in., breadth 24 ft. 3 in., and the depth 12 ft. 1½ inch?
9. In August 1821, bills on London bore at Boston a premium of 8½ per cent.; what is the amount of a bill of exchange of 250 £. at this rate, in Federal money, and what is the value of a pound sterling at this course of exchange?

10. If 12 yards of cloth are sold at 15 s. per yard, and there is 7 £. 10 s. loss per cent. in the sale, what is the prime cost of the whole?

11. A has linen cloth, at 30 c. per yard, ready money—in barter, 36. B has 3610 yards of ribbon, at 22 c. per yard, ready money, and would have of A $200 ready money, and the rest in linen cloth; what rate does the ribbon bear in barter per yard, and how much linen must A give B?

12. Given the last term 39, the number of terms 19, and the sum of the series 399, to find the common difference.

13. Given the common difference 2, the last term 39, and the sum of the series 399, to find the first term and the number of terms.

14. If the ratio be 4, the number of terms 6, and the greatest term 3072; what is the sum of the series?

15. Suppose the density of the moon 464, and that of the earth 392.5: Required the proportion between the quantity of matter in the earth and that of the moon, allowing the earth's diameter to be 7964.12, and the moon's 2180 miles, and supposing the earth a complete sphere?

16. If a weight of 1440 lbs. were to be raised with a lever 10 feet long, and the prop fixed one foot from the weight, what power or weight, applied to the other end of the lever, would balance it?

17. There is a cubical vessel whose side is 2 feet; what is the side of a vessel, which shall contain 3 times as much?—4 times as much?—10 times as much?

18. In what time will a musket ball, dropped from the top of a steeple 484 feet high, reach the ground, falling 16 ft. the first second, 3 times as far the second, 5 times as far the third, and so on?

19. A bullet, discharged from a gun perpendicularly into the air, was found to return to the earth in just 12 seconds; how high did it ascend?

20. If 65 and 56 are $\frac{1}{3}$, what is $\frac{1}{5}$ of the same number?

21. A and B have between them a number of guineas, which are to be so divided, that the sum of their squares may be 208, and the difference of their squares 80; supposing A's the greater number, how many has he more than B?

22. A, B and C, are to share 100 £. in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ respectively; but C dying, it is required to divide the whole sum properly between the other two.

23. Paid 1012 £. 10 s. for a principal of 750 £. taken in 7 years before; at what rate per cent. per annum did I pay interest?

24. Suppose there are 4 companies, in each of which there are 9 men; it is required to find how many ways 4 men may be chosen, one out of each company.

25. How many different numbers can be made of the following figures: 1220005555?

26. A and B began to play together with equal sums of money; A first won 20 guineas, but afterwards lost back $\frac{2}{3}$ of what he then had, when B had 4 times as much as A; what sum did each begin with?

27. What is the 30th power of 4?

28. A person after spending $\frac{1}{3}$ and $\frac{1}{4}$ of his money, has yet remaining 160 £.; what had he at first?

29. How much gold of 15, 17, and 22 carats fine, must be mixed with 5 oz. of 18 carats fine, so that the composition may be 20 carats fine?

30. If 18 ells of stuff that is $\frac{3}{4}$ yd. wide cost 39 s. 6 d., what will 50 ells, of the same quality, cost, being a yard wide?

31. The diameters of two concentric circles being 10 and 6, required the area of the ring contained between their circumferences.

32. What is the area of a triangle, whose base is 20 rods, and perpendicular 14 rods?

33. What is the area of a parallelogram, whose base is 12 rods, and perpendicular 40 feet?

34. The sun's diameter is 883217.58 English miles; Jupiter's is 89170.81; Saturn's 79042.35; Georgium Sidus's 35109; Mercury's 3222.48; Venus's 7687.85; Earth's 7964.12; Mars's 4189.69; and the Moon's 2180. Required the comparative magnitude between each of these bodies and the Earth.

# PART III.

## EXPLANATION OF CHARACTERS.

= The sign of equality: as 12 d.=1 shilling, signifies that 12 pence are equal to one shilling.

+ Sign of addition: as 5+7=12, that is 5 added to 7 is equal to 12.

— Sign of subtraction; as 12—7=5; that is, 12 decreased by 7 is equal to 5. Read 12 minus 7, or 12 less 7 equal to 5.

× Sign of multiplication: as 8×4=32; that is, 8 multiplied by 4 is equal to 32.

÷ or 5)30 Sign of division: as 30÷5=6; that is, 30 divided by 5 is equal to 6.

$\frac{240}{16}$ Numbers placed in the manner of fractions, also denote division. The numerator, or upper number, is the dividend, and the denominator, or lower number, is the divisor: thus $\frac{240}{16}$ is the same as 240÷16=15.

: :: : Sign of proportion; thus, 2 : 4 :: 8 : 16; that is, as 2 is to 4, so is 8 to 16.

∺ Sign of Geometrical progression.

9—2+6=13; that is, the difference between 9 and 2 added to 6 is equal to 13. The horizontal line over the 9 and 2 is called a *vinculum*. It connects all numbers over which it is drawn.

12—$\overline{3+4}$=5 Signifies, that the sum of 3 and 4 taken from 12 leaves, or is equal to 5.

$\surd$ Prefixed to any number signifies that the square root of that number is required.

$\sqrt[3]{}$ Prefixed to any number signifies that the cube root of that number is required.

## MONEY, WEIGHTS, MEASURES, &c.

Before proceeding to the succeeding sections, the learner should make himself familiar with the following tables of Money, Weights, and Measures, &c. By doing this, he will greatly facilitate his future progress, and render the exercises more pleasant and interesting.

### 1. *Federal Money.*

| | | | Marked. | Mills. |
|---|---|---|---|---|
| 10 Mills | make one | Cent | m. c. | 10= 1 cent. |
| 10 Cents | make one | Dime | d. | 100= 10= 1 dime. |
| 10 Dimes | make one | Dollar | $ | 1000= 100= 10= 1 dollar. |
| 10 Dollars | make one | Eagle | E. | 10000=1000=100=10=1 Eagle. |

The right hand division of this table shows the number of mills, cents, &c. contained in a dime, &c.; thus 10000 mills, 1000 cents, or 100 dimes, or 10 dollars are equal to 1 eagle.

Three kinds of metal are coined in the United States, gold, silver, and copper. The coins are the following:—

Gold. Eagle, Half Eagle, Quarter Eagle.

Silver. Dollar, Half Dollar, Quarter Dollar, Dime, Half Dime.

Copper. Cent, Half Cent.

### 2. *English Money.*

| | | | | |
|---|---|---|---|---|
| 4 Farthings | make one | Penny, | marked | qrs. d. |
| 12 Pence | make one | Shilling, | " | s. |
| 20 Shillings | make one | Pound, | " | £. |

Farthings.
4= 1 Penny.
48= 12= 1 shilling.
960=240=20=1 Pound.
A *groat* is 4 d.

### 3. *Troy Weight.*

| | | | | |
|---|---|---|---|---|
| 24 Grains | make one | Pennyweight, | mark'd | grs. pwt. |
| 20 Pennyweights | make one | Ounce, | " | oz. |
| 12 Ounces | make one | Pound, | " | lb. |

Grains.
24= 1 Pennyweight.
480= 20= 1 Ounce.
5760=240=12=1 Pound.

*Note.*—By this weight are weighed gold, silver, jewels, electuaries, and all liquors.

An ounce of gold is divided into 24 parts, called carats; and an ounce of silver into 20 parts, called pennyweights; therefore, to distinguish the fineness of metals, such gold as will lose nothing by fire, is said to be 24 carats fine. But if it lose 1 or more carats by fire, it is called 23 carats fine, or less, according to the loss sustained.

Silver, which loses nothing by fire, is 12 ounces fine. If it lose 1 or more pennyweights in a pound by trial, it is said to be 11 oz. 19 pwts. fine, or less as the case may be.

Alloy is some base metal, with which gold or silver is mixed to abate its fineness: 22 carats of gold, and 2 of copper are the standard of gold coin in England; and 11 oz. 2 pwts. of fine silver melted with 18 pwts. of copper, make the standard of silver coin.

### 3. *Avoirdupois Weight.*

| | | | |
|---|---|---|---|
| 16 Drams | make one | Ounce, | marked dr. oz. |
| 16 Ounces | | Pound, | lb. |
| 28 Pounds | | Quarter of a hundred weight, | qr. |
| 4 Quarters | | Hundred wt., or 112 pounds, | cwt. |
| 20 Hundred weight | | Ton, | T. |

Drams.
16= 1 Ounce.
256= 16= 1 Pound.
7168= 148= 28= 1 Quarter.
28672= 1792= 112= 4= 1 Hundred wt.
573440=35840=2240=80=20=1 Ton.

By this weight are weighed all coarse and drossy goods, and metals, except gold and silver.

| | |
|---|---|
| 12 particular things make | 1 dozen, |
| 12 doz. | 1 gross, |
| 144 doz. | a great gross, |
| 20 | 1 score. |

| | |
|---|---|
| A barrel of pork weighs | 200 lbs. |
| " beef | 200 |
| " Flour | 196 |
| " Soap | 256 |
| " Anchovies | 30 |
| " Raisins | 112 |
| A Firkin of Foreign Butter | 56 |
| " " Soap | 94 |
| A quintal of fish | 112 |
| A Punch of foreign Prunes | 1120 |
| A Tother of foreign Lead | 19½ cwt. |
| A Stone of Iron, shot, or horseman's wt. | 14 lbs. |
| " Butcher's Meat | 8 |

| | |
|---|---|
| A gallon of Train Oil | 7½ lbs. |
| " Molasses | 11 |
| A Tod is | 28 |
| A Weigh | 182 |
| A Sack | 364 |
| A Last | 4368 |

*Note.*—175 Troy ounces are just equal to 192 Avoirdupois ounces; and 175 Troy pounds are equal to 144 Avoirdupois. 1 lb. Troy=5760 grs.; and 1 lb. Avoirdupois=7000 grains.

### 5. *Apothecaries Weight.*

| | | | |
|---|---|---|---|
| 20 Grains | make one | Scruple, | marked gr. ℈ |
| 3 Scruples | | Dram, | ʒ |
| 8 Drams | | Ounce, | ℥ |
| 12 Ounces | | Pound. | lb. |

Grains.
20= 1 Scruple,
60= 3= 1 Dram.
480= 24= 8= 1 Ounce.
5760=288=96=12=1 Pound.

All weights used by Apothecaries above grains are Avoirdupois. The Apothecaries' ounce and pound are the same as the Troy, only differently divided.

### 6. *Cloth Measure.*

| | | | |
|---|---|---|---|
| 2 Inches and one fourth | make one | Nail, | marked in. na. |
| 4 Nails, or 9 inches | | Quarter of a yard, | qr. |
| 4 qrs. of a yard, or 36 inches | | Yard, | yd. |
| 3 qrs. or 27 in. | | Ell Flemish, | E. Fl. |
| 5 qrs. or 45 in. | | Ell English, | E. E. |
| 6 qrs. or 54 in. | | Ell French, | E. Fr. |
| 4 qrs. 1 in. and one fifth, or 37 inches and one fifth | | Ell Scotch, | E. Sc. |
| 3 qrs. and two thirds | | Spanish Var. | |

Scotch and Irish linens are bought by the English or American yard, which is the same, and Dutch linens by the Ell Flemish; but are sold in America by the American yard, and in England by the Ell English. The Scotch and Irish linens are sold there as in America. The Scotch allow one English yard in every score of yards.

### 7. *Long Measure.*

| | | | |
|---|---|---|---|
| 3 Barley corns | make one | Inch, | marked bar. in. |
| 12 Inches | | Foot, | ft. |

| | | |
|---|---|---|
| 3 Feet | Yard, | yd. |
| 5½ Yards, or 16½ ft. | Rod, Perch, or Pole, | pol. |
| 40 Rods | Furlong, | fur. |
| 8 Furlongs | Mile, | mile. |
| 69½ Statute miles, nearly, | Degree of the circle of the Earth, | deg. |
| 360 Degrees | Great circle of the Earth. | |

In measuring distances the following measures are commonly used.

| | | |
|---|---|---|
| $7\frac{92}{100}$ Inches | make one | Link. |
| 25 Links | | Pole. |
| 100 Links | | Chain. |
| 10 Chains | | Furlong. |
| 8 Furlongs | | Mile. |

Bar. corns 3= 1 Inch.
36= 12= 1 Foot.
108= 36= 3= 1 Yard.
594= 198= 16½= 5½= 1 Pole.
23760= 7920= 660= 220= 40=1 Furlong.
190080=63360=5280=1760=320=8=1 Mile.

Inches. $7\frac{92}{100}$=1 Link.
198= 25= 1 Pole.
792= 100= 4= 1 Chain.
7920=1000= 40=10=1 Furlong.
63360=8000=320=80=8=1 Mile.

| | | |
|---|---|---|
| 60 geometrical miles | make | one degree. |
| 4 inches | | a hand. |
| 5 ft. | | a geometrical pace. |
| 6 points | | a line. |
| 12 lines | | an inch. |
| 6 ft. | | a French toise or fathom. |

By Long Measure are measured lengths, or distances without regard to breadth.

## 8. *Time.**

| | | | |
|---|---|---|---|
| 60 Seconds | make one | Minute, | marked s. m. |
| 60 Minutes | | Hour, | h. |
| 24 Hours | | Day, | d. |
| 7 Days | | Week, | w. |
| 4 Weeks | | Month, | m. |
| 13 Months | | Julian Year, | yr. |

* By the Calendar, the year is divided in the following manner:

Thirty days hath September,
April, June, and November;
February, twenty-eight alone,
And all the rest have thirty-one.

When the year of our Lord can be divided by 4 without a remainder, it is Biasextile, or Leap Year, in which February has 29 days. Example: 1832 can be thus divided.

The civil solar year of 365 days, being short of the time, by 5 h. 48 m. 48 sec.

Seconds. 60= 1 Minute.
3600= 60= 1 Hour.
86400= 1440= 24= 1 Day.
604800=10080=168= 7=1 Week.
2419200=40320=672=28=4=1 Month.
sec. min. h. d. h. w. d. h.
31,557,600=525960=8766=365 6=52 1 6=1 Julian Year.

## 9. *Motion.*

60 Seconds make one Prime minute, marked ″
60 Minutes Degree, °
30 Degrees Sign, s.
12 Signs, or 360 degrees { The whole great circle of the Zodiac.

Seconds. 60= 1 Minute.
3600= 60= 1 Degree.
108000= 1800= 30= 1 Sign.
1296000=21600=360=12=Zodiac.

The Zodiac is the great circle of the sphere, containing the 12 Signs, or constellations, through which the Sun appears to pass annually.

## 10. *Land, or Square Measure.*

144 Inches make one Square Foot.
9 Feet Yard.
30¼ Yards, or 272¼ Feet Pole.
40 Poles Rood.
4 Roods, or 160 Rods, or 4840 Yards Acre.
640 Acres Mile.

Inches. 144= 1 Foot.
1296= 9= 1 Yard.
39204= 272¼= 30¼= 1 Pole.
1568160= 10890= 1210= 40= 1 Rood.
6272640= 43560= 4840= 160= 4=1 Acre.
4014489600=27878400=3097600=102400=2560=640=1 Mile.

## 11. *Solid Measure.*

1728 Inches make one Foot.
27 Feet Yard.
40 Feet of round Timber, or 50 Feet of hewn Timber, Ton or Load.
128 Feet (or a pile 8 ft. long, 4 wide, and 4 high) Cord of Wood.

---

occasioned the beginning of the year to run forward in the season, nearly one day in four years. Julius Cæsar therefore ordered, that one day should be added to February, every fourth year, by reckoning the 24th day twice; and because this day was the sixth, (sextilis,) before the kalends, or first of March, there were two of these sextiles in this year, from which the name *Bissextile* is derived. The year thenceforward being reckoned at 365 d. 6 h. was called the Julian year.

## 12. *Wine Measure.*

| | | | |
|---|---|---|---|
| 2 Pints | make one | Quart, | marked pts. qts. |
| 4 Quarts | | Gallon, | gal. |
| 10 Gallons | | Anchor of Brandy, | anc. |
| 18 Gallons | | Runlet, | run. |
| 31½ Gallons | | Half a Hogshead, | ½ hhd. |
| 42 Gallons | | Tierce, | tier. |
| 63 Gallons | | Hogshead, | hhd. |
| 2 Hogsheads | | Pipe or Butt, | P. or B. |
| 2 Pipes | | Tun | Tun. |

Cubic Inches. 28¾ = 1 Pint.
57½ = 2 = 1 Quart.
231 = 8 = 4 = 1 Gallon.
9702 = 336 = 168 = 42 = 1 Tierce.
14553 = 504 = 252 = 63 = 1½ = 1 Hogshead.
19404 = 672 = 336 = 84 = 2 = 1½ = 1 Puncheon.
29106 = 1008 = 504 = 126 = 3 = 2 = 1½ = 1 Pipe.
58212 = 2016 = 1008 = 252 = 6 = 4 = 3 = 2 = 1 Tun.

Spirits, Perry, Cider, Mead, Vinegar, and Oil are measured by Wine Measure.

## 15. *Ale or Beer Measure.*

| | | | |
|---|---|---|---|
| 2 Pints | make one | Quart, | marked pts. qts. |
| 4 Quarts | | Gallon, | gal. |
| 8 Gallons | | Firkin of Ale in London, | A. Fir. |
| 8½ Gallons | | " Ale or Beer | |
| 9 Gallons | | " Beer in London, | B. fir. |
| 2 Firkins | | Kilderkin, | kil. |
| 2 Kilderkins | | Barrel, | bar. |
| * 1½ Barrel or 54 gals. | | Hogshead of Beer, | hhd. |
| 3 Barrels, or 2 hhds. | | Butt, | butt. |

BEER.

Cubic In. 35¼ = 1 Pint.
70½ = 2 = 1 Quart.
282 = 8 = 4 = 1 Gallon.
2538 = 72 = 36 = 9 = 1 Firkin.
5076 = 144 = 72 = 18 = 2 = 1 Kilderkin.
10152 = 288 = 144 = 36 = 4 = 2 = 1 Barrel.
15228 = 432 = 216 = 54 = 6 = 3 = 1½ = 1 Hogshead.
30456 = 864 = 432 = 108 = 12 = 6 = 3 = 2 = 1 Butt.

ALE.

Cubic Inches. 35¼ = 1 Pint.
70½ = 2 = 1 Quart.
282 = 8 = 4 = 1 Gallon.
2256 = 64 = 32 = 8 = 1 Firkin.
4512 = 128 = 64 = 16 = 2 = 1 Kilderkin.
9024 = 256 = 128 = 32 = 4 = 2 = 1 Barrel.
13536 = 384 = 192 = 48 = 6 = 3 = 1½ = 1 Hogshead.

* Sometimes 2 barrels or 72 gallons.

Milk is sold by the Beer quart.

In Massachusetts, the Mackerel, or other fish Barrel, must contain 30 gallons. In New-York and Connecticut, the Shad and Salmon barrel must contain 200 lbs.

In England a barrel of Salmon or of Eels is 42 gallons; and a barrel of Herrings is 32 gallons. In Ireland, the gallon for measuring all liquids must contain $217\frac{6}{10}$ cubic inches.

### 16. *Dry Measure.*

| | | | |
|---|---|---|---|
| 2 Pints | make one | Quart, | marked pts. qts. |
| 4 Quarts | | Gallon, | gal. |
| 2 Gallons | | Peck, | pk. |
| 4 Pecks | | Bushel, | bu. |
| 8 Bushels | | Quarter, | qr. |
| 4 Quarters | | Chaldron, | ch. |
| $4\frac{1}{2}$ Quarters | | Chaldron in London | |
| 5 Quarters | | Wey | wey. |
| 2 Weys | | Last | last. |

*Note.*—The Wey and Last are but little used in this country.

Cubic inches.

$268\frac{4}{5}$ = 1 Gallon.
$537\frac{7}{5}$ = 2 = 1 Peck.
$2150\frac{2}{5}$ = 8 = 4 = 1 Bushel.
$17203\frac{1}{5}$ = 64 = 32 = 8 = 1 Quarter.
86016 = 320 = 160 = 40 = 4 = 1 Wey.
172032 = 640 = 320 = 80 = 10 = 2 = 1 Last.

By this measure any goods, such as corn, fruit, seed, &c. are measured.

A Winchester bushel is $18\frac{1}{2}$ inches in diameter, and 8 inches deep.

## SECTION I.

[Sufficient illustrations of NOTATION were given in Sec. 1. Part I.]

## SECTION II.

[The attentive learner will find sufficient assistance in understanding NUMERATION, by Sec. 2. Part I.]

## SECTION III.

### FUNDAMENTAL RULES.

The whole of arithmetical science depends on four primary *rules*, to acquire a knowledge of which is the first business of the learner. The *fundamental* rules are of course to be first considered in this part of our work.

### SIMPLE ADDITION.

SIMPLE ADDITION has for its object the uniting of several numbers, of the same denomination, into one number.* It is founded on the self evident truth, that "the whole is equal to the sum or amount of all its parts." As; 5 added to 7 produces the number 12. Hence we may see the reason for giving the following

#### RULE.

Write the numbers to be added under each other, in such a manner, that units may stand under units, tens under tens,

* Addition is only an abbreviation of the formation of numbers by the successive union of units. For instance, if it be required to add 5 to 7, seven is taken for a starting point, and five units are added to it, one at a time; thus 7 and 1 are 8, and 1 are 9, and 1 are 10, and 1 are 11, and 1 are 12. This is the course which the *child* universally pursues in his *first* attempts at adding numbers; but by practising in this way he soon learns the amount of any two of the small numbers. The application of this process to large numbers is made easy, by the very nature of numbers; it being just as easy to add seven tens and five tens, as it is to add the same number of units. The same is true of hundreds, thousands, millions, &c. (*Lacroix.*)

&c. First find the number of units in the right hand column, and if it be less than ten, write the sum at the foot of the column; if it be ten or more than ten, reserve the tens to be reckoned in with the other tens in the next column, and write the *excess* of units. Proceed in the same way with the columns of tens, hundreds, &c. till you come to the left hand column, under which write the full sum.

EXAMPLES *for illustrating the Rule.*

1. I paid 35 dollars for a cow, 74 dollars for a horse, and 152 dollars for a carriage. How much money did the whole cost? If I write them thus:

35 dollars, price of the cow;
74 dollars, price of the horse;
152 dollars, price of the carriage,
261 Ans.

I have the units standing under units, and tens under tens, &c. I am prepared to learn the amount of the whole by adding the right hand column, the amount of which is eleven. This is one more than ten, therefore I write 1 in the place of units, and add the one ten to the five tens in the next column. After adding the tens together, I find the amount is 16, which is one hundred, and six tens over. I write 6 under the tens, and add the one hundred to the next figure which is in the place of hundreds. I write 2 under the column to which I added the one hundred, and find the number of dollars expended is 261. I shall obtain the same result if I write 152 and add to that 74, which will make the amount 226, and then add to this the other number 35. The amount is 261.

2. "A merchant bought ten barrels of cider for 35 dollars; 7 barrels of flour for 42 dollars; a hogshead of molasses for 33 dollars; a chest of tea for 86 dollars; and 3 hundred weight of sugar for 24 dollars. What did the whole amount to?

| | |
|---|---|
| Cider | 35 |
| Flour | 42 |
| Molasses | 33 |
| Tea | 86 |
| Sugar | 24 |
| | 220 |

In this example there are five numbers to be added, and each number has two figures. First add together all the units. 4 and 6 are 10, and 3 are 13, and 2 are 15, and 5 are 20 units or two tens. These tens may be added with the other tens in the next column, and as there is no excess of units, we place a cipher under the unit column. Then the 2 tens which are reserved and 2 are 4,

and 8 are 12, and 3 are 15, and 4 are 19, and 3 are 22 tens or 2 hundred and 2 tens, which are placed to the left hand of the cipher; and thus the whole sum is made out 220."

3. Add together 527, 2519 and 9812.

| | |
|---|---|
| | 527 |
| | 2519 |
| | 9812 |
| 12 | 858 |

In this example the sum of the units is 18; we write down the 8 units, and reserve the ten to be joined with those of the next column, which thus increased contains 5 tens, which we write down, and add the next column, which amounts to 18 hundreds. We write down the 8 hundreds, and carry the thousand to the left hand column, which thus increased amounts to 12; we write down the 2 thousands under the column, and place the 1 ten thousands, one place to the left hand; i. e. we write the full number. (*Lacroix.*)

The reserving of the tens, hundreds, &c. and adding them with the other tens, hundreds, &c. is called *carrying*, and is more fully illustrated by the following example, furnished by Mr. Colburn.

4. "A merchant had all his money in bills of the following description, one dollar bills, ten dollar bills, hundred dollar bills, thousand dollar bills, &c.; each kind he kept in a separate box. Another merchant presents three notes for payment, one 2673 dollars, another 849 dollars, and another 756 dollars. How much was the amount of all the notes; and how many bills of each sort does he pay, supposing he paid it with the least possible number of bills?

| Thous. | Hunds. | Tens. | Ones. |
|---|---|---|---|
| 2 | 6 | 7 | 3 |
| | 8 | 4 | 9 |
| | 7 | 5 | 6 |
| 4 | 2 | 7 | 8 |

The first note would require 2 of the thousand dollar bills, 6 of the hundred dollar bills, 7 of the ten dollar bills, and 3 of the one dollar bills; the second note would require 8 of the hundred dollar bills, 4 of the ten dollar bills, and 9 one dollar bills; the third note would require 7 of the hundred dollar bills, 5 ten dollar bills, and 6 one dollar bills. Counting the one dollar bills, we find 18 of them. This may be paid with 1 ten dollar bill and 8 one dollar bills; putting the ten with the other ten dollar bills, we find 17 of them; this may be paid with 1 hundred dollar bill and 7 ten dollar bills; putting the 1 hundred dollar bill with the other hundred dollar bills, we find 22 of them. This may be paid with 2 of

the thousand dollar bills and 2 hundred dollar bills; putting the 2 thousand dollar bills with the other thousand dollar bills, we find 4 of them. Hence the three notes may be paid with 4 of the thousand dollar bills, 2 of the hundred dollar bills, 7 ten dollar bills, and 8 one dollar bills; and the amount of the whole is 4278 dollars."

5. A farm consists of 40 acres of plough-land, 135 of pasture, 48 of mowing, 20 of orchard, and 273 of wood-land; how many acres in the farm?

40 acres.
135 acres.
48 acres.
20 acres.
273 acres.
516 acres in the farm.

PROOF.

In order to detect mistakes which may have been made in performing the operation, it is necessary to resort to some method of proof. What is called casting out the 9's is an easy method, and preferable to any other.

Add the figures in the upper number, rejecting the 9's as often as they occur, and set the remainder opposite the number on the right hand; do the same with each of the other numbers; then if the excess of nines in the sum of the remainders be equal to the excess of nines in the sum total, the work is supposed to be right. (*Pierce.*)

*Illustration.*

| | | |
|---|---|---|
| 94346 | excess of 9's | 8 |
| 42130 | " " | 1 |
| 61679 | " " | 2 |
| 198155 | " 2 equal | 2 |

We here begin at the left hand of the first number; the first figure being 9, we reject it, and begin with the 4, which we add to the next figure, 3,—4 and 3 are 7 and 4 are 11, which is 9 and 2 over; reject the 9 and add the excess 2 with the 6,—2 and 6 are 8, which we set down as the remainder of the first number. Proceeding in the same way with the second number, 4 and 2 are 6 and 1 are 7 and 3 are 10, which is 9 and 1, we get 1 for the remainder of the second number. The remainder of the third number, 2, is obtained in the same way. We then add these remainders,—2 and 1 are 3, and 8 are 11, which is 9 and 2 over.

We then add the sum total, the same as before; 1 and 9 are 10, which is 9 and 1 over,—1 and 8 are 9,—1 and 5 are 6 and 5 are 11, which is 9 and 2 over, corresponding with the last remainder.

*This method of proof may be thus explained:*

As a figure, by its removal from one place to the next higher, is increased by nine times its value, it follows, that any figure, however far removed from the place of units and divided by 9, (which is the same thing in effect as rejecting the nines,) will have only a remainder of itself; therefore any number, and the sum of the figures which compose that number, when divided by 9, will have equal remainders.

"This additional value taken by figures by their removal to a higher place, gives to 9 the power of producing an effect which cannot belong to any other of the digits except 3, and to this only as it is a component part of 9.

"Take any number, as 3467; this, separated into its several parts, becomes $3000+400+60+7$; but $3000=3\times1000=3\times(999+1)=3\times99+3$. In like manner $400=4\times99+4$, and $60=6\times9+6$. Therefore, $3467=3\times999+4\times99+6\times9+3+4+6+7$; and $3467\div9=(3\times99+4\times99+6\times9+3+4+6+7)\div9$. But $3\times999+4\times99+6\times9$ is evidently divisible by 9; therefore, if 3467 be divided by 9, it will have the same remainder as $3+4+6+7$ divided by 9, and the same will hold for any other number whatever." (*Pierce.*)

This method of proof by casting out the 9's may *fail* in one respect, as the figures may be *transposed* and still the operation *appear* to be right.

| | |
|---|---|
| 4952 - - 2 | In this example, the operation *appears* |
| 3781 - - 1 | by this method of proof to have been |
| 2468 - - 2 | rightly performed, though there is an |
| ——— — | error of 900 in the answer. The actual |
| 12101 5=5 | amount of the numbers here added is |

11201, and it will be observed that the false answer contains the same figures as the true one, though

QUESTIONS.—Why is 9 taken, rather than 7, or 8, in order to prove a sum, as above directed? Can you prove a sum by casting out the 3's? Can you prove it by casting out the 18's, 27's, &c.? Why not by casting out the 6's or 7's?

they are placed in a different order. But it is not *easy* to make an error of this kind, unless it be done *designedly*.

A *true* sum will always appear so by this method of proof; and to make a false one appear true, there must be at least *two* errors, which are directly opposite to each other; and if there be more than two errors, they must balance among themselves; but the chance against this particular circumstance is so great, that we may as safely trust to this proof as any other, except, indeed, when a person who knows the method purposely transposes the figures. (*Bonnycastle.*)

### USE OF ADDITION.

Addition of numbers is necessary in all the departments of life, and is important to every person. The merchant, cashier, teller, &c. spend a considerable portion of their time in performing operations in this rule. Without it, the laborious practice of counting units, till the whole amount can be ascertained, must be resorted to. This must, after all, be an uncertain method of ascertaining the desired answer, on account of exposure to forgetfulness of the former numbers.

### MANNER OF STUDYING ADDITION.

The first inquiry with the learner should be, What is the nature of the operation? The next, How can it be performed in the most expeditious and correct manner?

With regard to the *first*, any tyro can furnish an answer. The *latter* requires more thought. The fact, that numbers of any amount, if properly written, may be united into one sum, is demonstrated on the preceding pages. The best manner of writing numbers for adding, is to place those of corresponding value directly under each other, and then a certain number of units in one column will equal one in the next. It becomes perfectly convenient, under these circumstances, to place numbers with those of their own character, and to write them so as to be read and understood with ease.

The only things necessary are, that the learner ascertain the reason for every direction given, and continue to practise till the operations are familiar.

## SECTION IV.

## SIMPLE SUBTRACTION.

Simple Subtraction teaches to take one number from another of the same kind, that is greater; or, which amounts to the same thing, to separate this last into two parts, one of which shall be the given number.

If, for instance, we have the number 9 and wish to take 4 from it, we shall, in so doing, separate it into two parts, which, by addition, would be the same again.

To take one number from another when they are small, it is necessary to pursue a course opposite to that pursued in the addition of small numbers; that is, in the series of names of numbers, we ought to begin from the greater of the numbers in question, and descend as many places as there are units in the smaller, and we shall come to the name given to the difference required. Thus in descending four places below 9, we come to 5, which expresses the number that must be added to 4 to make 9; or which shows how much 9 is greater than 4. When numbers are large, the subtraction is performed part at a time, by taking successively from the units, tens, hundreds, &c. of the greater number, the corresponding units, tens, hundreds, &c. of the smaller number, as in this example:— (*Lacroix.*)

From 587 take 345.

587
345
242 Ans.

Five units taken from seven units leave 2; 4 tens from 8 tens leave 4; and 3 hundreds from 5 hundreds leave 2. So the remainder is 2 hundreds, 4 tens, and 2 units, or 242, which shows how much 587 is greater than 345.

This process evidently gives a true result, because in taking from the greater of the two numbers all the *parts* of the smaller, we certainly take from it the *whole* of the smaller. When the units, tens, hundreds, &c. in the larger number are smaller than the corresponding figures in the smaller number, this process requires some modification.

If, for instance, 397 is to be taken from 524:—

524
397
127

In performing this operation, we cannot at first take the units in the lower, or smaller number from the units in the upper or larger number; but we may do this, by taking one of the two tens in the upper number and joining it with the 4 units;

we have taken 14 units, which is the same as 4 units and 1 ten. Then 7 units taken from 14 leave 7. We have now left in the upper number but 1 ten, from which we cannot subtract 9, but we may take 1 hundred from the 5 and join it with the 1 ten; we shall then have 4 hundreds and 11 tens; taking from these tens the 9 tens in the lower number, and 2 remain.

*Remaining* or *remainder* is a term frequently used, and it requires some explanation.

Take for illustration a sum and separate its parts, or decompose it.

From 125
Take 69
56

We cannot take 9 units from 5 units, and of course it becomes necessary to take one of the tens from the place of tens and reduce it to units, and add to the 5 which will increase it to 15, from which 9 can be taken and 6 remain. The 1 hundred in the next place borrowed and reduced to tens, will make, with the one that remains, eleven tens, from which 6 tens can be taken, and 5 tens remain.

Thus 125 is equal to 11 tens and 15 units;
And 69 is equal to 6 tens and 9 units.
56 There remain 5 tens and 6 units.

When there are ciphers between the figures of the larger number, it is necessary to go to the first figure on the left to borrow the ten that is wanted. See an example:—

7002
3495

As we cannot take the 5 units of the lower number from the 2 of the upper, we borrow 10 units from the 7000, denoted by the figure 7, which leaves 6990; joining the 10 we borrowed to the figure 2, the upper number is now decomposed into 6990 and 12; as may be seen in this manner.

In like manner decompose the lower number.

6000+900+90+12=7002
3000+400+90+ 5=3495
3000+500+00+ 7=3507

Taking the 5 (units) of the lower number, we obtain 7 for the units of the remainder. Then take 9 (tens) from 9 (tens) and nothing remains; 4 (hundreds) from 9, (hundreds) and 5 (hundreds) remain; 3 (thousands) from

6, (thousands) and 3 (thousands) remain. Then the whole remainder is 3 thousand 5 hundred, 0 tens, and 7 units, or 3507. (*Lacroix.*)

EXAMPLES.

1. What is the difference between the numbers 7005 and 467?

7005
467
6538 Ans.

2. America was discovered by Columbus in 1492; the first permanent settlement was made at Jamestown in 1607: how many years between these two events?

1607
1492
115 Ans.

3. From 1832 take 1699. What remains? Ans. 133.

4. What number must be added to 425 to make 793?

793
425
368 Ans.

5. If 368 be taken from 793, how many will remain?
Ans. 425.

6. How much greater is the number 2741 than 1417?
Ans. 1324.

7. From 1000000 take 999999. What remains?
Ans. 1.

*Note.* For greater convenience, when it is necessary to decrease the upper figure by unity, we can suffer it to retain its value, and add this unit to the corresponding lower figure, which thus increased gives as is wanted, a result one less than would arise from the written figures. (*Lacroix.*)

Adding one to the lower figure is the same in effect as taking one from the upper.

The foregoing explanations prepare the way for the following

RULE.

1. Place the less number under the greater, so that units may stand under units, tens under tens, &c. and draw a line under them.

2. Beginning at the right, take each figure in the subtrahend from the figure over it, and set the remainder under the line.

3. If the lower figure be greater than the one over it, add ten to the upper figure, from which figure so increased, take the lower, and write the remainder, carrying one to the next figure in the lower line, and thus proceed till the whole is finished.

### METHOD OF PROOF.

1. Take the excess of 9's in the less from the excess of 9's in the greater; their difference will be equal to the excess of 9's in the remainder. But if the excess of 9's in the smaller number exceed that in the greater, add 9 to the excess in the greater, and proceed as before.

*Illustration.*

| | |
|---|---|
| 473214 | 3 |
| 398765 | 2 |
| 74449 | 1=1 |

Let A represent the number of nines in the *minuend*, and B the number of nines in the *subtrahend;* C the excess of nines in the minuend, and D the excess of nines in the subtrahend. A and B then being an even number of nines, the difference between them must of course be an even number of nines. All that remain of the given numbers are the two excesses C and D; and it is evident that the difference between these excesses will be equal to the excess of nines in the difference of the two given numbers; for, in fact, it is the same thing, as may be shown :—

A — C
B — D
E — F

Let A, B, C, and D be the same as before; let E represent the number of nines in the remainder, and F the excess of nines in the remainder. Since A, B, and E, each represent an even number of nines, they may be rejected; then C will represent the remainder, D the subtrahend, and F the remainder, and F must be equal to itself.

### RULE.*

2 Reject the 9's from the minuend, subtrahend, and re-

* There are other methods of proving subtraction. 1. The remainder added to the least number, exactly gives the greatest. (*Lacroix.*) 2. Add the remainder to the subtrahend, and if their sum be equal to the minuend, the work is right. (*Thompson.*)

mainder; then subtract the excess of 9's in the remainder from the excess of 9's in the minuend, and the remainder will be equal to the excess of 9's in the subtrahend.

If the excess of 9's in the remainder be greater than the excess of 9's in the minuend, add 9 to the latter and subtract as before.

EXAMPLE.

| | |
|---|---|
| 8146 | 1 |
| 7062 | 8 |
| 884 | 2=2 |

USE OF SUBTRACTION.

The constant application of this rule in the transactions of business, sufficiently exhibits its usefulness.

---

## SECTION V.

### SIMPLE MULTIPLICATION.

Simple Multiplication teaches how to find the amount of any given number of one denomination, repeated a certain number of times.

It is a compendious method of performing Addition. The number to be multiplied is called the *Multiplicand*. The number by which we multiply is called the *Multiplier*. The number found after the work is finished is called the *Product*.

Both the Multiplicand and Multiplier are commonly called *terms* or *factors*.

"When numbers to be added are equal to each other, addition takes the name of *multiplication*, because in this case the sum is composed of one of the numbers repeated as many times as there are numbers to be added."

How much will 5 books come to at 25 cents for each?

| |
|---|
| 25 |
| 25 |
| 25 |
| 25 |
| 25 |
| 125 |

This example is easily performed by addition: but if it had been required to find the price of 30, 50, or 100 books, the operation must have been very tedious, on account of the number of times, which the price of one book, 25 cents, must have been written down.

In performing this example by adding the number 25, written 5 times, we find that the units

amount to 25; and the tens to 10, which, increased by 2 tens from the units, is 12. We have learned then, that 5 repeated 5 times, or 5 times 5 are 25; and also, that 2 repeated 5 times, or 5 times 2 are 10.

25 is called the multiplicand.
5 the multiplier.
125 the product.

5 times 5 are 25; 2 times 5 make 10, and 2 are 12=125.

The MULTIPLICATION TABLE, attributed to Pythagoras, should be thoroughly learned by every scholar.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 |
| 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |

"To form this table, the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, are written first on the same line. Each of these numbers is then added to itself, and the sum written in the second line, which thus contains each number of the first line doubled, or multiplied by 2. The third line is then formed by adding each number in the second line to the number immediately over it in the upper line. By adding the numbers in the third line to those in the first the fourth line is formed, and so on.

"If the *formation* of this table is understood, the mode of using it is apparent. If, for instance, the product of 7 by 5 is required, looking at the fifth line, and directly under 7, we find 35, which is 7 repeated 5 times. In the same manner any product of the 12 first numbers may be found.

"It will be seen by looking at the table, that the product

of 7 by 5 is the same as the product of 5 by 7; the product of 8 by 9 is the same as the product of 9 by 8; and the same is true of all numbers, as may be easily demonstrated. 7 times 5 means 7 times all the units in the 5; but 7 times 1 are 7, and 7 times 5 are 7 times as many; that is, 5 times 7. Hence the product of any two numbers is the same, whichever be made the multiplier." (*Lacroix.*)

1. What will 24 sheep cost, at 5 dollars for each? It is plain, that if 1 cost 5 dollars, 24 will cost 24 times as many.

| | |
|---|---|
| 24 | First multiply the 4 units in the multiplicand, |
| 5 | by the multiplier, 5. 5 times 4 are 20, which is |
| ——— | just 2 tens. Write a cipher, reserving the 2 |
| 120 | (tens) to be carried to the place of tens. Multiply |

the 2 (tens) in the multiplicand by the multiplier; the product is 10 (tens,) which, increased by the 2 (tens) before reserved, becomes 12 (tens,) or 1 hundred and 2 tens; and the whole product is 120. This mode is always used when the multiplier is one figure. It is varied, when two or more are used.

2. What will 24 horses come to, at 67 dollars apiece?

| | |
|---|---|
| 67 | In this example the multiplier consists of |
| 24 | two figures, instead of one as in the preced- |
| ——— | ing. We multiply first by the units in the |
| 268 | multiplier, as in the preceding case, and obtain |
| 134 | 268 as the result, which is the multiplicand, 67, |
| ——— | repeated 4 times, or the price of 4 horses. In |
| 1608 | multiplying by the next figure, 2, we must bear |

in mind, that it is 2 tens, or 20; twice 7 are 14, i. e. 14 tens; hence the 4 must be put in the place of tens, and the 1 (hundred) reserved to be reckoned with the other hundreds; 2 times 6 are 12, and the 1 reserved are 13. We then add these two products together and get 1608, which is 67 repeated 24 times.

From the preceding examples and illustrations, we give the following general

RULE.

1. Place the multiplier under the multiplicand, putting units under units, tens under tens, &c.

2. When the multiplier does not exceed 12, begin at the right hand of the multiplicand, and multiply each figure by

the multiplier, setting down the unit figure under units, carrying for tens to the next place, as in addition, and the work is done.

3. When the multiplier exceeds 12, multiply each figure of the multiplicand by each figure of the multiplier, placing the first figure of each product directly under that by which you multiply. Add these several products together as they stand, and the sum will be the total product.

The most simple number expressed by several figures, is 10, 100, 1000, &c. When the multiplier is such a number, if it is recollected that the same figure is increased in value ten times by every remove towards the left, we shall soon perceive, that to multiply by 10, we must make each of its orders of units ten times greater; and this is effected by placing a cipher on the right of the number proposed, because then all its significant figures will be advanced one place towards the left.

For the same reason, to multiply any number by 100 we should place two ciphers on the right; for since it becomes 10 times greater by the first cipher, the second will make it ten times greater still, and consequently it will be 10 times 10, or 100 times, greater than it was at first.

Hence, when the multiplier is 10, 100, 1000, or 1 with any number of ciphers, annex as many ciphers to the multiplicand, as there are ciphers in the multiplier, and the number thus produced will be the product.

Multiply 1428 by 100. Ans. 142800.

If the multiplier is 20, 30, 40, 200, 300, 2000, 4000, 20000, &c., as the ciphers are of no value except as they remove the significant figures towards the left, it is sufficient to multiply by the significant figures only, and place the ciphers on the right hand of the product.

EXAMPLE.

What will 900 hogsheads of wine come to, at 67 dollars a hogshead?

$$\begin{array}{r} 67 \\ 900 \\ \hline \text{Ans. } 60300 \text{ dollars.} \end{array}$$

It is plain also, that if there are ciphers at the right hand of the multiplicand, they may be omitted until the multiplication has been performed, and then may be annexed to the product.

EXAMPLE.

Multiply 2500 by 24.

```
  2500
    24
 ------
  100
  50
 ------
 60000
```

If there are ciphers *between* the significant figures of the multiplier, neglect them, observing to place the first figure of every product directly under the figure by which you are multiplying.

EXAMPLE.

What is the product of 8057069 multiplied by 70050?

```
   8057069
     70050
 ----------
  40285345
 56399483
 ----------
 564397683450
```

What will 32 acres of land come to at 57 dollars per acre?

```
  57          57
   4           8
 ----  or   ----
 228         456
   8           4
 ----        ----
 1824        1824
```

In this example, we first multiply the price of one acre by 4, which evidently gives the price of 4 acres. Now if the price of 4 acres be multiplied by the number which represents the number of times that 4 is contained in 32, it will obviously give the price of 32 acres. This number is 8. Hence we multiply the price of 4 acres by 8, which gives 1824 dollars as the price of the land. The second operation is explained on the same principle; for multiplying the price of 1 acre by 8, gives the price of 8 acres; and 4 times 8 are 32.

Any number which, like 32, is produced by the multiplication of 2 or more numbers, is called a *composite* number. Thus 24 is a composite number, and 4 and 6, or 3 and 8, are its component parts, because either of these, multiplied together, will produce 24.

From a review of the remarks made in connexion with the last operation, we give the following RULE:—

"When the multiplier is a composite number, we may multiply first by one component part, and that product by the other."

Or if there be more than two component parts, "multiply by each of those parts separately, instead of the whole number at once."

If the multiplier is 9 or a number of 9's, the work may be conveniently performed by adding a cipher or ciphers, according to the number of 9's, to the multiplicand, and then subtracting it from the number so increased.

What will 252 yards of broadcloth come to at 9 dollars per yard?

2520
252
2268

We have already seen that placing a cipher at the right hand of a number is the same as multiplying it by 10. Now in this example a cipher is annexed to the multiplicand, but as the multiplier is 9 instead of 10, it becomes necessary to take 252, the original multiplicand, from 2520, and this leaves 252 increased 9 times. So in the following example.

3262 Multiplicand.
999 Multiplier.

| | |
|---|---|
| 32620000 | 3262 |
| 3262 | 999 |
| 3258738 | 29358 |
| | 29358 |
| | 29358 |
| | 3258738 |

The general reasoning in all these operations is simply this:—If the price of a particular thing is a certain amount, the price of two must be twice as much—the price of 50 must be 50 times as much, &c.

What will 50 men earn in a week, if each man earn 12 dollars?

If one man earns 12 dollars, 50 men must earn 50 times 12 dollars, or 12 times 50 dollars, which will be the same = 600.

### METHOD OF PROOF.

Cast out the 9's from the two factors, as in addition. Multiply the two remainders together, and cast out the 9's

from the product. If the excess of 9's in their product be equal to the excess of 9's in the total product, the work is right.

EXAMPLE.

| | |
|---|---|
| 5728 | 4=excess of 9's in the multiplicand. |
| 625 | 4=ditto in the multiplier. |
| 28640 | 4×4=16. 1+6=7. |
| 11456 | |
| 34368 | |
| 3580000 | 7=excess in the product. |

This method of proof is liable to the same objections here as in addition.

Multiplication may also be proved by dividing the product by the multiplier. If the work be right, the quotient will be equal to the multiplicand.

USE OF MULTIPLICATION.

The learner can hardly need to be told, that operations in adding together the several sums are rendered very easy by this rule; while by addition it would be very long and tiresome. It facilitates the transaction of business in a very high degree.

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## SECTION VI.

## SIMPLE DIVISION.

Simple Division teaches how to divide a given number into a certain number of equal parts.

The number to be divided is called the *Dividend.*

The number by which we divide, or which shows into how many parts the number is to be divided, is called the *Divisor.*

The number which shows how many each of the parts contains, is called the *Quotient.*

Lacroix remarks, that "the manner of decomposing one number by another, in order to know how many times the last is contained in the first, is called *Division,* because it serves to divide, or portion out, a given number into equal parts, of which the number or value is given.

"For instance, if we wish to divide the number 64 into 4 equal parts, our object is to ascertain how many times 4 is

contained in 64; that is, what number, multiplied by 4, will give a product of 64."

There are several ways in which a question of this kind may be solved.

1. We may solve it by addition; that is, we may add the number 4 to itself continually till the number 64 is produced: thus, 4+4+4+4+4+4+4+4+4+4+4+4+4+4+4+4 +4+4+4=64; then, by counting the number of times which 4 is written, we ascertain how many times this number is contained in 64, which we find to be 16 times.

The same result may be obtained by subtracting 4 from 64 continually till nothing remains; thus, 64—4=50—4=56 —4=52—4=48—4=44—4=40—4=36—4=32—4=28— 4=24—4=20—4=16—4=12—4=8—4=4—4=0. Here again, if we count the number of times which 4 has been taken from 64, we shall find it 16, as before.

But it will be seen at once, that it would be very inconvenient in practice, to make use of repeated additions or subtractions for finding how many times one number is contained in another, especially if one of these numbers is much larger than the other. It then becomes necessary to have recourse to an abbreviation of this method.

From the example already given, it may be seen, that the quotient, or number of times which 4 is contained in 64, is a number which, if multiplied by the divisor, 4, will make just 64; thus 16+4=64.

"If the dividend do not exceed 144, and the divisor do not exceed 12, the quotient may be found at once from the table given in Multiplication; since that table contains the product of all factors which do not exceed 12. If it were asked, for instance, how many times 8 is contained in 56, it would be necessary to go down the eighth column to the line in which 56 is found; the figure 7 at the beginning of this line, shows the second factor of the number 56, or how many times 8 is contained in this number.

"We shall see from this table, that there are numbers which cannot be exactly divided by others. For instance, as the seventh line, which contains all the multiples* of 7 below 84, has not 40 in it, it follows, that 40 is not divisible by 7."

(*Lacroix.*)

* The product of any number, multiplied by 2, 3, 4, 5, &c., is called a *multiple* of that number. A *multiple* is that which can be measured, or divided by another number without a remainder.

When the number to be divided, or the dividend, is larger than 144, we must seek the quotient by some other means than by the table.

Let it be required to divide 4375 dollars equally among 25 men; how many dollars will each receive?

It will be borne in mind, that our object is to ascertain how many times 25 are contained in 4375; or in other words, to find a number, which, multiplied by 25, shall produce 4375; or, to divide 4375 into 25 equal parts.

That we may better understand the operation, we will write the thousands, hundreds, &c. of the dividend, independently of each other.

```
25)4000(100
  25
  ---
 1500
  300
  ---
25)1800(70
 1750
 ----
   50
   70
    5
   --
25)125(5
  125
  ---
  000
```

It is evident, in this case, that the first figure of the quotient cannot be thousands, because 25 is not contained in 4, but if we call the 4 thousands 40 hundreds, we perceive, that the divisor, 25, is contained in this number, which is only a part of the dividend, less than 200 times, and more than 100 times; we accordingly take 100 times 25 or 2500 from this part of the dividend, and the remainder is 1500, with which we join the 300, making 1800. As 1800 contains 25 less than 100 times, it is plain, that the next quotient figure must be tens; and in 1800, 25 is contained more than 70 and less than 80 times; hence we take 70 times 25, or 1750, from this part of the dividend, and 50 remains. With this remainder we join the 7 (tens) and 5 (units,) which make up the last portion of the dividend, 125 (units,) in which 25 is contained 5 times, making the quotient 175; for 100+70+5=175.

How many dictionaries can be purchased for 675 dollars, at 5 dollars each?

If the dictionaries were only one dollar apiece, we could buy 675; if they were 2 dollars, one half as many; if 3 dollars, one third; if 4 dollars, one quarter; but as they are 5 dollars, we can buy only one fifth as many; that is, as many as the number of times which 5 is contained in 675.

To make the manner of ascertaining how many times 5 is contained in 675 more plain, we may separate the number into four parts,

$$675 = 500 + 100 + 50 + 25.$$

5 is contained in 5 once; and in 500 one hundred times; in 10 twice, in 10 times 10, i. e. 100, 20 times; in 50, 10 times; in 25, 5 times; and $100 \div 20 + 10 + 5 = 135$ the answer.

Thus by dividing the parts of the whole separately, and adding the several quotients together, we obtain the number of times which one number is contained in another.

If it be required to divide 1656 by 3, this question may be changed into another form, viz: To find such a number, that multiplying its units, tens, hundreds, &c. by 3, the product of these units, tens, hundreds, &c. may be the dividend, 1656.

This number will not have units of a higher order than hundreds; for if it had but *one* thousand, the product would contain at least 3, which is not the case. It appears, then, that the thousand in the dividend is a number reserved, when the hundreds of the quotient were multiplied by 3, the divisor.

The figure occupying the place of hundreds in the required quotient, ought to be such, that when multiplied by 3, its product may be 16, or the greatest multiple of 3 less than 16. This restriction is necessary, on account of the reserved numbers which the other figures of the quotient may furnish when multiplied by the divisor, and which should be united to the product of the hundreds.

"The number which fulfils this condition is 5; but 5 hundreds multiplied by 3 gives 15 hundreds, and the dividend, 1656, contains 16 hundreds; the difference, 1 hundred, must have come then from the reserved number, arising from the multiplication of the other figures of the quotient by the divisor. If we now subtract the partial product, 15 hundreds, or 1500, from the total product 1656, the remainder 156 will contain the product of the units and tens of the quotient by the divisor; and the question will be reduced to finding a number, which, multiplied by 3, gives 156, a similar question to that which presented itself above; and the process is repeated again and again, till the dividend is exhausted.

"The operation just described may be disposed of in this way:

```
3)1656(552
  15
  --
   15
   15
   --
    06
```

The divisor is placed on the left hand, separated by a line, and a similar line is placed at the right of the dividend to separate the quotient from it. We then take the 16 (hundreds) on the left hand of the dividend, and dividing this number by 3, we get 5 (hundreds,) which is placed for the first figure in the quotient; then taking the product of the divisor by this figure, and subtracting it from 16 (hundreds) the partial dividend, we write the remainder 1 (hundred) below, by the side of which we place the 5 (tens) of the dividend. We divide this number by the divisor, and obtain 5 (tens) for the second figure of the Quotient; we then take the product of this figure by the divisor, and subtracting it from the partial dividend, get 0 for the remainder. We then bring down the last figure of the dividend, 6 (units;) divide this by 3, and get 2 (units) for the last figure of the quotient.

"It is manifest, that if we find a partial dividend that cannot contain the divisor, it must be because the quotient has no units of the order of that dividend, and that those which it contains, arise from the products of the divisor by the units of the lower orders in the quotient. It is necessary, therefore, whenever this is the case, to put a cipher in the quotient to occupy the place of the order of units that is wanting.

Let 1632 be divided by 4.

```
4)1632(408
  16
  --
   032
    32
    --
    00
```

By this example we see, that the divisor is contained just 4 (hundred) times in the 16 (hundreds) of the dividend, without leaving any remainder. The 3 (tens) in the dividend will not contain 4; there can, therefore, be no tens in the quotient; hence we put a cipher in the place of tens in order to represent the true value of the other quotient figures."

If the divisor consists of several figures, it is not always easy at first to tell how many times the partial dividend will contain it. The following example and illustration will aid the learner to obviate this difficulty.

In 21755 hours many days?

```
24)21755(906 dys.
   216
   ---
   155
   144
   ---
    11 hours.
```

Here we perceive that 24 is not contained in 21; we must therefore take 217 for the partial dividend. To ascertain how many times the divisor is contained in this, the best way is to find how many times the first figure of the divisor is contained in the first, or two first of the partial dividend. The quotient figure thus obtained, or diminished by one or two will generally be the figure required. But if, after subtracting the product of the divisor and this quotient figure, the remainder equals or exceeds the divisor, the quotient figure may be increased accordingly.

When there is no remainder after the division, the quotient is the absolute and perfect answer. But if there is a remainder, a part of the dividend is left undivided.

EXAMPLE.

If 2261 dollars be divided equally among 34 men, how many will each receive?

```
34)2261(66
   204
   ---
   221
   204
   ---
    17 dollars remain undivided.
```

In this example, after giving each man 66 dollars from the dividend, 17 dollars remain, which are evidently part of the dividend, that has not been divided.*

It is very often true, that the dividend does not contain the divisor any exact number of times; that there is no exact number, which, multiplied by the divisor, will produce the dividend. In the above example, 17 remains. Multiply 66 by 34 and the product will be 2244; to this add 17 and the sum will be 2261, the dividend.

The number which remains undivided after division, is called the *remainder*.

* The proper method of dividing the remainder in division, or of expressing its true value, will be taught when we come to the consideration of Fractions.

## Rule.

"1. Draw a line on the right and left of the dividend, and write the divisor on the left.

"2. Find how many times the divisor is contained in as many figures as are just necessary, and place the number on the right.

"3. Multiply the divisor by this number, and place the product under the figures of the dividend above mentioned.

"4. Subtract this product from that part of the dividend under which it stands, and bring down the next figure of the dividend, or more if necessary, to the right of the remainder.

"5. Divide this number so increased as before, and so on, till the whole is finished.

"*Note.* If is necessary to bring down more than *one* figure to the remainder, in order to make it larger than the divisor, a cipher must be placed in the quotient for every figure so brought down." (Bonnycastle.)

This rule is derived from the principles and illustrations before given. It will be easily understood by those who have attended to them. So many of the varieties of Division as are important, follow. To the more advanced learner they may be unnecessary; but they will be found convenient to the beginner.

### VARIETIES IN DIVISION.

When the divisor does not exceed 12, the quotient may be obtained without writing all the work as in the former examples.

Find how many times the first or two first left hand figures of the dividend will contain the divisor, and place the result underneath, as in subtraction. If the product of this figure and the divisor be not equal to the partial dividend, consider the remainder as standing at the left hand of the next figure of the dividend, for a new dividend. Divide this as before. This method is called *Short Division.*

### EXAMPLE.

In 1407 shillings how many dollars?

6)1407

234 and 3 remainder.

We say 6 in 14 tiwce, and 2 over. As this 2 is evidently 200, we suppose it written in the place of hundreds;

that is, at the left of the cipher, which occupies the place of tens; then we say 6 in 20, 3 times, and 2 over. As this 2 is 20 or two tens, we suppose it written in the place of tens; that is, at the left of the 7 (units;) then 6 in 27, 4 times, and 3 over. This 3 is 3 units, or 3 shillings, which remain of the dividend above an even number of dollars; and is the same as remainder in *long division;* or where the whole process is written out.

The process of division may also be shortened when both factors, that is, the divisor and dividend, are terminated by ciphers, by dropping all the ciphers at the end of that factor which contains the least number, and as many from the other.

Let 64000 be divided by 400. From the divisor 400, we can drop the two ciphers, which leaves 4 for the divisor; and from the dividend we can also drop two ciphers, which will leave 640 for the dividend; the quotient will be the same as if we used the whole numbers; for we have only to change the name of the units. Instead of 64000, or 64 thousands, and 400, or 4 hundreds, we have 640 units, and 4 units. The quotient of 640 and 4, or of any two given numbers, is always the same, whatever may be the denomination of their units.

| | | |
|---|---|---|
| 400)64000(160 | 4)640 | 4\|00)640\|00 |
| 400 | | |
| 2400 | 160 | 160 |
| 2400 | | |
| 00000 | | |

If the divisor only is terminated by ciphers, cut them off, and cut off as many figures from the right hand of the dividend. Divide the remaining figures of the dividend by the significant figures of the divisor as usual. If there is any remainder, place the figures cut from the dividend, at the right of it, and it will be the true remainder. If there is no remainder after division, the figures cut off are the remainder.

| 1. Divide 5239 by 80. | 2. Divide 341617 by 800. |
|---|---|
| 8\|0)523\|9 | 8\|00)3416\|17 |
| 65 — 39 Rem. | 427 — 17 Rem. |

In the first of these examples, we cut off the cipher from 80, which leaves the divisor 8; we also cut off the right hand figure of the dividend, leaving it 523. Now it is evident we have diminished the value of both these numbers equally, (that is, we have done the same as to divide them both by 10,) and consequently the value of the quotient will not be affected. But the 9 cut off from the dividend is just what would remain if this number were divided by 10; and the 3 which remains in dividing 523 by 8, is, in reality, 3 tens, and of course must be put in the place of tens, or at the left of 9, and thus the true remainder will be represented.

In the second example, it will be seen that there is no remainder in dividing 3416 by 8; consequently the 17 cut off is the true remainder, or just such a remainder as would be left if 341617 were divided by 800 in the usual way, as may be seen by the following:

800)341617(427 quotient.
3200
2161
1600
5617
5600
17

From the preceding remarks, it is obvious that if the divisor be 10, 100, 1000, or 1 with any number of ciphers annexed, we have only to cut off as many figures from the right hand of the dividend, as there are ciphers in the divisor; and those which remain will be the quotient; and those cut off must be the remainder.

EXAMPLE.

Divide 742946 by 1000.

1000)742|946

In this example it will be seen that cutting off the three right hand figures, is the dividing it by 1000; for 742946= 742000+946, and it is evident that 1000 is contained 742 times in 742000, and that 946 is a remainder, since it is less than 1000.

When the divisor is a composite number, that is, such a number as may be produced by the multiplication of two or more numbers; divide first by one of the component parts, and that quotient by the other.

EXAMPLE.

Divide 4729 by 48.

$48=8\times6$ 8)4729

6)591—1

98—3+8=24+1=25 remainder.

We first divide by 8, which is contained in 4729, 591 times, and there is one remaining. We then divide this quotient 591 by 6, which is contained 98 times and 3 remain; but it must be remembered that each unit in the first quotient is equal to 8 in the dividend; consequently every unit that remains will contain the same; therefore this remainder 3 must be multiplied by 8 in order to find the units it contains of the given dividend, to which the first remainder 1 must be added. Hence we have this rule for finding the true remainder in cases of this kind.

Multiply the last remainder by the first divisor, and add the first remainder (if any,) to the product. The sum will be the remainder.

### METHODS OF PROOF.

It has already been remarked, that the quotient in division is such a number as multiplied by the divisor will produce the dividend; hence it is evident that multiplication and division are exactly the reverse of each other, and of course will mutually prove each other.

Multiply the quotient and divisor together, and add the remainder, if any, to the product, and if the sum is equal to the dividend, the work is right.

Or thus: Add the remainder and all the products of the several quotient figures together, according to the order in which they stand in the work, and the sum will be equal to the dividend, if the work is right.

*Proof by rejecting the* 9's.

Cast out the 9's of the divisor and qoutient; multiply the excess, and add the remainder, if any, to the product. Cast out the 9's from this sum, and also from the dividend, and if the two excesses agree, the work is right.

### USE OF DIVISION.

In the transaction of business, it often becomes neces-

sary to divide sums of money, as dollars, cents, &c. or other articles into a certain number of equal parts. By division this can be effected with great facility.

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## SECTION VII.

### COMPOUND ADDITION.

Compound Addition is the addition of numbers of different denominations. The denominations, however, must be of the same kind. Pounds, shillings, &c. cannot be added to years, months, &c. Shillings, pence, &c. are different denominations from pounds, &c. but of the same kind; viz. that of money.

A butcher sold 3 pieces of beef; one piece weighed 12 lbs. 8 oz.; another 15 lbs. 3 oz.; the other 25 lbs. 15 oz. What was the weight of the whole?

| | |
|---|---|
| 12 pounds | 8 ounces. |
| 15 pounds | 3 ounces. |
| 25 pounds | 15 ounces. |
| — | — |
| 52 | 26 |

If we add the pounds as in simple addition, we find 52 of them; and by adding the ounces separately in the same manner, we find 26. But 16 ounces make a pound; so that 26 ounces contain 1 pound and 10 ounces; for 16 taken from 26, leaves 10. If we add this 1 pound with the others, we shall have 53 pounds and 10 ounces.

This operation may be more conveniently performed as follows:

| *Lbs.* | *oz.* |
|---|---|
| 12 | 8 |
| 15 | 3 |
| 25 | 15 |
| 53 | 10 |

Whe first add the ounces; 15 and 3 are 18, and 8 are 26, which contains 16 once and 10 besides. We write the 10 ounces under the other ounces, and carry the 1 (pound) to the next column, to be added with the other pounds, which thus increased, make 53, which we write under the column. Thus we find the whole weight to be 53 pounds and 10 ounces.

Add together 4 years, 6 months, 3 weeks, and 5 days; 8 yrs. 9 mo. 0 w. and 2 d.; and 7 yrs. 11 mo. 1 w. and 3 d.

| *Yrs.* | *mo.* | *w.* | *d.* | |
|---|---|---|---|---|
| 4 | 6 | 3 | 5 | |
| 8 | 9 | 0 | 2 | |
| 7 | 11 | 1 | 3 | |
| 21 | 3 | 1 | 3 | Amount. |

In Compound Addition, we proceed on the same principle as in Simple Addition, viz. of adding those numbers which are of the same denomination together, and when the sum of these is sufficient to make one or more of the next higher denomination, we write the excess (over the exact number which forms one or more of the next higher denomination) under the column added, and reserve the one or more of the higher denomination, to be added with the column which expresses it.

EXAMPLE *in pounds, shillings, and pence.*

| £. | *s.* | *d.* |
|---|---|---|
| 435 | 15 | 8 |
| 125 | 6 | 2 |
| 215 | 17 | 3 |
| 8 | 7 | 4 |
| 785 | 6 | 5 |

We first add the column of pence, which makes 17; this contains 12 (the number of pence in a shilling) once, and 5 excess, which we write under the column. By adding the 1 shilling reserved with the column of shillings, their sum is 46, which contains 20 (the number of shillings in a pound) twice, and 6 excess, which we write under the column. We now add the 2 pounds reserved, with the pounds; their sum is 785; and the whole amount therefore is 785 £. 6 s. and 5 d.

It will be observed, that in this process we added together the *units* and *tens* of the shillings as in simple addition; thus, 1 to carry to 7 is 8, and 4 are 15, and 6 are 21, and 5 are 26; 2 to carry to 1 is 3, and 1 are 4, which we conceive to be placed at the left hand of the 6, making 46 shillings. In the same manner also, we added the pounds.

From the preceding examples and illustrations, we derive the following

RULE.

To add compound numbers, write them under each other, placing those of the same denomination in the same column, and the lowest denomination at the right hand, and the higher in regular order towards the left. Add the column containing the lowest denomination. If the sum is sufficient to make one or more of the next higher denomination, write under the column the excess over the exact number which

makes one or more of the next, and reserve the one or more to be added with the column of the same name. Proceed in this manner with the next higher denomination, and so on till the work is finished.

Numbers are sometimes found where no even number of units makes one of the next greater: as $5\frac{1}{2}$ yards make one rod. In these cases the parts are to be written as fractions.

### Methods of Proof.

The method of proof by casting out the 9's is not applicable to compound numbers.

Compound Addition may be proved by cutting off the upper line, and adding the rest. If this sum, added to the upper line, is equal to the sum obtained by the first addition, the work is right.

Sums in this rule may be proved, also, by subtracting successively from the sum of the numbers added, all the parts of these numbers; and if the work is right, nothing will remain.

EXAMPLE.

| £. | s. | d. |
|---|---|---|
| 487 | 6 | 5 |
| 625 | 6 | 8 |
| 272 | 9 | 9 |
| 1335 | 2 | 10 |
| 111 | 1 | 0 |

We first add the hundreds in the column of pounds; the sum is 12, which taken from 13 the beginning of the result in the example, leaves 1, which was produced by what was reserved in the tens in performing the addition. Next add the tens; their sum is 12. Take from this the 3 (tens) increased by the 1 (hundred) that remains from the left hand column, considered as 10 tens, and the remainder is 1 (ten,) which was reserved from the column of units. The sum of the column of units is 14, which, taken from the 5 increased by the 1 (ten,) leaves 1, which was reserved from the column of shillings. The sum of the shillings is 21, which, taken from 2 increased by the 1 pound = 20 shillings, leaves 1, which was reserved from the column of pence. The sum of the pence is 22, which, taken from 10 increased by 1 shilling = 12 pence, leaves 0. The work is therefore right.

### Use of Compound Addition.

In the transaction of business, the amount of articles of different denominations, such as pounds, shillings and pence, pounds, ounces, &c. is often required. By this rule the object is attained.

## SECTION VIII.

### COMPOUND SUBTRACTION.

Compound Subtraction teaches how to find the difference between two sums of different denominations, which is done by taking the one sum from the other.

EXAMPLES.

| | £. | s. | d. |
|---|---|---|---|
| From | 823 | 7 | 3 |
| Take | 735 | 9 | 5 |
| Difference | 87 | 17 | 10 |

In this example, as we cannot take 5 pence from 3, we borrow from the minuend in the column of shillings 12 pence (1 shilling,) which we add to the 3 pence, and from the sum take the 5, and the remainder is 10. We therefore carry 1 (shilling) to the 9, which makes 10. Because 10 is greater than 7, we borrow 1 from the column of pounds, equal to 20 shillings, which added to 7, makes 27; from this take 10, and the remainder is 17. We then carry 1 (pound) to the subtrahend in the column of pounds, and subtract it as in simple addition. The reason for borrowing and carrying (tens) was explained in the simple rules. The principle here is the same, though the numbers vary from each other. When the lower figure exceeds the upper, borrow as many as will make one of the next higher denomination.

From 4 miles, 2 furlongs, 27 rods, 10 feet, and 4 inches, take 1 mile, 6 furlongs, 38 rods, 17 feet, and 6 inches.

| *M.* | *fur.* | *rods.* | *ft.* | *in.* |
|---|---|---|---|---|
| 4 | 2 | 27 | 10 | 4 |
| 1 | 6 | 38 | 14 | 6 |
| 2 | 3 | 28 | $11\frac{1}{2}$ | 10 |

In this example we borrow 1 foot, which is 12 inches, and add to the 4 inches; from the sum, 16, we take 6, and the remainder is 10. We add 1 to the 14 feet, or, what will amount to the same thing, consider the 10 above it as 9, to which add $16\frac{1}{2}$; the sum is $25\frac{1}{2}$; from this take 14, and it leaves $11\frac{1}{2}$. Add 1 to the 38 rods and take the sum from 67, (40 rods, = 1 furlong, being added to the 27,) and 28 remain. Carry 1 to 6 and subtract it from 10, (8 furlongs being added to the 2,) and the remainder is 3. Carry 1 to the 1, and subtract it from 4, and 2 remain.

In the result we have $11\frac{1}{2}$ feet and 10 inches. But $\frac{1}{2}$ foot

is 6 inches; add these to the 10, and the sum is 16, i. e. 1 foot and 4 inches. Add the foot to the other feet, and the result will read 12 ft. 4 in. instead of 11½ ft. 10 in.

| | *Bu.* | *pk.* | *qt.* | *pt.* |
|---|---|---|---|---|
| From | 27 | 3 | 5 | 1 |
| Take | 13 | 2 | 6 | 1 |
| | 14 | 0 | 7 | 0 |

From the preceding examples and illustrations we deduce the following

RULE.

Write the sum to be subtracted under the other as in addition, and subtract the lower sum from the upper. If the lower number of any of the denominations exceed the upper, add as many to the upper as make one of the next higher denomination; from the sum take the lower number, and carry one to the lower number of the next higher denomination before subtracting it.

In some instances it becomes necessary to borrow twice, or double the number that it takes to make one of the next higher, because it does not always take a simple number of units to make one of the next superior denomination.

*Example.*

| | *Rods.* | *yds.* | *ft.* | *in.* |
|---|---|---|---|---|
| From | 6 | 0 | 1 | 6 |
| Take | 3 | 5 | 2 | 9 |
| | 1 | 5 | 1 | 9 |

Having one to carry after we take the feet from the number above, it makes 6 yards. But 5½ yards make a rod, and 6 cannot be taken from 5½; we therefore take 2 rods from the next column, which is equal to 11 yards. From this number 6 can be taken, and 5 remain.

It will be seen by looking at the remainder, though but 1 rod is written in the place of rods, that the 5 yards &c. make more than another rod, and the true remainder is 2 rods, 0 feet, and 3 inches.

A few examples occur, where the subtraction cannot be performed, without resolving the given numbers into others, retaining the same value.

*Example.*

| | *Rods.* | *yds.* | *ft.* | *in.* |
|---|---|---|---|---|
| From | 20 | 0 | 0 | 0 |
| Take | 19 | 5 | 1 | 5 |

The difference in the value of the above numbers is 1 inch.

### Method of Proof.

Add the subtrahend and remainder together; if their sum is equal to the minuend, the work is right.

*Example.*

| | £. | s. | d. |
|---|---|---|---|
| From | 24 | 7 | 9 |
| Take | 16 | 10 | 11 |
| Remainder | 7 | 16 | 10 |
| Proof | 24 | 7 | 9 |

Since the remainder is the difference between the two given numbers, it is evident, that this difference, added to the smaller number, will produce the greater.

Or, subtract the remainder from the minuend, and if this last remainder is equal to the subtrahend, the work is right.

*Example.*

| | £. | s. | d. |
|---|---|---|---|
| From | 329 | 4 | 6 |
| | 20 | 8 | 9 |
| Rem. | 308 | 15 | 9 |
| Proof | 20 | 8 | 9 |

This method of proof is obvious; for if the difference between two numbers be taken from the larger number, the smaller will evidently remain after the subtraction.

### Use of compound Subtraction.

The difference between sums and quantities of different denominations is often required in the transaction of business. By this rule the result is readily accomplished.

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## SECTION IX.

## DECIMAL NUMBERS.

By Decimal Numbers, are meant any numbers which have a uniform mode of increasing by tens, i. e. ten of one denomination making one of the next higher. Thus, 10 ones, or units, make 1 ten; 10 tens make 1 hundred; 10 hundreds make 1 thousand, &c.

When the coin of the United States is examined, it will be found, that this is the manner of reckoning it: thus, 10 mills make one cent; 10 cents make one dime; 10 dimes make one dollar; 10 dollars make one eagle. (See table, page 172.)

If all the things, about which we have occasion to speak, were divided in this manner, it would greatly increase the facilities for the transaction of business. If, in long measure, a mile were divided into ten parts, and each of those into ten others, and each of these into ten others, &c. there would be far less liability to make errors in adding, subtracting, &c..

When things are thus divided, we call the parts *Decimal Fractions.* See Sec. 15.

Decimal numbers are added in the same manner as the numbers in Simple Addition, as may be seen by the following examples.

| Tens Thous. | Thous. | Hunds. | Tens. | Units. |
|---|---|---|---|---|
| | 6 | 3 | 7 | 6 |
| | 9 | 4 | 3 | 5 |
| | 1 | 2 | 6 | 4 |
| | 4 | 6 | 0 | 1 |
| 2 | 1 | 6 | 7 | 6 |

The same in U. S. coin.

| Eag. | dol. | di. | cts. |
|---|---|---|---|
| 6 | 3 | 7 | 6 |
| 9 | 4 | 3 | 5 |
| 1 | 2 | 6 | 4 |
| 4 | 6 | 0 | 1 |
| 2 | 1 | 6 | 7 | 6 |

The same remark applies to Subtraction, &c.

*Examples.*

| Thous. | Hunds. | Tens. | Units. |
|---|---|---|---|
| 6 | 3 | 5 | 8 |
| 3 | 6 | 7 | 9 |
| 2 | 6 | 7 | 9 |

In United States coin.

| Eag. | dol. | di. | cts. |
|---|---|---|---|
| 6 | 3 | 5 | 8 |
| 3 | 6 | 7 | 9 |
| 2 | 6 | 7 | 9 |

## SECTION X.

### FEDERAL MONEY.

The Denominations in Federal Money are Eagles, Dollars, Dimes, Cents, and Mills. The manner of division or increase is by a decimal proportion, as is seen in the last Section. An eagle divided into ten parts, each of those parts is a dollar; a dollar divided into ten parts, each of those parts is a dime, &c.

### Addition of Federal Money.

| Eagles. | Dollars. | Dimes. | Cents. | Mills. |
|---|---|---|---|---|
| 5 | 7 | 3 | 8 | 5 |
| 2 | 8 | 7 | 5 | 9 |
| 4 | 2 | 8 | 1 | 3 |
| 12 | 8 | 9 | 5 | 7 |

In this example we add these several sums of Eagles, Dollars, &c. as in Simple Addition. The amount is 12 Eagles, 8 Dollars, 9 Dimes, 5 Cents, and 7 Mills; or $128 95 cts. and 7 m., which is the more usual mode of reading sums in Federal Money, separating them by points, thus $128.95.7.

### Subtraction of Federal Money.

A man had 15 eagles, 6 dollars, 5 dimes, 7 cents and 5 mills. He owed 12 eagles, 9 dollars, 8 dimes, 5 cents and 9 mills. How much will he have left after paying his debt?

| Eagles. | Dollars. | Dimes. | Cents. | Mills. |
|---|---|---|---|---|
| 15 | 6 | 5 | 7 | 5 |
| 12 | 9 | 8 | 5 | 9 |
| 2 | 6 | 7 | 1 | 6 |

Rule.—Subtract as in Simple Subtraction. The remainder is 2 eagles, 6 dollars, 7 dimes, 1 cent and 6 mills, or $26, 71 cts. and 6 mills, = $26.71.6.

### Multiplication of Federal Money.

Eight men had each 27 eagles, 5 dollars, 7 dimes, 8 cents and 5 mills. How much had all?

| Eagles. | Dollars. | Dimes. | Cents. | Mills. |
|---|---|---|---|---|
| 27 | 5 | 7 | 8 | 5 |
| | | | | 8 |
| 220 | 6 | 2 | 8 | 0 |

Eight times 5 mills are 40 mills = 4 cts.; 8 times 8 cts. are 64, and the 4 reserved from the mills make 68 = 6 ds. and 8 cts. 8 times 7 dimes are 56, and 6 added make 62; 8 times 5 dollars are 40, and 6 added make 46 = 4 E. and $6; 8 times 7 are 56, and 4 added make 60; 8 times 2 are 16, and 6 added make 22. All the men therefore have 220 eagles, 6 dollars, 2 dimes, and 8 cents = $2206.28. Here it will be seen that we multiply just as in Simple Multiplication.

### Division of Federal Money.

Eight men have 220 eagles, 6 dollars, 2 dimes, and 8 cents. How much money has each?

| | Eagles. | Dollars. | Dimes. | Cents. | Mills. |
|---|---|---|---|---|---|
| 8) | 220 | 6 | 2 | 8 | 0 |
| | 27 | 5 | 7 | 8 | 5 |

220 eagles divided by 8, is 27, and 4 E. = to 40 dollars remain. To these add the 6 dollars; 8 in 46, 5 times, and 6 dollars = to 60 dimes remain; to these add the 2 dimes, 8 in 62, 7 times, and 8 dimes = to 80 cents remain; to these add the 8 cents; 8 in 68, 8 times, and 4 cents = to 40 mills remain; 8 in 40, 5 times. Observe, that the quotient is of the same denomination as the dividend. Each man, then, has 27 eagles, 5 dollars, 7 dimes, 8 cents, and 5 mills, or $275.78.5.

In every sum in Federal Money which has mills, the right hand figure denotes mills, the two next cents, or cents and dimes, and the rest dollars, or dollars and eagles.

By the preceding examples in Addition, Subtraction, Multiplication and Division of Federal Money, it will be seen that all operations in this coin are performed just as in Simple Addition, and of course need no farther illustrations.

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## SECTION XI.

### EXCHANGE

*From one Currency to another.*

By *Currency* is meant the mode of calling or counting money. In different countries these modes vary. *Sterling* is money reckoned by pounds, shillings, pence and farthings. Federal money is reckoned by eagles, dollars, &c. In all places it is agreed to divide a pound into 20 parts, and to call each of these parts a shilling. A shilling is divided into 12 parts, and each part is called a penny. But the worth of a pound in one place is not the same as in some other places. Hence some other standard must be used, in order to know the value of a pound in different places. The law establishes Federal Money as the currency of the United States. Still there are several currencies which are nominal in different parts. The pound is not often used; but shillings and pence are as commonly spoken of as dimes and cents. A shilling in Canada is a fifth part of a dollar; in New-England it is a sixth part of a dollar; in New-York, an eighth part; in

Pennsylvania it is two fifteenth parts. In Canada 20 cents is the value of a shilling; in New-England, 16 cents and 7 mills, nearly; in New York, 12 cents and 5 mills; and in Pennsylvania, 13 cents and 5 mills, nearly. In reading the price of a ton, yard, &c. when given in shillings, it is necessary to know what is the currency of the place in order to have a knowledge of the real worth of the article. The most important principles or rules are those which direct how to bring pounds, shillings, &c. to Federal Money, and the contrary. If I wish to know how many dollars, cents, &c. are contained in 23 £. 7 s. and 9 d., I can ascertain in several ways. As 6 s. make a dollar in New-England, and there are 20 s. in 1 £., there must be one sixth as many dollars as there are shillings; one sixth of 20 = 3 and one third. If there are 3⅓ dollars in a pound, there are 23 times as many in 23 £. = 76 and two thirds dollars = $76.66.6 +. In 7 shillings there is 1 dollar, and 1 shilling remains = 16 cents and 7 mills, nearly. As 9 d. is three fourths of 12 d. or 1 s., it is equal to 12 cents and 5 mills. The whole sum is $76.66.6 + $1.16.7 + 12 cents and 5 mills = $77.95.8.

But a much shorter way is that of multiplying the pounds by 10, which makes the value of a unit then worth one third of a dollar.

In 2 £. how many dollars?

2 × 10 = 20. Each unit is worth one third part of a dollar, because 20 is one half the number of shillings in 2 £., and of course a unit is worth 2 s., and 2 s. is equal to one third of 6 s. or $1.

If in a proposed sum there are shillings, pence and farthings, the following may be adopted as general

## Rules.

1. Write down the pounds, and at the right hand put half the number of shillings.

If there is an odd shilling, reduce it to farthings, and add to these the farthings in the pence and farthings. (A unit of this number will be as near the value of ⅓ of 1 cent, as 48 is to the number 50. If the odd shilling is called 50 instead of 48, and a proportionate addition made to the farthings in the given pence and farthings, and then ⅓ part of the amount be taken, it will be cents.)

If the change is to be made to New-England currency,

divide by 3, but if to New-York currency, divide by 4, and the true answer is obtained.*

2. If pounds only are given, place a cipher at the right hand, and divide as before. The answer will be dollars.

3. If shillings, pence and farthings only are given, the answer may be obtained by dividing the farthings by 4, and putting the quotient at the right hand of the pence; then dividing this number by 12, and placing the quotient at the right hand of the shillings, and dividing these by 20; and then divide the quotient as before directed.

EXAMPLE.—Reduce 1 s. 6 d. to cents.

12 | 6
20 | 1.50
3 ) 75 ( 25 cents, Ans.

4. The following Rules may be convenient.

In changing N. E. to N. Y. currency, add $\frac{1}{3}$.
" " N. Y. to N. E. currency, subtract $\frac{1}{4}$.
" " N. E. to Penn. currency, add $\frac{1}{4}$.
" " Penn. to N. E. currency, subtract $\frac{1}{5}$.
" " N. Y. to Penn. currency, subtract $\frac{1}{16}$.
" " Penn. to N. Y. currency, add $\frac{1}{15}$.
" " N. E. to Canada currency, subtract $\frac{1}{6}$
" " Canada to N. E. currency, add $\frac{1}{5}$.

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## SECTION XII.

### COMPOUND MULTIPLICATION.

Compound Multiplication teaches how to find the amount of a given sum of different denominations, multiplied by a simple number.

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* Ten thirds of a dollar are equal in value to one £.; 100 thirds of a dime, 1000 thirds of a cent, and 10,000 thirds of a mill, are all of the same value.

Ten fourths of a dollar make a pound *New-York* currency; 100 fourths of a dime, &c.

It is commonly the most simple method to consider the multiplication of each denomination of the compound number by the simple factor, as a distinct question; and the several products thus obtained will be the whole amount sought.

What is the weight of 5 casks of raisins, each weighing 2 cwt. 3 qrs. and 25 lbs.?

| *cwt.* | *qrs.* | *lbs.* |
|---|---|---|
| 2 | 3 | 25 |
| 5 | 5 | 5 |
| 10 | 15 | 125 |

It is evident that the number of cwt. in one cask repeated as many times as there are casks, will give the number of cwt. in all the casks; the same is true of the quarters, and of the pounds. Hence 10 cwt. 15 qrs. and 125 lbs. expresses the true weight of the 5 casks. But 125 lbs. is equal to 4 qrs. and 13 lbs.; then we may reckon the 4 qrs. with the other quarters and merely write 13 lbs. under the column of pounds. Then 15 qrs.+4=19, which are equal to 4 cwt. and 3 qrs.; then 10 cwt.+4=14 cwt. We have then 14 cwt. 3 qrs. and 13 lbs., as seen in the following example:—

| | | |
|---|---|---|
| 2 | 3 | 25 |
| | | 5 |
| 14 | 3 | 13 |

## Rule.

1. The multiplier may be written under the lowest denomination. Multiply each denomination successively beginning with the lowest, setting down the excess, and carrying from each denomination to the higher, as in Compound Addition.

2. When the multiplier is a composite number, the multiplication may be made by the component parts, as in the following example:

What is the weight of 36 bales of cotton, each weighing 3 cwt. 2 qrs. and 17 lbs.?

| *cwt.* | *qrs.* | *lbs.* |
|---|---|---|
| 3 | 2 | 17 |
| | | 6 |
| 21 | 3 | 18 |
| | | 6 |
| 131 | 1 | 24 |

Here we first multiply by 6, which gives the weight of 6 bales, and this product by 6, which evidently gives the weight of 36 bales, 6 times 6=weight of 36 bales.

What will 47 yards of cloth cost, at 1 £. 8 s. 9 d. per yard?

| £. | s. | d. | |
|---|---|---|---|
| 1 | 8 | 9 | |
| | | 6 | |
| 8 | 12 | 6 | |
| | | 8 | |
| 69 | 0 | 0 | |
| 1 | 8 | 9 | |
| 67 | 11 | 3 | |

In this case the multiplier 47 is not a composite number; that is, it cannot be produced by the multiplication of any other numbers; but we may multiply the price of one yard by 6, and that product by 8, which will give the cost of 48 yards; then if we subtract the price of one yard from this product, the cost of 47 yards will remain.

| £. | s. | d. | |
|---|---|---|---|
| 1 | 8 | 9 | |
| | | 9 | |
| 12 | 18 | 9 | |
| | | 5 | |
| 64 | 13 | 9 | Cost of 45 yds. |
| 2 | 17 | 6 | Cost of 2 yds. |
| 67 | 11 | 3 | Ans. |

Or we may multiply the price of one yard by 9, and this product by 5, which will give the price of 45 yards; then multiplying the price of one yard by 2, will give the price of 2 yards, which added to the cost of 45, will give the cost of 47 yards as before. Hence,

2. When the multiplier cannot be produced by the multiplication of small numbers, two such numbers as come nearest to it may be taken; and then the value of the odd parts may be found and added to the amount; or if too large, subtracted from it.

What is the cost of 3475 yards of cloth, at 2 £. 3 s. 5 d. per yard?

| £. | s. | d. | | |
|---|---|---|---|---|
| 2 | 3 | 5 | × 5 | |
| | | 10 | | |
| 21 | 14 | 2 | × 7 | |
| | | 10 | | |
| 217 | 1 | 8 | × 4 | |
| | | 10 | | |
| 2170 | 16 | 8 | | |
| | | 3 | | |
| 6512 | 10 | 0 | Price of 3000 yds. | |
| 868 | 6 | 8 | " 400 yds. | |
| 151 | 19 | 2 | " 70 yds. | |
| 10 | 17 | 1 | " 5 yds. | |
| 7543 | 12 | 11 | | |

Here we first multiply by 10, which of course gives the price of 10 yards; the price of 10 yards multiplied by 10 gives the price of 100 yards, which, multiplied by 10, gives the price of 1000 yards; and this multiplied by 3 gives the price of 3000 yards. The price of 100 yards multiplied by 4 gives the price of 400 yards; the price of 10 yards multiplied by 7 gives the price

of 70 yards, and the price of 1 yard multiplied by 5, gives the price of 5 yards. Add these several amounts to the price of 3000 yards and we have 7543 £. 12 s. 11 d., the price of 3475 yards. Hence

3. When the multiplier is larger than 12, multiply the given price or quantity by 10, which gives the amount of 10, this product multiplied by 10 gives the amount of 100, &c. Multiply the given price or quantity by the units, and the price or amount of 10 by the tens in the multiplier; and so on as to the 100's. The several products added together will be the amount sought.

What is the cost of 9 cwt. of tea at 7 s. 6 d. per pound?

| £. | s. | d. |
|---|---|---|
| | 7 | 6 |
| | | 7 |
| 2 | 12 | 6 |
| | | 8 |
| 21 | 0 | 0 |
| | | 2 |
| 42 | 0 | 0 |
| | | 9 |
| 378 | 0 | 0 |

In this example we first multiply by 7, which gives the price of 7 lbs.; this product multiplied by 8 gives the price of 56 lbs. $= \frac{1}{2}$ cwt. This multiplied by 2 gives the price of 1 cwt. which multiplied by 9 gives the price of 9 cwt. The same result may be obtained by multiplying first by 7, then by 4, which gives the price of a quarter; and this, multiplied by 4, gives the price of 1 cwt.

## SECTION XIII.

### COMPOUND DIVISION.

Compound Division teaches the method of finding how many times a given simple number is contained in any sum or quantity of different denominations.

A compound number may be divided by a simple number by regarding each denomination of the former as a distinct dividend.

If 3 £. 13 s. and 8 d. be divided equally between 2 men, how much will each receive?

If the pounds, shillings, and pence in this question could be separately divided by 2 without a remainder, the division would be perfectly easy. But 3 £. cannot be divided by 2 without a remainder, neither can the 13 s. But if we take

one pound from the 3, 2 will remain, which may be equally divided between two men, by giving them 1 each. We may now call the 1 £. 20 s. and put it with the 13 s., making 33 s., which cannot be divided by 2 without a remainder of 1, which we may call 12 d. and reckon it with the 8 d. making 20 d.; this number can be divided by 2 without a remainder, so that the whole is divided into two equal parts.

| | £. | s. | d. |
|---|---|---|---|
| 2) | 2 | 32 | 20 |
| | 1 | 16 | 10 |

But this change of the numbers may be made more conveniently as we proceed in the operation.

| | £. | s. | d. |
|---|---|---|---|
| 2) | 3 | 13 | 8 |
| | 1 | 16 | 10 |

From the preceding example and illustration, we deduce the following

RULE.

1. Write the numbers as in Simple Division, and divide the several denominations of the dividend successively by the divisor.

2. When there is a remainder after dividing any denomination, reduce it to the next lower and add it to the number in that denomination in the given sum. Divide these, and if there is a remainder, reduce it to the next lower denomination and proceed as before till the work is finished. If the divisor can be separated into its component parts, divide by each of them separately.

EXAMPLE.

If 27 lambs cost 10 £. 2 s. 6 d. what will 1 cost?

| | £. | s. | d. |
|---|---|---|---|
| 9) | 10 | 2 | 6 |
| 3) | 1 | 2 | 6 |
| | | 7 | 6 |

Here we divide first by 9 and that quotient by 3, because 3 times 9 are 27. If the same money had been paid for 9 lambs, they would have cost 1 £. 2 s. 6 d. each; but since there are 3 times 9 lambs, it is evident, that one will cost only a third part of 1 £. 2 s. 6 d. = to 7 s. 6 d.

Divide 12 £. 16 s. 6 d. by 57.

| £. | s. | d. | £. | s. | d. |
|---|---|---|---|---|---|
| 57) 12 | 16 | 6 ( | 0 | 4 | 6 |
| 20 | | | | | |
| )256 shillings. | | | | | |
| 228 | | | | | |
| 28 | | | | | |
| 12 | | | | | |
| )342 pence. | | | | | |
| 342 | | | | | |
| 000 | | | | | |

As there are no numbers, which by multiplying, will produce 57, we must divide by the whole number. But 57 is not contained in 12 the number of pounds; hence we reduce the pounds to shillings, by multiplying them by 20, and add the 16 s. to the product, which makes 256 s. in which 57 is contained 4 times and 28 s. remain. Reduce these to pence, and add the 6 d.; the amount is 342 d. in which 57 is contained exactly 6 times.

In some instances, the divisor is not given directly, but must be ascertained from the condition of the question.

EXAMPLE.

Divide 189 dollars among 3 men, A, B, and C; give B twice as much as A, and C 3 times as much as B. How much will each receive?

| | |
|---|---|
| 1 A 1 part. | |
| 2 B 2 parts. | |
| 6 C 6 parts. | |
| — | |
| 9 | |
| 9)189 | |
| 21 A's share. | |
| 2 | |
| 42 B's share. | |
| 3 | |
| 126 C's share. | |

We first give one dollar to A, 2 dollars, (or twice as much) to B, and 6 dollars, (or 3 times as much as B receives,) to C. Thus $1+2+6=9$ dollars are disposed of. We now wish to know how many times we can repeat this process. This is ascertained by dividing the 180 dollars by 9, which gives the quotient of 21. Now as A receives 1 dollar at each division, it is evident that 21 dollars is his share. B receives twice as much, $21 \times 2 = 42$; and C 3 times as much as B, $42 \times 3 = 126$. This is proved to be correct by adding the several shares together; thus, $21 + 42 + 126 = 189$ dollars. Hence we deduce the following rule.

RULE.

"When the shares of partners are unequal, find how many of the least share are contained in the whole number of shares.

Divide the given sum by the whole number of shares, and the quotient will be the value of the least share.

Multiply the quotient by the number of shares belonging to each partner separately; and the product will be the required answer."

METHOD OF PROOF.

Compound Multiplication and Division mutually prove each other.

EXAMPLES.

What is the weight of 6 loads of hay, each weighing 18 cwt. 3 qrs. 15 lbs.?

| *cwt.* | *qrs.* | *lbs.* |
|---|---|---|
| 18 | 3 | 15 |
| | | 6 |
| 113 | 1 | 6 |

*Proof.*

| *cwt.* | *qrs.* | *lbs.* |
|---|---|---|
| 6)113 | 1 | 6 |
| 18 | 3 | 15 |

Divide 40 £. 7 s. 4 d. equally among 8 men. What will each one receive?

| £. | *s.* | *d.* |
|---|---|---|
| 8)40 | 7 | 4 |
| 5 | 0 | 11 |

*Proof.*

| £. | *s.* | *d.* |
|---|---|---|
| 5 | 0 | 11 |
| | | 8 |
| 40 | 7 | 4 |

## SECTION XIV.

## REDUCTION,

OR

## COMBINATION OF THE PRECEDING RULES.

Reduction teaches the method of bringing numbers of one denomination into another retaining the same amount of value.

"*Reduction Descending*"* is the bringing of a higher into a lower denomination, as pounds into shillings, pence, and farthings; and is performed by multiplication.

"*Reduction Ascending*,"* is the bringing of a lower into a higher denomination, and is performed by division. It is exactly the reverse of the former.

When a question to be reduced contains numbers of different denominations, we may reduce them all to one denomination, and then perform the operation by the simple rules. If the question is, How many pence in 6 £. 14 s. and 7 d. we reduce the whole sum to pence, by multiplying the pounds by 20, and the shillings by 12.

EXAMPLE.

| £. | s. | d. |
|---|---|---|
| 6 | 14 | 7 |
| 20 | | |
| 134 | | |
| 12 | | |
| 1615 | | |

As 1 £. is 20 s., 6 £. are 6 times 20 s. = 120 s.; to these add the 14 s. and the sum is 134 s. A shilling is 12 pence; hence 134 s. are 134 times 12 d. = 1608 d. to these add the 7 d. and the sum is 1615 d. Hence we derive the following

RULE.

To reduce a sum of different denominations to the lowest contained in it, multiply the highest by that number which it takes of the next lower denomination, to make one of that number, and to the product add the number belonging to the lower. Proceed with the other denominations in the same manner till the whole is reduced to the denomination required.

How many inches are there in 2 rods, 6 ft. and 4 inches? Ans. 472.

If we divide the 1615 pence obtained in the first example by 12, we shall evidently reduce them to shillings; and by dividing these by 20, we shall reduce them to pounds.

---

* The terms *Reduction Ascending*, and *Descending* are objectionable; but as they are common, it is thought better to use them. Reduction combines several of the other rules, and involves no different principles.

OPERATION.

12)1615
20)134 — 7 d. remain.
6 £. 14 s. remain.

Hence the following

RULE.

When lower denominations are to be brought into higher, divide the lower by that number which it takes to make one of the next higher. Proceed in the same manner through all the denominations. The remainder after each division will be of the same denomination as the dividend.

In 21758 grains how many pounds, &c.?

OPERATION.

24)21758
2|0)90|6 — 14 grs.
12)45 — 6 pwts.
3 — 9 oz.

Ans. 3 lbs. 9 oz. 6 pwts. 14 grs.

PROOF.

*lbs. oz. pwts. grs.*
3 9 6 14
12
45
20
906
24
3624
1812
14
21758 grs.

In 960 £. 18 s. how many dollars?

£. s.
960 18
20
6)19218
3203 dollars.

The object of this question is to reduce pounds and shillings to dollars, which is done by reducing the pounds to shillings, and dividing these shillings by 6, because 6 s. = $1.

METHOD OF PROOF.

Change the order of the question and divide the last product by the last multiplier; or if the sum is in Reduction Ascending, multiply the last quotient by the last divisor, and so on, till all the denominations are divided or multiplied.

EXAMPLES.

In 6 rods, 10 feet, and 9 inches, how many inches?

| *rods.* | *ft.* | *in.* |
|---|---|---|
| 6 | 10 | 9 |
| $16\frac{1}{2}$* | | |
| 109 | | |
| 12 | | |
| 1317 inches. | | |

PROOF.

12)1317

$16\frac{1}{2}$)109 — 9 in.

6 r. — 10 ft.

In 24 £. 2 s. how many dollars?

| £. | s. |
|---|---|
| 24 | 2 |
| 20 | |
| 6)482 | |
| $80 | 2 s. |

PROOF.

$80 2 s.

6

2|0)48|2

24 £. 2 s.

In some instances, the conditions of the question to be answered require a preparation to be made before the direct answer can be obtained.

EXAMPLE.—I have 72 shillings which I am to divide between John, George, and William, so that George shall have 4 cents as often as John has 2, and William 6.

In 72 s. there is $\frac{1}{6}$ part of 72 dollars, because 6 shillings make one dollar.

6)72(12 which is the number of dollars to be divided. In one dollar are 100 cents; $100 \times 12 = 1200$ cents, the number to be divided.

| | | | |
|---|---|---|---|
| John receives | 2 | 12)1200 | 100 |
| George " | 4 | 100 | 2 |
| William " | 6 | | 200 John's share. |
| | 12 | 100 | 100 |
| | | 4 | 6 |
| | | 400 Geo.'s share. | 600 William's share. |

## SECTION XV.

### FRACTIONS.

*Fractions* denote a part or parts of a unit; though sometimes the term *fraction* denotes numbers larger than unity. They are of two kinds, Vulgar and Decimal.

* To multiply by $\frac{1}{2}$, $\frac{1}{3}$, &c. is only to take $\frac{1}{2}$, $\frac{1}{3}$, &c. of the multiplicand.

A *Vulgar Fraction* is expressed by two numbers, written one above the other, thus, $\frac{1}{2}$, $\frac{3}{4}$, &c. The upper number is called the *numerator*, and the lower, the *denominator*. The denominator denotes into how many parts the unit is supposed to be divided; and the numerator, how many of those parts are reckoned.

A *proper fraction* has the numerator less than the denominator; as $\frac{1}{3}$.

An *improper fraction* is the reverse; as $\frac{6}{5}$. This is more in value than a unit.

A *compound fraction* is a fraction of a fraction; as $\frac{1}{3}$ of $\frac{1}{2}$.

A *mixed number* is a whole number and a fraction; as $7\frac{6}{7}$.

The *common measure* of a fraction is a divisor which will divide both the numerator and denominator, without a remainder.

The *greatest common measure* is the greatest number which will divide them. 12 is the greatest common measure of $\frac{12}{48}$.

The *common multiple* of two or more numbers is a number which may be divided by each of them without a remainder. 24 is a common multiple of 3, 4, 6, and 8, as it can be divided by each without a remainder.

The *least common multiple*, is the least number which can be thus divided.

A *perfect number* is one which is equal to the sum of its aliquot parts. Thus, 6 is equal to its half (3,) its third (2,) and its sixth (1.)

Decimal Fractions have the numerator only expressed. The denominator, always being 1, with as many ciphers as there are figures in the numerator, is known without being written. The numerator is written with a point at the left hand; thus .1 is the same as $\frac{1}{10}$; $.21 = \frac{21}{100}$; $.324 = \frac{324}{1000}$; that is, one tenth; twenty-one hundredths; three hundred and twenty-four thousandths.

When a sum contains whole numbers and decimals, it is divided by a period; thus, 25.13 is read twenty-five and thirteen hundredths. The point between the whole number and decimals is called the *separatrix*.

Decimals *decrease* towards the right hand in the same proportion as whole numbers *increase* towards the left; that is, in a ten-fold proportion, according to the following

## TABLE.

| 100 Millions. | 10 Millions. | Millions. | 100 Thousands. | 10 Thousands. | Thousands. | Hundreds. | Tens. | Units. | Tenth parts. | Hundredths. | Thousandths. | 10 Thousandths. | 100 Thousandths. | Millionths. | 10 Millionths. | 100 Millionths. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1. | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Ciphers at the right hand of decimals do not alter their value; but at the left hand, they diminish in a ten-fold proportion. Thus .5, .50, and .500, are each equal to $\frac{1}{2}$. The first expression is 5 tenths; the second is 50 hundredths; and the third is 500 thousandths. In each case it will be seen, that the numerator is just half of the denominator which is understood. But .5, .05, .005, have very different values; the first is $\frac{1}{2}$; the second, $\frac{5}{100}$ or $\frac{1}{20}$; the third, $\frac{5}{1000}$ or $\frac{1}{200}$.

### 1. ADDITION OF DECIMALS.

What is the sum of 23.4 rods, 5.6 rods.

| | |
|---|---|
| 23.4 | Here, besides the whole numbers, we have |
| 5.6 | 4 tenths and 6 tenths, which make 10 tenths; |
| Ans. 29.0 | which are evidently equal to 1 whole number, and may be added with the whole numbers. |

What is the sum of 325.6, 72.83, 7.325, 35.7285, 22.3, 18.15, 53.521, and 89.3162?

| | |
|---|---|
| 325.6 | As decimals decrease towards the |
| 72.83 | right hand in a ten-fold proportion, |
| 7.325 | tenths should evidently be added to |
| 35.7285 | tenths, hundredths to hundredths, &c. |
| 22.3 | Hence the following |
| 18.15 | |
| 53.521 | |
| 89.3162 | |
| Ans. 624.7707 | |

RULE.

Write the numbers to be added under each other, according to their value. Add them as in Simple Addition; and point off as many for decimals as there are decimals in any one of the given numbers.

### 2. Subtraction of Decimals.

From 32.8723 take 19.3821.

| | |
|---|---|
| 32.8723 | By this example it will be seen that |
| 19.3821 | the operation is performed as in whole |
| Rem. 13.4902 | numbers. Hence the following |

RULE.

Write the less number under the greater with one decimal point directly over the other; then subtract as in whole numbers, and point off the decimals as in addition.

### 3. Multiplication of Decimals.

Multiply 56.25 by 3.5.

| | |
|---|---|
| 56.25 | In both factors there are three deci- |
| 3.5 | mals; therefore we point off three deci- |
| 28125 | mals in the product. |
| 16875 | |
| Prod. 196.875 | |

Multiply .05 by .03.

| | |
|---|---|
| .05 | In this example there are four decimal |
| .03 | places in the two factors; but the product |
| Prod. .0015 | of the significant figures contains but two; |

therefore we prefix as many ciphers as there are in the two factors. Multiplying a fraction by a fraction is taking only a part of a part. Hence the product is less than the multiplicand. Thus $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$; .5 of .5, or .5 multiplied by .05=25 hundredths, or $.5 \times .03 = 15$ ten thousandths, or .0015. Hence the following

RULE.

Multiply as in whole numbers, and point off as many figures from the right hand of the product for decimals as there are decimals in both factors.

When there are not so many figures in the product, prefix ciphers to supply the deficiency.

## Division of Decimals.

Divide 134.196 by 5.3.

```
5.3)134.196(25.32
    106
    ---
     281
     265
     ---
      169
      159
      ---
       106
       106
```

As division is just the reverse of multiplication, evidently there must be as many decimals in the divisor and quotient as there are in the dividend. Hence the following

### RULE.

Divide as in simple numbers, and point off as many figures for decimals in the quotient as the decimals in the dividend exceed those in the divisor. If there are not so many figures in the quotient, prefix ciphers to supply the deficiency.

Divide .008 by .2.

```
.2)008(.04
```

The quotient would be simply 4; one cipher must therefore be prefixed.

## Reciprocals.

When two numbers multiplied together give 1 as the product, they are called *reciprocals* to each other.

What is the reciprocal of 25?

```
25)1.00(0.4 Ans.
   100
   ---
   000
```

As the object is to find a number, which multiplied by 25, shall produce 1, the answer can be obtained by dividing 1 by 25. But as we cannot divide 1 by 25, we first reduce the 1 to hundredths by annexing ciphers, and the quotient is of the same denomination. Hence the following

### RULE.

To find the reciprocal of any number:

Divide 1 by the given number, and the quotient will be the answer.

## Reduction of Decimals, &c.

### I. *Reduction of Vulgar Fractions to Decimals.*

Reduce $\frac{1}{2}$ to an equivalent decimal.

2)10
.5 Ans.

By annexing a cipher to the numerator, it is reduced to tenths, i. e. 10 tenths = 1; as the required decimal is only one half, dividing these tenths by 2, the denominator will give 5 tenths, the answer.

Reduce $\frac{1}{4}$ to a decimal.

4)10(.25 Ans.
8
20
20

In this example the required decimal is one fourth. We therefore divide the numerator, 1, reduced to tenths, by 4; the quotient is 2, and 2 tenths remain; these may be reduced to hundredths by annexing a cipher; then dividing these by 4, the quotient is 5; and nothing remains. The answer is 2 tenths and 5 hundredths, or 25 hundredths. Hence the following

RULE.

Annex a cipher to the numerator and divide it by the denominator. If there is a remainder, annex a cipher to it and divide as before, and so on till nothing remains, or until a sufficient number of decimals are produced.

### II. *Reduce numbers of different denominations to their equivalent decimal value.*

Reduce 5 s. 3 d. and 2 qr. to the decimal of a pound.

4)2
12)3.5
20)5.29166
Ans. .264583

As 2 qrs. are = $\frac{2}{4}$ of a penny, we may reduce them to tenths, by annexing a cipher, and then dividing them by 4 the denominator; the quotient is 5 tenths of a penny. But $3\frac{1}{2}$ d. or 3.5 d. are equal to $\frac{3.5}{100}$ of a penny, = $\frac{3.5}{1200}$ of a shilling, which reduced to a decimal is .29166 +; 5 s. $3\frac{1}{2}$ d. may therefore be written 5.29166 s. = $\frac{5.29166}{10000}$ of a shilling = $\frac{5.29166}{200000}$ of a pound. Reduce this to a decimal, and it becomes .264583 £. Hence the following

RULE.

Write the numbers perpendicularly under each other, in order, with the least at the top. Divide each denomination by that number which it takes to make one of the next greater, and place the quotient at the right hand of the denomination next greater than that divided, beginning with the least and proceeding in order to the greatest. The last quotient will be the required decimal.

III. *Find the value of a decimal in the terms of an integer or whole number.*

What is the value of .264583 of a pound?

.264583
20
5.291660
12
3.499920
4
1.999688

Ans. 5s. 3d. 1.99968 qrs.

This is the reverse of the preceding case. In dividing 3.5 by 12 we obtained the decimal .29166; but there was still a remainder, 8, by annexing ciphers to which and then dividing by 12, would always leave the same remainder. Such a decimal is called a running decimal. But the value of it, after the decimal has been carried to 5 or 6 figures, is so small, that it may be neglected. Hence the answer in this example is less than the given sum in the preceding, by a small part of a farthing.

RULE.

Multiply the decimal by that number which it takes of the next less denomination to that in which the decimal is given. From the product cut off as many figures as there are in the given decimal for a remainder. Multiply the remainder and cut off as before through all the denominations. The figures cut off at the left hand are the answer, as seen in the preceding example.

## SECTION XVI.

## VULGAR FRACTIONS.

Though Fractions properly denote parts of a unit, sometimes the term is used to denote numbers larger than unity.

Vulgar Fractions, as before stated, are used to denote various irregular divisions of a unit. The lower number is called the *denominator*, because it shows into how many equal parts the unit is divided; and the upper number is called the *numerator* because it shows how many of those parts are taken, or denoted by the fraction.

Let a dollar be divided into 8 equal parts; and one of these be given to A, 3 to B, and 4 to C; then A has $\frac{1}{8}$, B $\frac{3}{8}$, and C $\frac{4}{8}$ of a dollar.

In performing division we often find a remainder, the value of which is fractional. For instance, in dividing 239 by 8, 7 remain undivided. In this example it will be perceived that 29 times 8 have been taken from 239, and 7 remain undivided; and as we cannot divide it by 8, we may express its value by placing it for the numerator of a fraction, the denominator of which is 8, the divisor; thus $\frac{7}{8}$, i. e. seven eighths of another 8, besides the 29 times 8; for 8 units of the dividend are just equal to one of the quotient; then 7 of the dividend are equal to 7 eighths of 1 in the quotient; and the quotient in this case therefore, is $29\frac{7}{8}$. $\frac{7}{8}$ is a fraction, the denominator of which, 8, represents the number of parts into which each of the quotient figures must be divided in order to produce the dividend, and the numerator, 7, expresses the number of these parts which are contained in the remainder, after the 29 times 8 have been taken from the dividend.

```
8 ) 2 3 9 ( 2 9 7/8
    ------
      7 9
      7 2
    ------
        7
```

Let a dollar be divided into 10 equal parts; they will of course be dimes. $\frac{1}{10}$ of a dollar then is 10 cents, and $\frac{2}{10} = 20$ cents; $\frac{3}{10} = 30$ cents, &c. Hence,

*If the numerator of a fraction be increased, while the denominator remains the same, the value of the fraction is proportionally increased; and the reverse.*

Again let the fraction of a dollar be expressed thus $\frac{1}{20}$; this is only equal to 5 cents; and $\frac{1}{25}$ to only 4. Hence

*If the denominator of a fraction be increased, the value of the fraction is decreased in the same proportion.*

Most of the operations in fractions depend on these two principles.

Again take $\frac{1}{10}$ of a dollar $= 10$ cents. Multiply both the numerator and denominator by the same number, 2 for in-

stance: $\frac{2}{20}$. Here we have 2 *twentieths* of a dollar, which are equal to 1 *tenth*, for a twentieth is one half of one tenth.

*Hence if both the numerator and denominator be multiplied, or divided by the same number, the value of the fraction will not be altered.* Fractions of the same value may therefore be expressed in an infinite number of ways: $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{10}{20} = \frac{50}{100}$, &c. In each of these the numerator is just half of the denominator. The most simple of the above fractions is $\frac{1}{2}$, and the others may be reduced to it by dividing both terms by any number that will divide both without a remainder.

In reducing fractions to their most simple terms, it is very convenient to know the largest number which will divide both terms without a remainder.

What is the greatest number that will divide both terms of the fraction, $\frac{143}{637}$, without a remainder?

143)637(4
572
65

It is evident that any number can be divided by itself; but by no larger number; and the quotient in such cases is always 1. If then 637 the denominator can be divided by 143 the numerator, this is the greatest common measure of the fraction. But 637 divided by 143 leaves 65 remainder. Every common measure of 143 and 637 will also measure 65; for $637 = 143 \times 4 + 65$. As 65 is the greatest measure of itself, the common measure of 65 and 143 cannot be greater than 65. But $143 \div 65$ leaves 13 remainder; hence 65 is not the common measure sought. Every common measure of 65 and 143 must also measure 65 and 13, for $143 = 65 \times 2 + 13$. The common measure of 65 and 13 cannot be greater than 13. We therefore ascertain whether 13 will measure 65, and find it will. 13, therefore, is the greatest common measure sought. The whole operation may be shown together, thus:—

143)637(4
572
65)143(2
130
13)65(5
65
00

To make the reason of this operation more obvious, observe, as 13 measures 65 and 13, it will also measure 143, which is twice $65 + 13$; and as it measures 65 and 143, it will also measure $637 = 143 \times 4 + 65$. Hence we deduce the following

RULE.

To find the greatest common measure of any two numbers, divide the greater by the less, and the less by the remainder of the first division; then this remainder by the remainder of the second division, then this second remainder by the third, and so on till nothing remains. The last remainder used as a divisor will be the common measure required.

If the greatest common measure of more than two numbers be required, find the greatest common measure of two of them as before; then of that common measure and one of the other numbers, and so on through the whole. The greatest common measure last found will be the one sought.

It is very convenient also, in many instances, to know the least common multiple of two or more numbers, that is, the least number that can be divided by each of those numbers without a remainder.

What is the least common multiple of 4, 6, 8, and 10?

| | | | | |
|---|---|---|---|---|
| 4) | 4 | 6 | 8 | 10 |
| 2) | 1 | 6 | 2 | 10 |
| | 1 | 3 | 1 | 5 |

$4\times2\times1\times3\times1\times5=120$, Ans.

As $4\times6\times8\times10=1920$, this sum is evidently a common multiple of each of these numbers, but 8 is a multiple of 4 (the first divisor in the above examples;) therefore we may take the quotient of 8 divided by $4=2$; and $4\times6\times2\times10=480$, which is also a multiple of 4, 6, 8, and 10. Also 10 is a multiple of 2, (the second divisor,) and 6 of 2 likewise; therefore $3\times5\times2\times4=120$, can be measured by 4, 6, 8, and 10, and is the least number that can be measured by them. Hence we derive the following

RULE.

To find the least common multiple of two or more numbers, divide them separately by any number that will divide two or more of them without a remainder, and set the quotients and the undivided numbers also in a line below. Again divide as before, and so on till no two numbers can be divided by any common divisor. Then multiply all the undivided numbers by all the divisors, and the product will be the required least common multiple, as seen in the example.

## Reduction of Vulgar Fractions.

Reduction of Vulgar Fractions is the bringing of them from one form or denomination into another, to prepare them for the operations of addition, subtraction, &c.

### I.

Reduce $\frac{288}{480}$ to its lowest terms.

As dividing the two terms of a fraction by the same number does not alter its value, we may here divide the 288 and 480 by any number that will divide both without a remainder.

Thus $8)\frac{288}{480} = \overset{12}{\frac{36}{60}} = \frac{3}{5}$, Ans. We first divide by 8, and the quotients by 12 and obtain $\frac{3}{5}$. No number will divide 3 and 5 without a remainder; hence $\frac{3}{5}$ are the lowest terms of the fraction $= \frac{288}{480}$.

Or we may find the greatest common measure of the numbers, thus:

$$\begin{array}{r} 288)480(1 \\ 288 \\ \hline 192)288(1 \\ 192 \\ \hline 96)192(2 \\ 192 \\ \hline 000 \end{array}$$

then $96)\frac{288}{480} = .\frac{3}{5}$

Greatest common measure,

To reduce fractions to their lowest terms, divide both terms by any number that will divide them without a remainder; divide the quotients in the same manner and so on till no number will divide them without a remainder. The last quotients will be the terms sought.

In the first instance we divided by 8 and 12, which gave the answer; in the second, after finding the greatest common measure 96, (in the manner before explained,) we divided both terms by it, which gave the same result, as must necessarily be the case, for $8 \times 12 = 96$.

Every number which terminates in 0, 2, 4, 6, or 8, can be successively divided by 2.

Every number terminating in 0, or 5, is divisible by 5.

Every number terminating in 0, is divisible by 10.

If the sum of the significant figures of any number is divisible by 3, the number itself can be divided by 3, and if the sum of the figures is divisible by 9, the number itself is also.

Thus 6342 is divisible by 3, because $6+3+4+2=15$; and 7236 is divisible by 9, because the sum of its figures is 18.

Reduce $\frac{29}{765}$ to its lowest terms.

29)765(26
58
185
174
11)26(2
22
4)11(2
8
3)4(1
3
Greatest common measure, 1)3(3
3
0

In this operation we see that the greatest common measure is 1; of course the fraction is already in its lowest terms; for any number divided by 1, gives a quotient equal to itself. Consequently 29 and 765 are *prime to each other;* as also are all fractions in their lowest terms.

EXAMPLES *for Practice.*

What is the greatest common measure of 28 and 42? Ans. 14.

Reduce $\frac{28}{42}$ to its most simple terms. Ans. $\frac{2}{3}$.

What is the greatest common measure of 49 and 147? Ans. 7.

Reduce $\frac{49}{147}$ to its most simple terms. Ans. $\frac{7}{21}$.

II.

Reduce $3\frac{4}{7}$ to an improper fraction.

It will be recollected that an improper fraction is one, the numerator of which is greater than the denominator, and has a value of course greater than unity. As 3 in this example is a whole number, it must be reduced to sevenths, because

the fraction is expressed in sevenths; this can be done by multiplying 3 by 7, the denominator of the fraction; $3 \times 7 = 21$. As 7 sevenths are equal to 1, evidently 3 times 7 sevenths, are equal to 3. We then have 21 sevenths $+ 4$ sevenths, $= 25$ sevenths or $\frac{25}{7}$. Ans.

Hence, to reduce a mixed number to an improper fraction, multiply the whole number by the denominator of the fraction, and to the product add the numerator; this sum written over the denominator will form the fraction required.

In $16\frac{3}{8}$ dollars how many eighths? Ans. $\frac{131}{8}$.
Reduce $75\frac{18}{25}$ to an improper fraction. Ans. $\frac{1743}{25}$.

## III.

Reduce $\frac{131}{8}$ to its equivalent whole or mixed number.

8)131
Ans. $16\frac{3}{8}$

We have here 131 eighths of a unit, and evidently the eighth part as many units, or whole numbers. Hence

To reduce an improper fraction to its equivalent whole or mixed number, divide the numerator by the denominator; the quotient will be the whole number, and the remainder, if any, will form the numerator of a fraction, which must be written over the denominator, and placed at the right hand of the whole number.

## IV.

Reduce 5 to an equivalent fraction, having 4 for a denominator.

5
4
20 and $\frac{20}{4}$ Ans.

The denominator here being 4, denotes that the units are to be divided into 4ths, or quarters; and 20 quarters are equal to 5 units; therefore

To reduce a whole number to an equivalent improper fraction, having a given denominator, multiply the whole number by the given denominator, and write the product over the given denominator.

This case is the reverse of the preceding.

## V.

A man had $\frac{1}{2}$ of $\frac{1}{3}$ of an eagle; what part of an eagle had he?

$\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$ Ans. If we divide the eagle into 5 equal parts, i. e. fifths, and each of these into 2 parts, (or, which is the same thing, multiply the fifths by 2,) we shall evidently have 10 halves of fifths = tenths. $\frac{1}{2}$ of $\frac{1}{5}$ of an eagle is therefore $\frac{1}{10}$ = 1 dollar.

A lady bought $\frac{2}{3}$ of $\frac{3}{4}$ of a yard of silk; how much did she buy?

$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$ Ans. Two thirds of three quarters is evidently 2 quarters = $\frac{1}{2}$.

A man, owning $\frac{7}{8}$ of a ship, sold $\frac{3}{5}$ of his share; what part of the ship did he sell?

$\frac{7}{8} \times \frac{3}{5} = \frac{21}{40}$ Ans. We have already seen, that multiplying or increasing the denominator of a fraction while the numerator remains the same, diminishes the value of the fraction in the same proportion. Thus in this example $\frac{1}{5}$ of $\frac{1}{8}$ is $\frac{1}{40}$; that is, the ship is supposed to be divided first into 8 equal parts, and each of these into 5, which makes the whole necessarily divided into 40 equal parts. We have also seen, that multiplying or increasing the numerator of a fraction, while the denominator remains the same, increases the value of the fraction; hence in this case, since the man owned 7 eighths, and sold 3 fifths of these 7 eighths, these two figures multiplied together will show the number of parts which he sold, from the 40 parts, into which the ship is supposed to be divided.

Hence, to reduce a compound fraction to an equivalent simple one, multiply all the numerators together for a new numerator, and all the denominators together for a new denominator.

## VI.

Reduce $\frac{1}{2}$ and $\frac{3}{8}$ to equivalent fractions having a common denominator. Ans. $\frac{3}{8}$ and $\frac{4}{8}$.

We have seen that multiplying both terms of a fraction by the same number or numbers, does not alter the value. If therefore we multiply the 1 and 2 by 3, the denominator of the second fraction, it will be $\frac{3}{8} = \frac{1}{2}$; and if we multiply the 2 and 3 by 2, the denominator of the first, it will be $\frac{4}{8} = \frac{2}{4}$.

Reduce $\frac{2}{5}$, $\frac{3}{4}$ and $\frac{1}{3}$ to equivalent fractions having a common denominator.

$$\frac{2 \times 4 \times 3 = 24}{5 \times 4 \times 3 = 60} = \frac{2}{5}.$$

$$\frac{3 \times 5 \times 3 = 45}{4 \times 5 \times 3 = 60} = \frac{3}{4}$$

$$\frac{1 \times 5 \times 4 = 20}{3 \times 5 \times 4 = 60} = \frac{1}{3}$$

It will be seen that we multiplied each numerator into all the denominators except its own for a new numerator; and all the denominators together for a common denominator; that is, both terms were multiplied by the same numbers.

$\frac{24}{60}$, $\frac{45}{60}$, $\frac{20}{60}$, Ans.

Hence, to reduce fractions of different denominators to equivalent fractions having a common denominator, multiply each of the numerators into all the denominators except its own for a new numerator, and all the denominators together for a common denominator.

When the denominators of fractions are not prime to each other, there is a more simple way to obtain a common denominator.

Reduce $\frac{2}{3}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ to fractions having a common denominator.

It is necessary to find the least multiple of 3 which will also be a multiple of 4, 6, and 8. This is 24. We have now only to change the fractions into 24ths of a unit, which can be done by dividing 24 by 3, 4, 6, and 8, and multiplying each term of the respective fractions by the respective quotients. Thus $24 \div 3 = 8$, and $8 \times 2$ the numerator of the first fraction, is equal to 16, the new numerator; and $8 \times 3 = 24$ the common denominator. Again, $24 \div 4$, the denominator of the second fraction, is 6, and $6 \times 3$, the numerator of the second fraction, is 18, the new numerator; and $6 \times 4 = 24$, the common denominator. In this manner we obtain $\frac{16}{24}$, $\frac{18}{24}$, $\frac{20}{24}$, and $\frac{21}{24}$.

Hence, to reduce fractions of different denominators to equivalent fractions having a common denominator,

Find the least common multiple of the several denominators, and it will be the common denominator required.

Divide the common denominator by the denominator of each fraction and multiply the quotient by the numerator, and the several products will be the numerators of the fractions required.

## VII.

What is the value of $\frac{3}{7}$ of a pound, Troy weight?

3
12
—
7)36(5
35
—
1
20
—
7)20(2
14
—
6
24
—
7)144(20$\frac{4}{7}$
14
—
4

Ans. 5oz. 2pwts. 20$\frac{4}{7}$grs.

Evidently $\frac{3}{7}$ of a pound are $\frac{3}{7}$ of 12 ounces; 1 seventh of 12 ounces is 12 sevenths of 1 ounce; (i. e. if each ounce be divided into 7 equal parts, and 1 part belonging to each ounce be taken, 12 parts, or sevenths, will be taken;) then 3 sevenths of 12 ounces are 3 times 12 = 36 sevenths of 1 ounce; hence we multiply the numerator 3 by 12. Again, 36 sevenths contain as many whole ones as the number of times which 7 is contained in 36; hence we divide the 36 sevenths by 7; the quotient is 5, and 1 seventh remains. One seventh of an ounce is 20 times 1 seventh of a penny-weight, or 2 penny-weights and 6 sevenths; the 6 sevenths of a penny-weight are 144 sevenths of a grain, or 20 grains and 4 sevenths of a grain. Hence

To find the value of a fraction in the known parts of an integer, Multiply the numerator by the parts of the next inferior denomination, and divide this product by the denominator; if any thing remain, multiply it by the next inferior denomination, and divide by the denominator as before; and so on, as far as necessary; the several quotients will be the answer required, in the denominations of which the several dividends are parts.

The numerator of a fraction may be considered as a remainder, and the denominator as a divisor. This rule therefore has its reason in the nature of compound division.

## VIII.

Reduce $\frac{4}{5}$ of a penny to the fraction of a pound.

A penny is $\frac{1}{12}$ of a shilling; and a shilling is $\frac{1}{20}$ of a pound. Hence $\frac{4}{5}$ of a penny is equivalent to $\frac{4}{5}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound. Reduce this compound fraction to a simple one, which will be the required fraction.

$$\frac{4}{5} \times \frac{1}{12} \times \frac{1}{20} = \frac{4}{1200} = \frac{1}{300} \text{ Ans.}$$

Hence, to reduce a fraction of one denomination to the fraction of a greater, but retaining the same value,

Reduce the given fraction to a compound one, by comparing it with all the denominations between itself and that to which it is to be reduced; then reduce the compound fraction to a simple one.

Reduce $\frac{3}{5}$ of a farthing to the fraction of a pound.

Three fifths of a farthing are evidently $\frac{3}{5}$ of $\frac{1}{4}$ of a penny, since 4 farthings make a penny; so $\frac{1}{4}$ of a penny is $\frac{1}{4}$ of $\frac{1}{12}$ of a shilling, since 12 pence make a shilling; and $\frac{1}{12}$ of a shilling is $\frac{1}{12}$ of $\frac{1}{20}$ of a pound, as 20 shillings make a pound. Hence the fraction evidently is $\frac{3}{5}$ of $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound, which must be reduced to a simple fraction according to Case 5.

$$\frac{3}{5} \times \frac{1}{4} \times \frac{1}{12} \times \frac{1}{20} = \frac{3}{4800} = \frac{1}{1600} \text{ Ans.}$$

## IX.

Reduce $\frac{1}{300}$ of a pound to the fraction of a penny.

$\frac{1}{300}$ of a pound is $\frac{1}{300}$ of 20 shillings, which, considered as a fraction, are $\frac{20}{1}$ of a shilling, which are 20 times 12 pence, or fractionally $\frac{12}{1}$ of a penny; then the fraction will be $\frac{1}{300}$ of $\frac{20}{1}$ of $\frac{12}{1}$, which must be reduced to a simple fraction, as before.

$$\frac{1}{300} \times \frac{20}{1} \times \frac{12}{1} = \frac{240}{300} = \frac{4}{5} \text{ Ans.}$$

Hence, to reduce a fraction of one denomination to the fraction of another, but less, retaining the same value,

Reduce the fraction to a compound one by comparing it, in an inverted order, with all the denominations as before; then reduce the compound fraction to a simple one.

Or, Multiply the numerator by that number which it takes of the less denomination to make one of the greater, and so on through all the denominations, which will make the numerator to the given denominator.

## X.

What fractional part of a pound are 6 s. 7 d. $3\frac{3}{5}$ farthings?

| *s.* | | *s. d. qrs.* |
|---|---|---|
| 20 | integral part | 8 7 $3\frac{3}{5}$ |
| 12 | | 12 |
| 240 | | 79 |
| 4 | | 4 |
| 960 | | 319 |
| 5 | | 5 |
| 4800 | | 1598 |

In a pound there are 960 farthings, and 5 times 960 fifths of farthings = 4800. We wish now to know how many of these fifths are equal to 6 s. 7 d. $3\frac{3}{5}$ qrs.; this we can ascertain by reducing the sum to fifths of farthings; which we find to be 1598. The answer therefore is $\frac{1598}{4800} = \frac{799}{2400}$.

Hence, to reduce any given sum or quantity to the fraction of any greater denomination, of the same kind,

Reduce the given sum to the lowest denomination expressed, for a numerator; then reduce the integral part to the same denomination for a denominator.

If the given sum itself contain a fraction, it must be reduced to the fractional part expressed by the denominator.

## XI.

Reduce $\frac{3}{5}$ to a fraction of the same value, having 12 for a numerator.

As the value of fractions depends on the proportion which the terms bear to each other, the required numerator must be as many times greater than 5, as 12 is greater than 3; but $12 = 3 \times 4$; therefore $5 \times 4 = 20$ is the required denominator; and the required fraction is $\frac{12}{20}$.

Hence, to reduce fractions to others having a given numerator,

Divide the given numerator of the required fraction by the numerator of the given fraction, and multiply the denominator of the given fraction by the quotient; the product will be the required denominator.

Or, Multiply the given numerator of the required fraction by the given denominator, and divide the product by the numerator of the given fraction; the quotient will be the required denominator.

Take the same example as above.

12
5
3)60
20 Ans. $\frac{12}{20}$ as before.

Reduce $\frac{8}{4}$ to an equivalent fraction having 34 for a numerator.

$$\begin{array}{r} 34 \\ 4 \\ \hline 3)136 \\ \hline 45\frac{1}{2} \end{array}$$

denominator. Ans. $\frac{34}{45\frac{1}{2}}$

## XII.

Reduce $\frac{12}{20}$ to a fraction of the same value, having 5 for a denominator.

As neither multiplying nor dividing both terms of a fraction by the same number alters the value of the fraction, if we divide 20 by 5 the quotient will be 4; and $12 \div 4 = 3$, and $20 \div 4 = 5$; the required fraction then is $\frac{3}{5}$.

Or, we may obtain the same result thus, $12 \times 5 = 60$, and $60 \div 20 = 3$, the required numerator.

Hence, to reduce fractions to others of the same value having a given denominator,

Divide the denominator of the given fraction by the given denominator of the required fraction, and the numerator of the given fraction by that quotient; the quotient of this last division will be the required numerator.

Or, Multiply the numerator of the given fraction by the given denominator of the required fraction, and divide the product by the denominator of the given fraction; the quotient will be the required numerator.

Reduce $\frac{7}{8}$ to a fraction of the same value, having 35 for a denominator.

$35 \times 7 = 245$, and $245 \div 8 = 30\frac{5}{8}$. Ans. $\frac{30\frac{5}{8}}{35}$.

From this and the preceding cases arises another kind of fractions $\left(\frac{30\frac{5}{8}}{35}, \frac{34}{35\frac{1}{2}}\right)$ called *mixed* fractions.

## XIII.

Reduce $\frac{30\frac{5}{8}}{45}$ to a simple fraction.

$30 \times 8 + 5 = 245$
$35 \times 8 \quad = 280 = \frac{7}{8}$ Ans.

In this example we multiply first the numerator by 8, the denominator of

its fractional part; we therefore multiply the denominator 35 by 8, so that the fraction may retain the same value. The 5, the numerator of the fractional part of the numerator, is multiplied by 8, simply by removing 8, its denominator; it must therefore be added to the product of 8 multiplied by 30.

Reduce $\frac{34}{45\frac{1}{3}}$ to a simple fraction.

$$\frac{34\times3}{45\times3+1} = \frac{102}{136} = \frac{3}{4} \text{ Ans.}$$

Hence, to reduce a mixed fraction to a simple one,

Multiply both terms of the fraction by the denominator of the fractional part, and to the product of that term which contains the fractional part, add its numerator.

Or, when the numerator is a mixed number, reduce it to an improper fraction, and its numerator will be the new numerator; then multiply the denominator of the fraction by the denominator of the fractional part, for a new denominator.

When the denominator is a mixed number, reduce it as before, then multiply the numerator of the fraction by the denominator of the fractional part, for a new numerator.

### Addition of Vulgar Fractions.

If the fractions had all the same denominators, it is plain that they might be added by simply adding their numerators, as $\frac{1}{4}$ and $\frac{2}{4}$ evidently make $\frac{3}{4}$, in the same manner as two quantities and one quantity of the same, make 3 of that kind.

But when numbers express different magnitudes or values, they cannot be added in the usual way, so as to express their separate values or magnitudes in the same sum; for instance, we cannot add together in the common mode of addition, 15 inches and 15 yards; or 5 dollars and 10 pounds; or 17 feet and 4 hours, or any other numbers of diverse kinds.

When fractions therefore have different denominators, the parts of which they are composed cannot be added together, because the parts are of different magnitudes. This difficulty can be obviated by reducing the parts to the same magnitude, or in other words, to a common value. If I bring dollars into shillings, and pounds into shillings, I then can add them together.

The process for reducing fractions to a common denominator has been explained, but will now be more fully illustrated.

A merchant sold $\frac{3}{5}$ and $\frac{3}{8}$ of a pound of tea. How much did he sell?

*Fifths* and *eights* are different parts; but if the fifths be divided each into 8 equal parts, they will be *fortieths;* and if the eights be divided each into 5 equal parts, they will be *fortieths* also. Of course the parts will be alike. In dividing them thus; $\frac{3}{5}$ are equal to $\frac{24}{40}$, and $\frac{3}{8}$ to $\frac{15}{40}$, and the sum of both is $\frac{39}{40}$.

What is the sum of $\frac{3}{7}$, $\frac{1}{2}$, and $\frac{3}{5}$?

These fractions reduced to a common denominator, become $\frac{30}{70}$, $\frac{35}{70}$, and $\frac{42}{70}$. The sum of the numerators is 212; and hence the sum of the fractions is $\frac{212}{70} = 3\frac{1}{35}$, Ans. The mode of finding the value of an improper fraction has been before illustrated.

What is the sum of $5\frac{3}{5}$ and $7\frac{3}{8}$?

As 5 and 7 are whole numbers of the same denomination, they can be added together; $5 + 7 = 12$; and $\frac{3}{5}$ and $\frac{3}{8}$ reduced to a common denominator become $\frac{24}{40}$ and $\frac{15}{40} = \frac{39}{40}$; and the sum of the two given numbers is $12\frac{39}{40}$.

The same result may be obtained by first reducing the mixed numbers to improper fractions; thus, $5\frac{3}{5} = \frac{28}{5}$ and $7\frac{3}{8} = \frac{59}{8}$. These improper fractions reduced to a common denominator, become $\frac{224}{40}$ and $\frac{325}{40} = \frac{477}{40} = 12\frac{39}{40}$. This latter method is sometimes more convenient.

What is the sum of $4\frac{1}{2}$, $\frac{8}{9}$, $\frac{7}{8}$, 5, and $14\frac{3}{8}$? Ans. $26\frac{1}{72}$.

A whole number may be reduced to an improper fraction simply by placing a unit under it; thus $5 = \frac{5}{1}$.

What is the sum of 2, $\frac{1}{2}$ of $\frac{3}{4}$ and $\frac{2}{3}$ of $\frac{7}{9}$?

$2 = \frac{2}{1}$
$\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$
$\frac{2}{3}$ of $\frac{7}{9} = \frac{14}{27}$

The compound fractions must be reduced to simple ones before reducing them to a common denominator.

Ans. $2\frac{190}{216}$.

What is the sum of $\frac{5}{6}$ of a ton, $\frac{2}{3}$ of a cwt. and $\frac{3}{4}$ of a pound?

There is a difficulty in finding the amount of these fractions, because they are of different denominations; but this difficulty may be obviated by reducing them all to fractions of the same denomination. We may reduce each to the fraction of a pound; thus, $\frac{5}{6}$ of a ton is $\frac{5}{6}$ of $\frac{2240}{1}$ of $\frac{1}{4}$ of $\frac{28}{1}$; and $\frac{2}{3}$ cwt. is $\frac{2}{3}$ of $\frac{1}{4}$ of $\frac{28}{1}$; and $\frac{3}{4}$ of a pound is simply $\frac{3}{4}$.

These fractions then reduced to simple ones, are $\frac{11}{6}\frac{2}{00}$ $\frac{2}{8}\frac{2}{4}$, and $\frac{3}{4}$ of a pound, which reduced to a common denominator, are $\frac{40}{216}\frac{32}{00}$, $\frac{5}{216}\frac{3}{16}$, and $\frac{162}{216}$. Their sum is $\frac{40}{216}\frac{86}{78}$ of a pound = 16 cwt. 3 qrs. 16 lbs. 4 oz. and $14\frac{2}{3}$ drams.

From the preceding illustrations we deduce the following

RULE.

Reduce compound fractions to simple ones; whole and mixed numbers to improper fractions; fractions of different denominations to those of the same, and all to a common denominator. The sum of the numerators placed over the common denominator will be the sum of the fractions.

SUBTRACTION OF VULGAR FRACTIONS.

A boy had $\frac{3}{4}$ of a pound of raisins, and he gave his sister $\frac{1}{4}$ of a pound; how many had he left?

In this case, taking 1 fourth from 3 fourths, evidently leaves 2 fourths.

From $\frac{3}{4}$ take $\frac{5}{9}$.

These fractions reduced to a common denominator, are $\frac{27}{36}$ and $\frac{20}{36}$; and 27 — 20 = 7, which written over the common denominator 36 is the remainder. The same reasons exist here for reducing the fractions to a common denominator as in addition. Hence the following

RULE.

Prepare the fractions as in addition. The difference of the numerators written over the common denominator is the remainder sought.

From $2\frac{8}{9}$ take $\frac{8}{9}$.

$2\frac{8}{9} = \frac{2}{9}^{2}$, which with $\frac{8}{9}$ reduced to a common denominator becomes $\frac{1}{7}\frac{9}{2}^{8}$ and $\frac{8}{7}\frac{4}{2}$; and 198 — 64 = 134, which, written over 72 the common denominator, forms the remainder $\frac{1}{7}\frac{3}{2}^{4} = 1\frac{31}{36}$.

A fraction may be more conveniently taken from a whole number by simply taking the numerator from the denominator, and placing the remainder over the denominator, and then taking one from the whole number; thus $7 - \frac{2}{9} = 6\frac{7}{9}$.

A mixed number may also be subtracted from a mixed number, when the fraction of the subtrahend is less than that of the minuend. The fractions must be reduced to a

common denominator, if they are not already. Subtract the numerator of the subtrahend from the common denominator, and add the difference to the numerator of the minuend, and add one to the integer of the subtrahend before subtracting it.

EXAMPLE. From $15\frac{2}{5}$ take $9\frac{4}{5}$.

$15\frac{2}{5} - 9\frac{4}{5} = 5\frac{3}{5}$ remainder.

### MULTIPLICATION OF VULGAR FRACTIONS.

It has been already shown that the value of a fraction is increased by increasing its numerator, and diminished by diminishing its numerator while the denominator remains the same; and also that the value is increased by diminishing the denominator, while the numerator remains the same; thus, $\frac{3}{4}$ is greater than $\frac{2}{4}$.

Hence to repeat or multiply the numerator by any given number, without altering the denominator, increases the value of the fraction just so many times. In like manner dividing or diminishing the numerator, while the denominator remains the same, diminishes the value of the fraction the same number of times; thus, $\frac{1}{4}$ is less than $\frac{2}{4} = \frac{1}{2}$.

It is also evident, that dividing or diminishing the denominator, while the numerator remains the same, increases the value of the fraction in the same proportion, as $\frac{1}{2}$ is greater than $\frac{1}{4}$ and the contrary; for as the denominator shows into how many parts unity is divided, the greater the number of those parts, the less will be the value of each; and on the contrary, the less their number, the greater their value.

Now as multiplication is the repetition of a sum a given number of times, to multiply a fraction by a whole number we increase the value of the fraction as many times as that number contains units; thus, $\frac{1}{2} \times 2 = \frac{2}{2} = 1$ or unity; or we may divide the multiplier, $2 \div 2 = 1$, and $\frac{1}{1} = 1$. In both cases the value of the fraction is increased in a twofold proportion, that is, it is multiplied by 2. But as a proper fraction is always less than unity, in multiplying by a proper fraction, we repeat or take the value of the multiplicand less than once; thus $\frac{1}{2}$ multiplied by $\frac{1}{2}$, or $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$; and $\frac{2}{3} \times \frac{3}{4}$, or $\frac{2}{3}$ of $\frac{3}{4} = \frac{6}{12} = \frac{1}{2}$. In these cases the result is produced by multiplying the numerators together for the numerator of the product, and the denominators together for the denominator, according to the rule for reducing compound to simple fractions.

From the preceding illustrations it is evident that suppressing the denominator is multiplying the fraction by the number expressed by the denominator; thus $\frac{1}{2}$ with the denominator 2 suppressed, becomes 1.

The preceding propositions may be briefly stated, thus:

By multiplying } the numerator, the fraction is { multiplied.
By dividing } { divided.

By multiplying } the denominator, the fraction is { divided.
By dividing } { multiplied.

Hence altering the numerator alters the value of the fraction in the same way; and altering the denominator alters the value of the fraction in the contrary way.

From the preceding observations it will be seen that the term *multiplication* has a more general signification when applied to fractions, than when applied to whole numbers; since in the former case it does not always denote an increase of numbers, as it does when applied to whole numbers. A definition of *multiplication* in its most extensive application, may be thus stated: *the multiplication of one number by another, is to form a number by means of the first, in the same manner as the second is formed by unity; or which shall bear the same relation to the first, that the second does to unity.*

Thus, $4 \times 2 = 8$; that is, 8 is twice as large as 4, and so in like manner, 2 is twice as large as 1, or unity. But $4 \times \frac{1}{4} = \frac{4}{4} = 1$; that is, 1 is four times less than 4; and so in like manner, $\frac{1}{4}$ is four times less than 1. So if we wish to multiply 4 by $\frac{3}{4}$ the product is $\frac{12}{4} = 3$; that is, the product is three fourths as large as the multiplicand. By the first operation we obtain the improper fraction $\frac{12}{4}$, and then according to the rule before given, for finding the value of an improper fraction, we divide the numerator 12 by the denominator 4, and obtain the whole number 3 for the product. The same result would be obtained by first dividing the whole number 4 by the denominator 4, and multiplying the quotient by the numerator: $4 \div 4 = 1$ and $1 \times 3 = 3$. Hence,

To multiply a whole number by a fraction, multiply the whole number by the numerator, and divide the product by the denominator. Or, divide the whole number by the denominator, and multiply the quotient by the numerator.

### Examples.

1. Multiply 45 by $\frac{5}{9}$.

45 or 9)45

5 5

9)225 5

Ans. 25 Ans. 25

2. Multiply 65 by $\frac{3}{7}$.

65

3

7)195

27 and 6 remainder, which are 6 sevenths. Hence the Ans. $27\frac{6}{7}$. Or,

7)65

9 and 2 remainder, or $9\frac{2}{7}$, and $9\frac{2}{7} \times 3 = 27\frac{6}{7}$, as before.

To multiply a fraction by a whole number, multiply the numerator or divide the denominator by the whole number.

### Example.

Multiply $\frac{5}{12}$ by 6.

$5 \times 6 = 30$, that is, $\frac{30}{12} = 2\frac{1}{2}$.

Or thus: $12 \div 6 = 2$ which placed under the numerator 5, make $\frac{5}{2} = 2\frac{1}{2}$, as before.

When the denominator is not exactly divisible by the whole number, we always multiply the numerator by it.

As multiplying by a fraction is the taking from the multiplicand a part denoted by the multiplier, the operation may be properly expressed as a compound fraction, as has before been shown. Hence for the multiplication of vulgar fractions we have the following general

### Rule.

Reduce whole or mixed numbers to improper fractions, and compound fractions to simple, then multiply the numerators of the factors together for the numerator of the product, and the denominators for the denominator.

### Examples.

1. Multiply $\frac{9}{10}$ by $\frac{7}{5}$. $\frac{9}{10} \times \frac{7}{5} = \frac{63}{50} = 1\frac{13}{50}$, Ans.
2. Multiply 7 by $\frac{8}{9}$. $\frac{7}{1} \times \frac{8}{9} = \frac{28}{9} = 3\frac{1}{9}$, Ans.
3. Multiply $1\frac{2}{3}$ by 6. $1\frac{2}{3} \times \frac{6}{1} = \frac{7}{3} \times \frac{6}{1} = 5\frac{7}{13}$, Ans.
4. Multiply $\frac{2}{3}$ of $\frac{3}{4}$ by $\frac{8}{7}$. $\frac{2}{3}$ of $\frac{3}{4} = \frac{6}{12}$, and $\frac{6}{12} \times \frac{8}{7} = \frac{48}{84} = \frac{4}{7}$, Ans.
5. Multiply $3\frac{1}{2}$ by $2\frac{5}{6}$. $3\frac{1}{2} = \frac{7}{2}$ and $2\frac{5}{6} = \frac{17}{6}$ and $\frac{7}{2} \times \frac{17}{6} = \frac{119}{12} = 9\frac{11}{12}$, Ans.

6. Multiply $\frac{6}{7}$ by $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of $7\frac{1}{4}$. $7\frac{1}{4} = \frac{29}{4}$; then $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{29}{4} = \frac{174}{96}$, and $\frac{6}{7} \times \frac{174}{96} = \frac{1044}{672} = 1\frac{372}{672}$, Ans.

By these examples it will be seen that the above rule is applicable to all possible cases.

## Division of Vulgar Fractions.

As Division in all cases is exactly the reverse of Multiplication, it is evident that in the division of fractions, the quotient will often be greater than the dividend. Hence the dividend cannot in all cases properly be said to contain the divisor; yet the word is frequently so applied to the dividend by analogy, even when it does not contain the divisor.

Hence we may give the following general definition of division: *to divide one number by another, is to find a third number which shall bear the same relation to the first, as unity does to the second:* thus, $24 \div 4 = 6$, that is, 24 contains 4 six times, and 6 contains unity 6 times. So $3 \div \frac{1}{3} = \frac{3}{1} = 9$, that is, 3 contains one third 9 times, and 9 contains unity 9 times.

In like manner $4 \div \frac{2}{6} = \frac{24}{2} = 12$. In these two last examples it will be seen that the quotients are just as many times larger than the dividend, as the denominator of the fraction is larger than the numerator; and consequently the dividends are only one third of the quotient, for $\frac{3}{9} = \frac{1}{3}$ and $\frac{4}{12} = \frac{2}{6} = \frac{1}{3}$. In these cases it is evident that in 3 units there are nine thirds; and in 4 units there are 24 sixths, for each of the 4 units is repeated 6 times, that is, in both cases we multiply the whole number by the denominator of the fractional divisor. But as 24 is 6 times larger than 4 the dividend, instead of 3 times as it should be, we may consider 4 as an improper fraction, $\frac{4}{1}$. Now if we multiply this denominator 1, by 2 the numerator of the fractional divisor, and place the product 2 under the 24, it will be $\frac{24}{2} = \frac{12}{1} = 12$, the quotient sought. So likewise in the former case $3 = \frac{3}{1}$, and multiplying this by 1, the numerator of the divisor, the quotient is $\frac{9}{1} = 9$.

Let it be required to divide $\frac{8}{10}$ by $\frac{2}{5}$. It is evident that $\frac{1}{5} = \frac{2}{10}$ and $\frac{2}{5}$ are therefore equal to $\frac{4}{10}$. We wish now to ascertain how many times 4 tenths are contained in 8 tenths; and the answer is 2 times, for $8 \div 4 = 2$. Now if we multiply the numerator of the dividend by the denominator of the quotient, and the denominator of the dividend

by the numerator of the divisor, we shall have $\frac{4}{2}\frac{0}{0} = \frac{4}{2} = 2$. Or it may be thus illustrated. As the dividend is $\frac{2}{5}$ of the quotient, and $\frac{1}{5}$ being $\frac{1}{2}$ of $\frac{2}{5}$, we shall obtain $\frac{1}{5}$ of the quotient by taking $\frac{1}{2}$ of the dividend, that is, dividing it by 2, which may be done by multiplying the denominator 10 by 2, the numerator of the divisor, then by multiplying $\frac{1}{5}$ of the quotient $\frac{8}{2\,0}$ by 5, we obtain the whole quotient, $\frac{4}{2}\frac{0}{0} = \frac{4}{2} = 2$. Hence the following

RULE.

To divide any sum by fractions, prepare the factors as in multiplication of fractions; invert the divisor, and proceed as in multiplication.

EXAMPLES.

1. Divide $\frac{5}{7}$ by $\frac{3}{5}$. $\frac{5}{7} \times \frac{5}{3} = \frac{2\,5}{2\,1} = 1\frac{4}{2\,1}$, Ans.
2. Divide $12\frac{3}{5}$ by $5\frac{7}{8}$. $12\frac{3}{5} = \frac{6\,3}{5}$ and $5\frac{7}{8} = \frac{4\,7}{8}$; then $\frac{6\,3}{5} \times \frac{8}{4\,7} = \frac{5\,0\,4}{2\,3\,5} = 2_{2\,3\,5}\frac{4\,7}{0}$, Ans.
3. Divide 8 by $\frac{3}{4}$. $8 = \frac{8}{1}$ and $\frac{8}{1} \times \frac{4}{3} = \frac{3\,2}{3} = 10\frac{2}{3}$, Ans.

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## SECTION XVII.

### CHANGING VULGAR TO DECIMAL FRACTIONS.

The only difference between Vulgar and Decimal Fractions, consists in the manner of dividing a unit, and the way of expressing them. The former divides it into any number of parts, and the latter into ten, and if a further division is made, each of these are divided into ten, and so on. To change vulgar to decimal, then, we have only to divide the unit into ten, a hundred, or a thousand parts, and observe what proportion of these parts is expressed by the vulgar fraction.

Reduce $\frac{1}{2}$ to decimals. We divide a unit into 10 parts and $\frac{1}{2}$ of 10 is .5.

Reduce $\frac{2}{2\,5}$ to decimals. Divide 1 into 100 parts; $\frac{1}{2\,5}$ of 100 is .4, and consequently, $\frac{2}{2\,5}$ is twice .4 = .8.

Hence the following general

RULES.

If the fraction to be reduced is a proper fraction, multiply the numerator by 10, 100, or 1000, &c. and divide it by the denominator,

2. If the fractions are mixed or compound, reduce them to simple ones and proceed as before.

It frequently is true, that the numerator thus multiplied, cannot be divided by the denominator without some remainder. In such cases no Decimal can be obtained which will express the whole value of the Vulgar Fraction. But 5 or 6 decimal figures will ordinarily be sufficient.

EXAMPLE. Reduce $\frac{1}{3}$ to a decimal.

$1 \times 10 \times 10 = 100.$ $\quad 3)100$

$.3333+$

By continuing to multiply by 10, or what is the same thing, suppose ciphers to be added, and carrying on the division by 3, the decimals might be obtained forever.

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## SECTION XVIII.

### PROPORTION, OR RULE OF THREE.

If a line is a foot in length, and another 2 feet, it is easy to understand, that two others may be proportioned to each other in a similar manner.

If 5 dollars will pay 8 men for labouring a day, 15 dollars will pay 3 times as many men, $= 24$, because 15 dollars is 3 times 5 dollars. The nature of proportion, &c. is well explained by Mr. Lacroix, to which the attention of the learner is directed.

"The *ratio*, or *relation* of two numbers, is the quotient arising from dividing one by the other.

We introduce some examples to illustrate the theory of ratios and proportions.

1. If 13 yards of cloth cost 130 dollars, what will be the price of 18 yards of the same cloth?

If we know the price of one yard of the cloth, it is plain that we can repeat this price 18 times and thus obtain the price of 18 yards. And since 13 yards cost 130 dollars, one yard must have cost the 13th part of 130 dollars; or, performing the division, we find the result 10 dollars, which multiplied by 18, gives 180 dollars as the price of 18 yards.

2. A courier, who travels always at the same rate, having gone 5 leagues in 3 hours, how many will he go in 11 hours?

He goes in one hour $\frac{1}{3}$ of 5 leagues, or $\frac{5}{3}$ of one league, and of course in 11 hours he will go 11 times as far, i. e. $\frac{5}{3}$ of a league multiplied by $11 = \frac{55}{3}$, that is, 18 leagues and 1 mile.

3. In how many hours will the courier of the preceding question go 22 leagues?

If we knew the time he would occupy in going one league, we should have only to repeat this number 22 times, and the result would be the number of hours required. And since it requires 3 hours to go 5 leagues, it will require only $\frac{1}{5}$ of the time, i. e. $\frac{3}{5}$ of an hour, to go one league; this number multiplied by 22 gives $\frac{66}{5} = 13$ hours and $\frac{1}{5}$, that is, 13 hours and 12 minutes.

In the preceding statements, the known numbers and those required depend on each other in a manner that it will be well to examine.

To do this we may resume the first question, in which it is required to find the price of 18 yards, of which 13 cost 130 dollars.

It is plain that the price of this piece would be double, if the number of yards were double; that the price would be triple if the number of yards were triple, and so on; also that for half or two thirds of the piece, we should have to pay but $\frac{1}{2}$ or $\frac{2}{3}$ of the whole price.

Hence if there be two pieces of the same cloth, the price of the second ought to contain that of the first as many times as the length of the second contains the length of the first, and this circumstance is stated in saying, that the prices are in proportion to the lengths.

The relation, or ratio of the lengths, then, is that number, whether whole or fractional, which denotes how many times one of the lengths contains the other. If the first piece had 4 yards and the second 8, the ratio of the former to the latter would be 2. In the given example the first piece had 13 yards and the second 18; the ratio of the former to the latter is then $\frac{18}{13}$, or $1\frac{5}{13}$.

As the prices have the same ratio to each other that the lengths have, 180 divided by 130 must give $\frac{18}{13}$, which is the case.

The four numbers 13, 18, 130, 180, written in this order, are such that the second contains the first as many times as

the fourth contains the third, and thus they form what is called a *proportion.*

*A relation is not changed by multiplying or dividing each of its terms by the same number.**

To denote that there is a proportion between the numbers 13, 18, 130, and 180, they are usually written thus, 13 : 18 : : 130 : 180, which is read 13 *is to* 18 *as* 130 *is to* 180; that is, 13 is the same part of 18 that 130 is of 180, or the ratio of 13 to 18 is the same as that of 130 to 180.

The first term of a relation is called an *antecedent*, and the second a *consequent.* In a *proportion* there are two antecedents and two consequents, viz. the antecedent of the first ratio, and that of the second; the consequent of the first ratio, and that of the second. In the proportion 13 : 18 : : 130 : 180, 13 and 130 are the antecedents, and 18 and 180 the consequents.

To ascertain that there is a proportion between the four numbers 13, 18, 130 and 180, we must see if the fractions $\frac{18}{13}$ and $\frac{180}{130}$ are equal,—and to do this, we reduce the latter to its lowest terms; and if they be equal, as is supposed by the nature of proportion, it follows that by reducing them to the same denominators the numerators will become equal, and consequently that 18 multiplied by 130 will give the same product as 180 by 13. This is actually the case, and proves that *if four numbers are in proportion, the product of the first and last, or of the two extremes, is equal to the product of the second and third, or the two means.*†

We see that if four numbers were not in proportion they would not possess the above property; for the fraction which expresses the first ratio not being equivalent to that which expresses the second, the numerator of the one will not be equal to that of the other, when they are reduced to a common denominator.

The order of the terms of a proportion may be changed, provided they be so placed that the product of the extremes be equal to that of the means. In the proportion 13 : 18 : : 130 : 180, the following arrangements may be made.

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* The learner will do well to remember this.

† All the operations in this rule are founded on this principle. The learner should fix it in mind.

13 : 18 : : 130 : 180
13 : 130 : : 18 : 180
180 : 130 : : 18 : 13
180 : 18 : : 130 : 13
18 : 13 : : 180 : 130
18 : 180 : : 13 : 130
130 : 13 : : 180 : 18
130 : 180 : : 13 : 18

In each one of these the product of the extremes is formed of the same factors, and the product of the means of the same factors. The second arrangement is one of those which most frequently occurs.

Since the product of the means is equal to that of the extremes, one product may be taken for the other; and, as in dividing the product of the extremes by one extreme, we must necessarily find the other as the quotient, so in dividing by one extreme the product of the means, we shall find the other extreme. For the same reason, if we divide the product of the extremes by one of the means, we shall find the other mean.

*We can then find any one term of a proportion, when we know the other three, for the term sought must be one of the extremes or one of the means.*

The operation by which, when any three terms of a proportion are given, we find the fourth, is called the *Rule of Three.* Writers have distinguished it into several kinds; but this is unnecessary, when the nature of proportion and the enunciation of the question are well understood.

Among four numbers which constitute a proportion, there are two of the same kind, ahd two others of the same kind but different from the first two.

If it were required to find how many days it would take 27 men to perform a piece of work, which 15 men, working at the same rate, would do in 18 days; we see that the number of days should be less, in proportion as the number of men is greater, and vice versa. There is still a proportion in this case, but the order of the terms is inverted. The first number of days would contain the second as many times as the second number of workmen contains the first. This order of the terms being the reverse of that assigned them by the enunciation of the question, we say that the number of workmen is in the *inverse ratio* of the number of days.

### GENERAL RULE.

Make the number which is of the same kind with the answer the third term, and the two remaining ones the first and second, putting the greater or less first, according as the third

is greater or less than the term sought; then the fourth term will be found by multiplying together the second and third, and dividing the product by the first." Or,

Besides the general rule there are four others, either of which, when applicable, performs the work more concisely. They follow,—

*Rule* 1.

By dividing the second term by the first, and multiplying the quotient by the third: Or

*Rule* 2.

By dividing the third term by the first, and multiplying the quotient by the second: Or

*Rule* 3.

By dividing the first term by the second, and the third by the quotient: Or

*Rule* 4.

By dividing the first term by the third, and the second by the quotient, the answer may be found.

[The distinction between direct and inverse proportion is not necessary, the latter being the proportion where more requires less, and the reverse. If 10 dollars will furnish 5 men with provisions 5 days, it is obvious at once that the same money must furnish 8 men less time, and the reverse.]

### Rule of Three in Vulgar Fractions.

Questions are stated as in whole numbers, after the fractions are prepared, and the operation is performed by inverting the first term, and then proceeding to multiply the numerators together, and also the denominators, as in the multiplication of vulgar fractions.

*Example.* If $\frac{2}{3}$ of a dollar buy $\frac{7}{8}$ of a yard of cloth, what will $\frac{1}{4}$ of a dollar buy?

$\frac{2}{3} : \frac{7}{8} :: \frac{1}{4}$ $\quad \frac{3}{2} \times \frac{7}{8} \times \frac{1}{4} = \frac{21}{64}$ yd. Ans.

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## SECTION XIX.

## COMPOUND PROPORTION.

It frequently becomes necessary to involve a larger number of circumstances in a proportion, than those found in the preceding examples.

1. If $100 gain $6 in 12 months, what will $200 gain in 8 months?

This may be resolved into two questions in simple proportion, and may be stated thus:

If 12 months give $6, what will 8 months give? = $4.

Then if $100 : $4 :: $200 : = $8.

But it is perfectly easy to combine them both into one operation, by multiplying the numbers together which compose the terms, that are involved in the parts of the proportion. Thus:—

If $100 \times 12 : 6 :: 200 \times 8$?:

As $100 for 12 months would be worth as much as $1200 for one month, these numbers may be multiplied together; and as $200 for eight months will be worth just as much as $1600 for one month, these numbers may be multiplied together. The proportion then may be expressed thus:

If $1200 gain $6, what will 1600 gain? The answer must be just as much larger than $6, as $1600 is larger than $1200, = $8.

2. If 6 men in 12 days build 25 rods of wall, how much will 12 men build in 50 days?

Here 6 men in 12 days must perform the same labor as one man in 6 times 12 days; and 12 men in 50 days must perform the work of one man in 12 times 50 days.

Hence the operation may be thus expressed:

If $\left\{\begin{matrix}6\text{ men}\\12\text{ days}\end{matrix}\right.$ = 72 days' labor, build 50 rods, $\left\{\begin{matrix}12\text{ men}\\50\text{ days}\end{matrix}\right.$ = 600 days' work, will build as much more than 50 rods, as 600 is greater than 72, = $416\frac{2}{3}$ rods.

Or, the usual form,—

6 men, rods, 12 men,
12 days : 50 :: 50 days : $416\frac{2}{3}$ rods, Ans.

This is denominated *Compound Proportion.*

The following may be adopted as a general

RULE.

"Make the number which is of the same kind with the required answer, the third term; and of the remaining numbers, take any two that are of the same kind, and place one

for a first term and the other for a second term, according to the direction in simple proportion; then any other two of the same kind, and so on till all are used; lastly, multiply the third term by the product of the second terms, and divide the result by the product of the first terms, and the quotient will be the fourth term, or answer required." (*Lacroix.*)

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## SECTION XX.

### CONJOINED PROPORTION.

The numbers of terms are frequently increased to a greater number than those mentioned in the last Section, and therefore such are denominated *Conjoined Proportion.* As the principles are the same as those already illustrated, it is necessary to furnish only a single example.

If 6 shillings in N. E. are equal to 8 in N. York, and 16 in N. Y. are equal to 15 in Pennsylvania, and 30 in Pennsylvania are equal to 20 in Canada, how many in Canada are equal to 18 in New-England?

| Ante. | | Con. | | |
|---|---|---|---|---|
| 6 | | 8 | | |
| 16 | : | 15 | :: | 18 : 15, Ans. |
| 30 | | 20 | | |

Or thus:—

$6 \times 16 \times 30 = 2880 : 8 \times 15 \times 20 = 2400 :: 18 : 15$, Ans.

The first numbers in each part of the question are antecedents, and the following are consequents. 6, 16, &c. are antecedents; 8, 15, &c. are consequents, as seen in the example above. Hence the following

#### RULE.

Multiply all the Antecedents for the first term, and all the Consequents for the second; the number that asks the question makes the third: then the product of the second and third terms, divided by the first, will give the desired answer.

## SECTION XXI.

### INTEREST, &c.

When one person employs the property for his own benefit, which belongs to another, it is evidently right that a proper reward should be given. The reward or pay for the use of money, is called *Interest*, of which there are two kinds, *Simple and Compound.*

Simple Interest is that which arises from the sum of money lent, which is called the *principal.*

The number of cents paid for the use of a dollar for a year, is called the *rate.*

The term *amount* means the sum of both principal and interest.

When an agreement is made to consider interest as payable at the close of every year, and the borrower does not pay the interest, but keeps it in his own hands, it is considered as added to the principal, or money lent, and interest is allowed for it the same as for the money lent. This is called *Compound Interest.*

The rate per cent of interest is different in different places, but it is commonly 6 per cent. This is established by the Congress of the U. States. Seven per cent is common in some of the states.

In calculating Interest, 12 months are considered a year, and 30 days a month.

What is the interest of 18 dollars and 24 cents, for 2 years and 6 months, at 6 per cent.?

| | |
|---|---|
| 18.24 | Since the rate is 6 cents on a dollar, or .06 |
| .06 | of a dollar, the principal multiplied by .06 and |
| 1.0944 | pointed according to the principles of decimal fractions, gives the interest for one year. |
| 18.24 | Instead, however, of multiplying by .06, we |
| 6 | may multiply by 6 merely, taking care to re- |
| 1.0944 | move the decimal point two figures from its natural place towards the left. |
| 1.0944 | If we multiply the interest of one year by |
| 2 | the number of years, we evidently get the in- |
| 2.1888 | terest for the given number of years. |

2)1.0944
.5472
2.1888
2.7360

If we take half of one year's interest, it will obviously be 6 months' interest, which added to 2 years' interest makes the required interest, for 2 years and 6 months.

18.24
6
2)1.0944
2
2.1888
.5472
2.7360 Ans.

The operation in all its parts, may be performed conveniently as seen here.

As the usual rate of interest is 6 cents for the use of a dollar 12 months, it must be $\frac{1}{12}$ part of 6 cents for one month, which is $\frac{1}{2}$ of a cent or 5 mills. For 2 months it is twice as much as for 1 month, equal to one cent. Hence it is easy to ascertain the interest for any number of months.

If, as we find the true interest is 5 mills for the use of a dollar, one month or 30 days, $\frac{1}{5}$ of thirty days will be required to earn 1 mill. One fifth of 30 is 6. Hence when the number of days is known, it is easy to find the amount of interest; as a dollar must gain $\frac{1}{6}$ part as many mills as there are days in the given time. When the interest of one dollar is found, the interest of 2, 5, or 50, &c. may be readily known, for 2 must gain twice as much as one; 5, five times as many mills; 50, fifty times as many, &c.

The foregoing illustrations will furnish the reasons for the following

### Rules.

Ascertain the number of days, divide by 6 and the interest of 1 dollar will be obtained in mills. Multiply this by the number of dollars, and the answer is found.

Example.—What is the interest of 5 dollars for 90 days? $90 \div 6 = 15$ mills, the gain of $1. And 15 m. $\times$ 5, the number of dollars, is $= 75$ mills $= 7\frac{1}{2}$ cents.

2. Or, ascertain the number of months and multiply the given sum by half of these. The answer is the number of cents.

3. Or, multiply the principal by the rate, and this is the interest for a year. Multiply the interest for one year by the number of years, and the answer is obtained.

Each of the above rules becomes necessary under different circumstances.

*How ought interest to be cast on notes which have many endorsements?* It seems obvious to the Author, that the only mode in casting simple interest, which is just to both the Dr. and Cr., is the following:—

Compute the interest on the whole amount of the Note, for the whole time; then cast the interest on each payment up to the time of general settlement, and add it to the sum of the endorsements.

By the above rule, justice is done to both the lender and borrower.

If the agreement is to pay annual interest, this is the same as Compound Interest, and will hereafter be explained.

4. If the principal is pounds, shillings, &c. multiply by the rate, and as this is a decimal, the two right hand figures must be cut off for decimal parts. The value of this fraction may be found by multiplying the decimals of a pound by the number of shillings in one pound, and adding the given shillings; the shillings are again to be multiplied by the number of pence in one shilling, &c.; in each case the decimals are cut off as before.

EXAMPLE.—What is the interest of 28 £. 10 s. 4 d. for 1 year?

| £. | s. | d. |
|---|---|---|
| 86 | 10 | 4 |
| | | 6 |
| 5.19 | 02 | 0 |
| 20 | | |
| 3.82 | | |
| 12 | | |
| 9.84 | | |
| 4 | | |
| 3.36 | | |

Ans. 5 £. 3 s. 4 d. 3 qrs.

## COMPOUND INTEREST.

If at the end of a year the interest on a given sum is added to the principal, it becomes a part of it, and interest afterwards is necessarily cast on it.

### Rule.

Find the interest of one dollar for the given time, and multiply that by the number of dollars, in the principal.—Or,

The compound interest at 6 per cent of any sum in Federal money, may be found by multiplying the sum by the following numbers. Multiply the given sum:—

| | | | |
|---|---|---|---|
| For 2 years by | 112.36 | 7 years by | 150.3630 |
| 3 | 119.1016 | 8 | 159.3848 |
| 4 | 126.2476 | 9 | 168.9478 |
| 5 | 133.8225 | 10 | 179.0847 |
| 6 | 141.8519 | 11 | 189.8298 |

"Point the product as in simple interest; it will then show the amount of principal and interest for the given number of years. Subtract the principal from the amount, and the remainder will be the compound interest.

When there are months and days, first find the compound interest for the given years; then for the months and days cast the simple interest, which, added to the amount, will give the answer."

A sum at compound interest will be doubled in 11 years, 10 months and 22 days.

## COMMISSION

"Is an allowance of so much per cent to a factor or correspondent abroad, for buying and selling goods for his employer."

## BROKERAGE

"Is an allowance of so much per cent to a person called a broker, for assisting merchants or factors in procuring or disposing of goods."

## INSURANCE

"Is an allowance of so much per cent given to certain persons or companies, who engage to make good the loss of ships, houses, merchandise, &c. which may happen from storms, fire, &c."

The method of working questions in these rules is the same as in simple interest, and needs no farther illustration.

## DISCOUNT.

*Discount* is an allowance made for the payment of any sum of money before it becomes due, according to a certain rate per cent. agreed on between the parties concerned. If I am to pay A 100 dollars in one year, it is plain that I ought not to pay so much, if it is paid to-morrow.

The *present worth* of any sum or debt due some time hence, is such a sum as, if put to interest for that time, at a certain rate per cent., would amount to the sum or debt then due.

That an allowance ought to be made for paying money before it becomes due, if it is not on interest till after it is due, is highly reasonable; for if I keep the money in my own hands till the debt becomes due, I may make an advantage of it, by lending it on interest for that time; but if paid before it becomes due, it is giving that benefit to another.

What is the discount of $846 for 6 months, at 6 per cent.?

```
  1 00
     3
  ----
  3.00

103 : 846 :: 3
     3
  ------
103)2538(24.64 8/103
   206
   ---
   478
   412
   ---
   660
   618
   ---
    420
    412
    ---
      8
```

We first find the interest of 100 dollars for the given time and rate, which added to $100, the amount is $103. We have then the following proportion: 103 : 846 : : 3 : , or 103 : 100 : : 846 : its present worth. The sum $24.64+ is to be taken from $846, in order to show its present worth.

### Rule.

1. "To find the discount, make a statement in the rule of three; As $100 or £., with its interest for the given rate and time, is to that interest; so is the given sum, to the discount required. And,

2. To find the present worth; As 100, with the interest for the given rate and time, is to 100; so is the given sum, to the present worth."

## SECTION XXII.

### PRACTICE.

Before the introduction of Federal money, this rule was very convenient and useful, and hence its name.

In the United States it is now seldom employed. But, as performing operations in it may be a useful exercise to the learner, it is thought best to introduce some of the prominent cases. Practice is a contraction of the rule of proportion, when the first term is a unit.

When the price only is a compound number, the answer is more easily obtained, than when the price and quantity are both compound numbers.

*Example.* What is the worth of 200 yards of cloth at 10 s. 6 d. a yard?

As 10 s. is $\frac{1}{2}$ a pound, if half the number of yards is taken, it will equal the number of pounds the cloth is worth at 10 s. a yard. And as 6 d. is $\frac{1}{20}$ of a pound, if that part of 200 is taken, it will be the price at 6 d. a yard. These added together, will make the price of the whole.

$\frac{1}{20}$) $\frac{1}{2}$)200

100

5

105 £. Ans.

An *aliquot part* of any number is such a part of it as, being taken a certain number of times, will exactly make that number. Hence the aliquot parts are used as divisors.

*Table of Aliquot Parts.*

Even parts of a shilling.

| *pence.* | *shill.* |
|---|---|
| 6 | $= \frac{1}{2}$ |
| 4 | $= \frac{1}{3}$ |
| 3 | $= \frac{1}{4}$ |
| 2 | $= \frac{1}{6}$ |
| $1\frac{1}{2}$ | $= \frac{1}{8}$ |
| 1 | $= \frac{1}{12}$ |

Even parts of a pound.

| *s.* | *d.* | £. |
|---|---|---|
| 10 | | $= \frac{1}{2}$ |
| 6 | 8 | $= \frac{1}{3}$ |
| 5 | | $= \frac{1}{4}$ |
| 4 | | $= \frac{1}{5}$ |
| 3 | 4 | $= \frac{1}{6}$ |
| 2 | 6 | $= \frac{1}{8}$ |
| 2 | | $= \frac{1}{10}$ |
| 1 | 8 | $= \frac{1}{12}$ |

GENERAL RULE.

1. Suppose the price of the given quantity to be 1 £. or 1 s. as is most convenient; then the quantity will itself be the answer at the supposed price.

2. Divide the given price into aliquot parts, either of the supposed price, or of another, and the sum of all the quotients will be the true answer required.

*Example.* What is the value of 526 yards of cloth at 3 s. 10½ d. per yard?

3 s. 4 d. is ⅙ of 1 £. )526

| | 87 £. | 13 s. | 4 d. | 0 £. | 3 s. | 4 d. |
|---|---|---|---|---|---|---|
| 4 d. is $\frac{1}{10}$ of 3 s. 4 d.) | 8 | 15 | 4 | 0 | 0 | 4 |
| 2 d. is ½ of 4 d.) | 4 | 7 | 8 | 0 | 0 | 2 |
| ¼ d. is ½ of 2 d.) | 0 | 10 | 11½ | 0 | 0 | 0¼ |
| | 101 £. | 7 s. | 3½ d. Price at | 3 s. | 10½ d. | |

In this example it is obvious that 526 £. would be the answer at 1 £. per yard; consequently, as 3 s. 4 d. is ⅙ of a pound, ⅙ part of that quantity, or 87 £. 13 s. 4 d., is the price at 3 s. 4 d. In like manner as 4 d. is $\frac{1}{10}$ of 3 s. 4 d., so $\frac{1}{10}$ of 87 £. 13 s. 4 d., or 8 £. 15 s. 4 d., is the price at 4 d. And by reasoning in this way, 4 £. 7 s. 8 d. will appear to be the price at 2 d., and 10 s. 11½ d. the price at ¼ d. Now as the sum of all these parts is equal to the whole price per yard, (3 s. 10½ d.,) so the sum of the answers belonging to each price will be the answer at the full price required. And the same will be true of any example whatever.

"As the following cases are all founded on the principles involved in this general rule, this illustration will be sufficient to explain them."

II. *When the price is less than a penny.* Divide the given number by the aliquot parts of a penny, and the quotient will be the answer in pence, which may be reduced to pounds."

*Example.* What cost 4506 yards of tape at ¾ of a penny per yard?

| | | |
|---|---|---|
| ½ d. | 2)4506 | 12)3379½ |
| ¼ d. | 2)2253 | 2\|0)28\|1 7½ |
| | 1126½ | 14 £. 1 s. 7½ d. Ans. |
| | 3379½ | |

III. When the price is an aliquot part of a shilling, divide the given number by that, and the answer will be the price

in shillings; but if it be not an even part, take such parts as will amount to the whole, and add the quotients together for the whole price. The same principles are to be observed when the price is shillings, &c.

If the price is pounds, shillings, &c., the number expressing the quantity may be first multiplied by the number expressing pounds, and the remaining part obtained as before.

IV. When there are parts of a yard, ton, &c. in the given sum, the price of the whole number may be obtained, and then the price of the part added to it.

*Example.* What will $5\frac{1}{2}$ yards of cloth cost at 4 s. 6 d. a yard?

$\frac{1}{4\,0}$ $\frac{1}{5}$)5 yds.
1

| | | | |
|---|---|---|---|
| | 2 | 6 | |
| $\frac{1}{2}$ yd. will cost | 2 | 3 | |
| | 1 £. | 4 s. | 9 d. Ans. |

---

## SECTION XXIII.

## FELLOWSHIP,

OR

## COMPANY BUSINESS.

When two or more men trade together, each advancing a certain sum, (to be continued for an equal time,) called stock, or capital, it is evident that each one's share of the gain or loss, must be in proportion to the stock which he puts in.

Three merchants, A, B, and C, entered into trade together. A advanced $2000, B $3000, and C $5000. They gained $3000. What is each man's share of the gain?

The whole stock is $10,000. Had one man put in this whole sum, the whole gain $3000 would have been his; but as this gain is to be divided among three men, it is evident that each one's share of the united stock must bear the same proportion to his gain, that the whole stock does to the whole gain. Therefore

$10000 : 3000 :: 2000 : to A's share of the gain.
10000 : 3000 :: 3000 : to B's share of the gain.
10000 : 3000 :: 5000 : to C's share of the gain.

By performing the operations of the above statements, we obtain the answers.

$600 A's share.
900 B's share.
1500 C's share.

*Proof* $3000

The above propositions may be stated in words, thus :—

The whole stock : to the whole gain :: each man's share of the stock : to his share of the gain.

It is evident, that the sum of the several shares of the gain will be equal to the whole gain. By adding the shares of the gain together, the work may be proved.

If each one's share of the stock had been the same, the shares of the gain would have been the same; to obtain which we should only have had to divide the gain by the number of shares. We may reduce the question to this in the present case, by dividing the whole stock $10000 into 10 partial stocks of $1000 each. The gain of each one of these will evidently be the 10th part of the whole gain; then by multiplying this part by 2, 3, and 5, which have the same relation to each other as the several shares of the whole stock, we shall obtain each man's share. The gain of $1000 = $300, and $300 \times 2 = $600, A's share; $300 \times 3 = 900, B's share; $300 \times 5 = $1500 C's share.

When the stocks of the partners are continued in trade an equal time, as in the preceding case, it is called *Single Fellowship.*

When the stocks are continued in trade unequal times it is called *Double Fellowship.*

A and B enter into partnership. A advanced 600 dollars for 6 months and B 1200 for 12 months. They gained $300. What was each one's share of the gain?

If the two shares of stock had been continued in trade the same time, it is evident, that B's share of gain could have been only double that of A. But as $1200 for 12 months is equal to 12 times 12 hundred for one month, and 6 hundred dollars for 6 months is equal to 6 times 6 hun-

dred for one month, A can have only $\frac{3600}{14400} = \frac{1}{4}$ of the whole gain. Then

A's part is $ 75
B's 225

The amount put in must be taken in connection with the length of time in which it is employed. Hence the

RULE.

Multiply each man's share of the capital by the time it is employed: then as the whole stock is to the whole gain, so is each man's share of stock to his share of the gain.

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## SECTION XXIV.

### LOSS AND GAIN.

If I buy 4 yards of cloth for ten dollars, and sell it at two dollars a yard, it is evident that I lose a part of the price. Questions in business often arise, on subjects connected with loss and gain in trade, and also the amount per cent. either lost or gained. If it were asked, how much per cent. was lost on the cloth above, I can ascertain by saying that as $10 lost $2, $1 must lose $\frac{1}{10}$ part of 2, which will of course be $\frac{2}{10}$ of $1 = 20$ cents. Then the loss will be 20 per cent.

If it be required to find the loss or gain in laying out $500, when the loss or gain is a certain per cent., say 15; I have only to apply the principles before explained, and say that if $1 gains or loses 15 cents, $500 will gain or lose 500 times as many = 7500. As the principles of proportion already explained are all which are required to perform these operations, the rule does not merit further notice in this place.

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## SECTION XXV.

### INVOLUTION AND EVOLUTION.

These terms are sufficiently defined in the corresponding section of Part I.

Any number whatever may be a root, and can be raised to the second, or any given power, by continued multiplication.

$2 \times 2 = 4$ is the 2d power or square of 2.
$2 \times 2 \times 2 = 8$ is the 3d power or cube of 2.
$2 \times 2 \times 2 \times 2 = 16$ is the 4th power, &c.

The following Table contains the powers of numbers as far as nine.

| 1st power or root. | 2d power. | 3d power. | 4th power. | 5th power. | 6th power. | 7th power. | 8th power. | 9th power. |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 4 | 8 | 16 | 32 | 64 | 128 | 256 | 512 |
| 3 | 9 | 27 | 81 | 243 | 729 | 2187 | 6561 | 19683 |
| 4 | 16 | 64 | 256 | 1024 | 4096 | 16384 | 65536 | 262144 |
| 5 | 25 | 125 | 625 | 3125 | 15625 | 78125 | 390625 | 1953125 |
| 6 | 36 | 216 | 1296 | 7776 | 46656 | 279936 | 1679616 | 10077696 |
| 7 | 49 | 343 | 2401 | 16807 | 117649 | 823543 | 5764801 | 40353607 |
| 8 | 64 | 512 | 4096 | 32768 | 262144 | 2097152 | 16777216 | 134217728 |
| 9 | 81 | 729 | 6561 | 59049 | 531441 | 4782969 | 43046721 | 387420489 |

Every number must have a root, that is, a number which, being multiplied into itself once, or a certain number of times, will produce it. But this is not always easily expressed by figures, without having recourse to fractions.

The least root which is a whole number is 1. The square of $1 \times 1 = 1$ has one figure less than the numbers in the factors; the cube of $1 \times 1 \times 1 = 1$ is less by two than the factors. The least root consisting of two figures is 10; and $10 \times 10 = 100$ has one figure less than the factors. The cube of $10 = 1000$ has two less. The greatest root consisting of one figure is 9. Its square, 81, contains just the number of figures found in its factors; its cube, 729, is just equal to the factors.

The greatest root consisting of two figures, is 99; and its square, 9801, contains just the number of figures in the fac-

tors.—Hence it will be seen that the second power can have no more than double the figures of its root, and in no case but *one less;* and that the third power can never have more than three times the number of figures of its root, and in no instance more than two less.

Then, to ascertain the number of figures in any required root, distinguish the given sum into parts or periods, by dots, putting into each period the number denoted by the index of the root required. If the 3d root is required, put 3 figures in a period; if the 4th, put in 4, and so on.

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## SECTION XXVI.

### SQUARE ROOT.

The square root of any number, is such a number as multiplied into itself will produce that number. The square root of 144 is of course 12, because $12 \times 12 = 144$. If 100 pieces of paper 1 foot square are to be placed in a square form, each side must measure 10 feet, because $10 \times 10 = 100$. The following example will exhibit the operation, and the reasons on which it is founded.

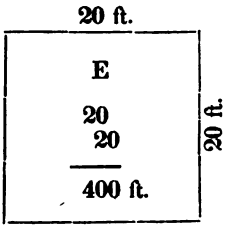
"If 529 feet of boards be laid down in a square form, what will be the length of the sides of the square? or, in other words, what is the square root of 529?

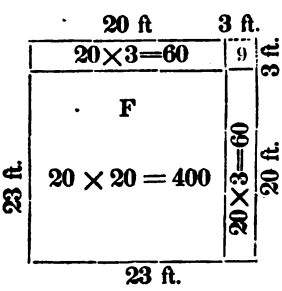
From what was shown in the last Section, we know that the root must consist of two figures, inasmuch as 529 consists of two periods. Now to understand the method of ascertaining these two figures, it may be well to consider how the square root, consisting of two figures, is formed. For this purpose we will take the number 23 and square it. By this operation it appears that the square of a number consisting of tens and units is made up of the square of the units, plus twice the product of the tens, by the units, plus the square of the tens. As $10 \times 10 = 100$, the square of the tens can never make a part of the two right hand figures

| | |
|---|---|
| 23 | |
| 23 | |
| 9 | square of units |
| 6 | twice the product of |
| 6 | the tens by units. |
| 4 | square of the tens. |
| 529 | square of 23 |

| | |
|---|---|
| . . | |
| 5 29 | [20 |
| 4 00 | |
| 129 | |

of the whole square. Hence the square of the tens is always contained in the second period, or in the 5 of the present example. The greatest square in 5 is 4, and its root 2; hence we conclude that the tens in the root are $2 = 20$, and $20 \times 20 = 400$. But as the square of the tens can never contain significant figures below hundreds, we need only write the square of the figure denoting tens under the second period. From what precedes, it appears that 400 of the 529 feet of boards are now disposed of, in a square form, E, measuring 20 feet on each side, and that 129 feet are to be added to this square in such manner as not to alter its form; and in order to do this the additions must be made upon two sides of the square E, $= 20 + 20 =$ 40 feet. Now if 129, the number of feet to be added, be divided by 40, the length of the additions, or dropping the cipher and 9, 12 be divided by 4, the quotient will be the width of the additions; and as 4 in 12 is had 3 times, we conclude the addition will be 3 feet wide, and $40 \times 3 = 120$ feet, the quantity added upon the two sides. But since these additions are no longer than the sides of the square, E, there must be a deficiency at the corner, as exhibited in F, whose sides are equal to the width of the additions, or 3 feet, and $3 \times 3 = 9$ feet, required to fill out the corner, so as to complete the square. The whole operation may be arranged as on the next page, where it will be seen that we first find the root of the greatest square in the left hand period, place it in the form of a quotient, subtract the square from the period and to the remainder bring down the next period, which we divide, omitting the right hand figure, by double the root, and place the quotient for the second figure of the root; and the square of this figure being necessary to preserve the form of the square, by filling the cor-



529[23
4
43]129
120
23×23=529 proof.

ner, we place it at the right of the divisor, in place of the cipher which is always understood there, and then multiply the whole divisor by the last figure of the root. As we may conceive every root to be made up of tens and units, the above reasoning may be applied to any number whatever, and may be given in the following general

RULE.

Distinguish the given numbers into periods; find the root of the greatest square number in the left hand period, and place the root in the manner of a quotient in division, and this will be the highest figure in the root required. Subtract the square of the root already found from the left hand period, and to the remainder bring down the next period for a dividend. Double the root already found for a divisor; seek how many times the divisor is contained in the dividend, (excepting the right hand figure) and place the result for the next figure in the root, and also on the right of the divisor. Multiply the divisor by the figure in the root last found; subtract the product from the dividend, and to the remainder bring down the next period for a new dividend. Double the root now found for a divisor, and proceed as before to find the next figure of the root, and so on, till all the periods are brought down." *(Thompson.)*

II. To extract the square root of Vulgar Fractions,

1. If the fractions are either proper or improper, extract the root of the numerator for a new numerator, and the root of the denominator for a new denominator, and it is the root required.

2. If the fractions are mixed, reduce them to improper fractions, and proceed as before. But if the exact root of either part of the fraction cannot be obtained, the fraction may be reduced to a decimal, and the root obtained as in whole numbers.

EXAMPLES.

What is the square root of $\frac{4}{9}$? Ans. $\frac{2}{3}$.
What is the square root of $27\frac{9}{16}$? Ans. $5\frac{1}{4}$.

III. It is frequently desirable to ascertain the length of one side of a triangle, when unknown; and the principles involved in the extraction of the square root furnish the

means. It is found, that the square root of the longest side of a right angled triangle, is just equal to the square root of the other two sides. Hence, if the length of the sides of a given triangle are 36 feet and 64 feet, the root extracted from these added together, will be equal to the root of the other side. That root multiplied into itself will be the length of the other side. If the root of the two shorter sides only is given, square each of these, and add them together. This sum will be the square of the longest side.

*Example.* There is a wall 36 feet high, and a ditch at the foot of it 64 feet wide. What is the length of a cord that will reach from the top of the wall to the opposite side of the ditch?

$64 + 36 = 100$. The root of $100 = 10$.
$10 \times 10 = 100$, the length of the cord.

This rule is very useful to carpenters, to enable them to know the length necessary to cut braces, &c. which are to reach from a sill or plate to a post, &c. The principle involved has been called the Carpenter's Theorem.

IV. If it is desired to find a mean proportional between any two numbers, it is easily obtained.

What is the mean proportional between 2 and 18?

$18 \times 2 = 36$. The square root of $36 = 6$ is the number sought. The number 2 has the same relation to 6, that 6 has to 18; that is, 2 is $\frac{1}{3}$ of 6, and 6 is $\frac{1}{3}$ of 18. Hence the

RULE.

Multiply the given numbers together, and the square root of their product is the mean proportional sought.

EXAMPLES.

1. What is a mean proportional between 2 and 72?

$72 \times 2 = 144$. The root of 144 is 12, the Ans.
2 is $\frac{1}{6}$ of 12; 12 is $\frac{1}{6}$ of 72.

2. What is the mean proportional between 16 and 64?

Ans. 32.

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## SECTION XXVII.

### EXTRACTION OF THE CUBE ROOT.

Extraction of the cube root, is finding a number, which multiplied into itself twice, will produce that number. Or, it

is finding the length of one side of a cube, of which the content is given.

EXAMPLES.

1. I have 64 blocks of wood, each containing 1 solid foot. How long is one side of the pile which they will make in a cubic form?

I find by extracting the root of 64, it is 4, or that $4\times4=16$, and $16\times4=64$. The pile of blocks must therefore measure 4 feet on each side.

2. I have in a cubic pile 1000 blocks of wood, each containing a cubic foot. It is evident, as $10\times10=100$, and $100\times10=1000$, that the pile measures 10 feet on each side.

In the examples above, it is easy to ascertain the root, because the numbers are such, that they are rendered familiar by the common operations of arithmetic. But this is not true of all numbers.

When a question is given for solution, the first thing to decide on, is how many figures the root will consist of. If it is the cube or third root, it is easily known by seeing how many periods of three figures each are contained in it. It is usual to put a dot over the first or last figure of each period.

*Example.* What is the cube root of $\dot{3}576\dot{3}42$?

Here the right hand period has the figures 342, the next, 576, the next, 3. Hence the root must have three figures. As the left hand period is 3, and as *one* is the largest root of it, I know at once, that the first figure of the root must be merely 1. As there must be two others, because there are two other periods, I may call the root 100, and if this multiplied twice by itself produces 3576342, it must be the whole root sought. But $100\times100\times100=1{,}000{,}000$. I know therefore that the root must be larger, and that, in place of ciphers in the root, there must be some figure of more value.

I then put the cube of 1 under the period 3, (and this is only 1, as it cannot be increased by being multiplied into itself,) then by taking this from 3, 2 remains, to which the next period may be joined. The second figure of the root must be such a figure as, being placed at the right hand of 1, the cube of both will be the largest cube found in the two first periods (having

$\dot{3}576\dot{3}42(15$
1
2576

3576
3375
201

3576342
3511804
64538

a root that does not embrace fractions.) If I place 6 there, I find the cube of 16 to be 4096, and this is larger than the two periods. I suppose that 5 must be the figure; the cube of 15 is 3375, and this less than the two periods, but is the largest cube in them, and 201 remains. Having found two figures of the root, I wish the third. This must be such that when put at the right of the 5, the whole cube will be the whole number of the 3 periods, or will leave a fraction only, which must be reduced to decimals, in order to obtain the exact root of the whole. I find this figure must be 2. Then the cube of 152=3,511,804. This is less than the three periods. But the cube of 153 is greater.

To the remainder, if a period of ciphers be annexed, the next figure of the root will be a decimal.

Farther explanation of the manner of extracting the cube root, can be more conveniently given by taking another example. The following by Mr Thompson is explicit.

"I have 12167 solid feet of stone, which I wish to lay up in a cubical pile; what will be the length of the sides? or, in other words, what is the cube root of 12167?

By distinguishing 12167 into periods, we find the root will consist of two figures. Since the cube of tens can contain no significant figures less than thousands, the cube of the tens in the root must be found in the left hand period. The greatest cube in 12 is 8, whose root is 2; but the value of 8 is 8000, and the 2 is 20, that is, 8000 feet of the stone will make a pile measuring 20 feet on each side, and (12167—8000=) 4167 feet remain to be added to this pile in such a manner as to continue it in the form of a cube. Now it is obvious that the addition must be made upon 3 sides; and each side being 20 feet square, the surface upon which the additions must be made will be (20×20×3=2×2×300=) 1200 feet; but when these additions are made, there will

. . .
12167(23 root.
2×2×2=8
2√300×2× 4167
30=1260
1200×3=3600
60×3×3=540
3×3×3=27
4167

evidently be three deficiencies along the lines where these additions come together, (20 feet long, or $20\times3=2\times30=$) 60 feet, which must be filled in order to continue the pile in a cubic form. Thus the points upon which the additions are to be made, are ($1200+60=$) 1260 feet and 4167 feet, the quantity to be added divided by 1260, the quotient is ($4167\div1260=$) 3, which is the thickness of the additions, or the other figure of the root. Now if we multiply the surface of the three sides by the thickness of the additions, the product, ($1200\times3=$) 3600 feet, is the quantity of stone required for those additions. Then to find how much it takes to fill the deficiencies along the line where these additions come together, since the thickness of the additions upon the sides is 3 feet, the additions here will be 3 feet square, and 60 feet, and the quantity of stone added will be ($60\times3\times3=$) 540 feet. But after these additions there will be a deficiency of a cubical form, at the corner, between the ends of the last mentioned additions, the three dimensions of which will be just equal to the thickness of the other additions, or 3 feet, and cubing 3 feet we find ($3\times3\times3=$) 27 feet of stone required to fill this corner, and the pile is now in a cubic form, measuring 23 feet on every side, and adding the quantities of the additions upon the sides, the edges, and at the corner together, we find them to amount to ($3600+540+27=$) 4167 feet, just equal to the quantity remaining of the 12167 after taking out 8000. To illustrate the foregoing operation, make a cubic block of a convenient size to represent the greatest cube in the left hand period. Make 3 other square blocks each equal to the side of the cube, and of an indefinite thickness, to represent the additions upon the three sides, then 3 other blocks, each equal in length to the sides of the cube, and their other dimensions equal to the thickness of the square blocks, to represent the additions along the edges of the cube, and a small cubic block with its dimensions each equal to the thickness of the square blocks, to fill the space at the corner. These placed together in the manner described in the above operation, will render the reason of each step in the process perfectly clear.

The learner may adopt the following general

### Rule.

1. Distinguish the given sum into periods of 3 figures each, beginning at the right hand.

2. Find the greatest cube in the left hand period, and write down its root in the place of the quotient in division.

3. Subtract the cube from the period, and bring down the next period at the right hand of the remainder; call this a dividend.

4. Square the quotient and multiply that product by 300. This is the triple square. Multiply the quotient by 30. This is the triple quotient. Add these products together for a divisor.

5. Ascertain how many times the divisor is contained in the dividend for the 2d figure of the root.

6. Multiply the triple square by the last quotient figure: then multiply the triple quotient by the square of the last quotient, and cube the last quotient, and add these products together and take the amount from the dividend. and to the remainder bring down the next period. Proceed as before, till the whole is finished.

1. What is the cube root of 15625?

| | |
|---|---|
| 15625(25 root | The greatest cube in 15, the left |
| 8 | hand period, is 8, of which the root is |
| ——— | 2; and $2\times2=4$, and $4\times300=1200$, |
| 7625 dividend. | the triple square: $2\times30=60$, the triple |
| ——— | quotient; their sum is 1260 the divisor, |
| 6000 | which is contained in the 7625, the div- |
| 1500 | idend, 5 times. $1200\times5=6000$; the |
| 125 | square of 5 is 25 and $60\times25=1500$, |
| ——— | and the cube of 5 is 125; the sum of |
| 7625=dividend. | these products is 7625=the dividend. |
| | The work is therefore done. The |
| | cube root is 25. |

2. What is the cube root of 2? Ans. 1.25+

In this example ciphers must be added as decimals to the 2.

II. Solids of the same form are to one another as the cubes of their diameters, or similar sides.

1. If a ball which weighs 72 lbs. is 8 inches in diameter, what is the diameter of one which weighs 9 lbs.?

$8\times8\times8=512$. 72 : 9 :: 512 : to the cube of the smaller ball = 64, the cube root of which is 4, the Answer.

2. A ball 4 inches in diameter weighs 6 lbs.; what is the weight of one 8 inches in diameter?

$4\times4\times4=64$ and $8\times8\times8=512$.

64 : 6 :: 512 : 48 lbs., the Ans.

To find *two* mean proportionals, between any two given numbers,

Divide the greater by the less, and extract the cube root of the quotient. Multiply the least given number by the root for the lesser, and this product by the same root for the greater of the two numbers sought.

1. What are the two mean proportionals between 2 and 16?

$16\div2=8$ $\sqrt[3]{8}=2$; and $2\times2=4$ the lesser, and $4\times2=8$, the greater. Proof. 2 : 4 :: 8 : 16.

2. What are the two mean proportionals between 6 and 162? Ans. 18 and 54.

[The learner may derive important assistance, in understanding the extraction of the square root, by using pieces of pasteboard, so cut as to enable him to make the additions to the sides of a square which are required, to continue it in that shape.

By the help of "the blocks," he will be better able to understand the principles involved in extracting the *cube root*, than he can, by any written demonstration.]

## EXTRACTION OF ROOTS IN GENERAL.

It is not very often required in business transactions, to find the roots of other powers. As it may be useful, however, to be acquainted with an expeditious mode of extracting any large root, the learner may attend to the following

### Rules.

1. Point off according to the root specified: if the fourth root is required, the period must have four figures; if the fifth, it must have five, &c.

2. Find the greatest root in the left hand period by trial; subtract its power from that period, and at the right hand of the remainder place the first figure of the next period. This is a dividend.

3. Involve the root already found to the power next less than that of which the root is required, multiply this by the number expressing the given power, and use this as the divisor of the dividend.

4. Ascertain how many times the dividend contains the divisor, and put the quotient at the right hand of the other figure of the root.

5. Involve the root already found to the given power, and subtract this from the given numbers. Bring down the first figure of the next period to the right hand of this remainder, for a new dividend, and proceed as before, till the work is finished.

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## SECTION XXVII.

### ARITHMETICAL PROGRESSION.

If any three numbers are in arithmetical progression, the sum of the two extremes must be equal to double the mean; and when four numbers are in progression, the sum of the two means must be equal to the sum of the two extremes. If a larger number are in progression, then the sum of the extremes must be equal to the sum of any two means equally distant from the extremes. For the reasons of which, see illustrations in the Rule of Three.

Arithmetical progression is a series of numbers either increasing or decreasing by a common difference. Five things are to be considered, viz. the first and last terms, the number, common difference, and sum of all the terms. If any three of these are given, the others can be readily found.

If I buy 6 apples, giving 2 cents for the first, 4 for the second, 6 for the third, and so on, increasing by a common difference of 2, what must I give for the last?—There are several ways in which I can obtain the answer. As the first costs 2 cents, the second 4, &c. I can add the price of the first to itself to make that of the second; the price of the first, or 2 cents, added to the price of the second, will make that of the third, and so on. But in large sums, this would be inconvenient.

As we have the first term, the number of terms, and the common difference given, we can in this, and all similar cases, find the answer by the following

#### Rule.

Multiply the number of terms less 1 by the common difference, and to this product add the first term. The sum will be the answer.

5 Number of terms less 1.
2 Common difference.

10
2 Cost of the first apple.

12 Ans.

2. If the first term be 8, common difference 4, and number of terms 10, what is the last term? Ans. 44.

RULE 2.

When the first term, the last term and number of terms are given, to find the common difference,—

Divide the difference between the first and last by the number of terms less 1, and the quotient must be the common difference.

*Example.* I bought 12 books. The cheapest cost 2 dollars; the dearest 35. What is the common difference in the price?

35
2

11)33(3 Ans.

RULE 3.

If the first and last terms and common difference are given, to find the number of terms,—

Divide the difference of the extremes (that is, the first and last terms) by the common difference, and the quotient increased by 1 will be the answer.

*Example.* If the first term be 5, the last 35, and the common difference 3, what is the number of terms?

35
5

3)30(10
1

11 Ans.

RULE 4.

When the first, last and number of terms are given, to find the sum of all the terms or series,—

Multiply half the sum of the extremes by the number of terms, and the product will be the answer.

## GEOMETRICAL PROGRESSION.

By Geometrical Progression is meant, a series of terms, which increase by a uniform multiplier, or decrease by a constant divisor. The multiplier or divisor is the *ratio.*

If I hire a man for six months and engage to pay him 3 dollars for the first month, and double the sum for the second, and double the second for third, and so on, what must I pay him for the last?

I can ascertain this by multiplying the wages of the first month to find the wages of the second, and so on.

| | | | | |
|---|---|---|---|---|
| 3 | Wages | of | the first month. | |
| 2 | | | | |
| 6 | " | " | second " | |
| 2 | | | | |
| 12 | " | " | third " | |
| 2 | | | | |
| 24 | " | " | fourth " | |
| 2 | | | | |
| 48 | " | " | fifth " | |
| 2 | | | | |
| 96. | " | " | sixth " | Ans. |

In this example, there are given the first term, the number of terms, and the ratio, to find the last.

It will be easily seen that, in producing each term of the series, the ratio is as many times a factor less 1, as the number of terms. (The ratio has been 5 times used to obtain the wages of the sixth month, 4 times to obtain the wages of the fifth month, &c.) The first term must always be a factor. Any term of the series is the product of the ratio raised to a power, whose index is one less than the number of the term, multiplied by the first term. Hence we may obtain the desired answer, in a more expeditious mode than by the process above.

Two being the ratio, if raised to the 5th power, (1 less than the number of terms,) it becomes 32, which multiplied by the first or least term is 96, the answer. Hence the

### Rule 1.

When the ratio, less extreme and number of terms are given, to find the last term or greatest extreme,—

Raise the ratio to a power one less than the number of

terms, multiply that power by the least extreme, and the product is the answer.

RULE 2.

When the first term, last term and ratio are given, to find the sum of the series,—

Multiply the last term by the ratio, and from the product subtract the first. Divide the remainder by the ratio less 1, and the quotient will be the answer.

*Example.* If the first term is 2, the last 4374, and the ratio 3, what is the sum of the series of numbers?

```
                4374  Greatest term.
                   3  Ratio.
               13122
                   2  Least term.
Ratio less 1 = 2)13120
                6560  Ans.
```

RULE 3.

If the first and last terms (the extremes) and the ratio are given, to find the number of terms,—

Divide the greatest term by the least, then find what power of the ratio will equal the quotient, and, to the index of that power add one, and this will be the number of terms.

*Example.* Least term 2)4374 Greatest term.

2187

3 involved to the 7th power is 2187. Then $7+1=8$ the number of terms.

RULE 4.

When the extremes and number of terms are given, to find the ratio;—Divide the greatest by the least term, and extract that root of the quotient whose index is denoted by the number of terms less 1, and the root will be the common ratio.

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## SECTION XXVIII.

### SINGLE POSITION.

This name is given to a rule, where the conditions of the question are to furnish the necessary data for obtaining the answer.

A said he had spent $\frac{1}{2}$ and $\frac{1}{3}$ of his money, which amounted to 72 dollars. What sum had he at first?

Suppose 150. Then half = 75
third = 50
125

Then 125 : : 150 : 72

72
300
1050
125)10800(86$\frac{2}{5}$ Ans.
1000
800
750
50 = $\frac{2}{5}$
125

*Proof.*
Half = 43$\frac{1}{5}$
Third = 28$\frac{4}{5}$
72

Here it will be easily seen, that 125 must bear the same relation to the supposed number, that 72 does to the true number.

The only object of using a supposed number, is to find such others, as are necessary in order to perform the operation.

But as questions of a character similar to the one above, are more easily answered by other means, the rule of Single Position is not important. To show this, the example may be resolved by Fractions.

A had spent $\frac{1}{2} = \frac{3}{6}$ and $\frac{1}{3} = \frac{2}{6}$ of his money, $= \frac{5}{6}$ in all. He said this amounted to 72 dollars. Then 72 is $\frac{5}{6}$ of the sum he had at first; $\frac{1}{6}$ of 72 must be $\frac{1}{5} = 12\frac{2}{5}$. If this is one sixth, the sum must have been 6 times as large $= 86\frac{2}{5}$ Ans.

The following is the general

RULE.

To solve a question by Single Position, suppose any number at pleasure, and pursue the course rendered necessary by the conditions of the question. Then the result of the supposition will bear the same relation to the supposed number, as the given number does to the answer.

## DOUBLE POSITION.

This rule is more complex than the last, requiring two supposed numbers. Questions, where the ratio between the required and supposed number differs from that of the given number to the required one, are readily answered by Double

Position. The demonstration of the rule is difficult without a knowledge of Algebra. The following is the general

RULE.

Suppose any two convenient numbers, and proceed with them as the conditions of the question require, and write down the errors. Multiply the first supposed number by the last error, and the last supposed number by the first error. If both supposed numbers are either too large or too small, divide the difference of the products by the difference of the errors. But if one is too large and the other too small, divide the sum of the products by the sum of the errors, and the quotient will be the answer.*

*Example.* What is that number which, on being increased by its half, fourth, and 5 more, will be doubled?

| Suppose 40. | | Supppse 60. | |
|---|---|---|---|
| Its half | 20 | Its half | 30 |
| Its fourth | 10 | Its fourth | 15 |
| And | 5 added | And | 5 added |
| | 35 | | 50 |
| 40 | | 60 | |
| 35 | | 50 | |
| 5 | 1st error. | 10 | 2d error. |

$$\begin{array}{cc} 40 & 5 \\ & \times \\ 60 & 10 \\ \hline 300 & 400 \end{array}$$

$$10-5=5)\overline{100}(20 \text{ Ans.}$$

$$\underline{300}$$

In this example both errors are too large.

Most questions of this kind may be answered by other modes, and it is not frequently necessary to resort to this rule.

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## SECTION XXIX.

## ALLIGATION MEDIAL.

If a merchant mixes 5 lbs. of tea worth 50 cents a pound, 6 pounds worth 75, and 8 pounds worth 90, what is a pound of this mixture worth?

* The true answer cannot be obtained by this rule, when the first error does not bear the same proportion to the second, as the difference between the true and first supposed number does to the difference between the true and second supposed number.

It is plain that if he ascertains the price of the whole, and then divides that amount by the whole number of pounds, the value of one pound of the mixture will be obtained.

5 lbs. at 50 = 250
6 " 75 = 450
8 " 90 = 720

19 | 19)1420(74$\frac{14}{19}$ Ans.
133
90
76
$\frac{14}{19}$

If it were required to find the price of 6 pounds of the mixture, this would evidently be 6 times the price of one pound. The price of any part is easily found.

RULE.

Find the value of all the quantities mixed together and then divide the whole price by the whole number of the quantities mixed, and this will be the price of a unit of that quantity.

## ALLIGATION ALTERNATE.

If I wish to mix together gold at 16, 18, 20 and 23 carats fine, to form a mixture of 19 carats fine. What quantity of each must I take?

It is evident that if I take 3 pounds at 23, and 4 pounds at 16, there must be 133 carats in the whole; and this number divided by 7, the number of pounds taken, will give 19 carats as the required fineness. A similar course with the others would produce a similar result. Hence the general

RULE.

Write the numbers under each other in the order of their value, and at the left hand place the given number. Join the numbers, so as to have one greater joined to one less than the given number. Take the difference between the given number and each price, placing the result opposite to the number by which it is obtained, and against that with which it is joined. The quantity then standing against each given number will be the quantity or amount of that number to be taken.

*Example.*

19 { 16 — 4 pounds at 16 carats fine.
18 — 1 " " 18 " "
20 — 1 " " 20 " "
23 — 3 " " 23 " "

It is evident that if these answers were divided by a common divisor, or multiplied by a common multiplier, the proportion must remain the same. Consequently, many answers may be obtained which will all be equally correct.

As the rule is of but little practical utility, it requires but illtle attention.

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## SECTION XXX.

## MISCELLANEOUS RULES,

### USEFUL IN THE VARIOUS AVOCATIONS OF LIFE.

### *Specific Gravity.*

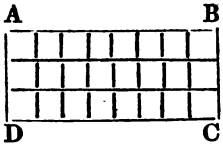
A cubic foot of water weighs 1000 ounces, avoirdupois. If a cubic foot of stone weighs 3000 oz. its Specific Gravity is 3, that is, 3 times as heavy as its bulk of water.

1. Then to find the specific gravity of any body, *that will sink in water*, weigh it in air, and write down the weight; then weigh it in water and write its weight; subtract its weight in water from its weight in air, and divide its weight in air by the difference. The product is the answer.

2. If the body be lighter than water, weigh it in air, then put it in the water and weigh the water which it displaces: divide its weight in air by the weight of the water, and this will give the answer.

### *To find the area of a Parallelogram.*

A Parallelogram is a four-sided figure, having its opposite sides equal; as ABCD.




Now if the line AB be supposed to contain 8 feet, and AD 3 feet, and from each of the dividing points in the line AB, lines be drawn parallel to AD, or BC, to DC; and also, from the dividing points in AD, lines be drawn parallel to AB, to BC, it will be evident that the parallelogram, ABCD, will thus be divided into as many square feet, as there are feet in AB×AD. Hence the following general

RULE.

To find the area of a Parallelogram,—*Multiply the length by the breadth, or height, and the product will be the area: as above,* 8 ft.×3=24, the area of ABCD.

To find the area of a Triangle,—*Multiply the base by the perpendicular height, and take half the product for the area.*

The reason for the above rule will be evident from the illustration of the preceding case, since the area of a triangle is equal to half the area of a parallelogram, of the same base and altitude.

*Circle.*

To find the area of a circle, *Multiply half of the circumference by half of the diameter, or multiply the circumference by the diameter, and take ¼ of the product for the area.*

EXAMPLE.—What is the area of a circle, whose diameter is 12 ft. and circumference 37 ft.?

37×12=444, which divided by 4=111, area.

The area of an *oval* is found by multiplying the longest and shortest diameters together, and then multiply the product by the decimal .7854.

*The Lever.*

It is an admitted principle in mechanics, that the power is to the weight, as the velocity of the weight, to the velocity of the power. Hence, to find what weight may be raised by a given power, we have the following

RULE.

*As the distance of the weight from the fulcrum, is to the distance of the power from the fulcrum; so is the power to the weight.*

EXAMPLE.—If a man weighing 120 lbs. rest on the end of a lever 8 ft. long, what weight will he balance on the other end, supposing the fulcrum 1 foot from the weight?

As 1 : 7 :: 120 : 840, Ans.

*The Axle.*

The proportion for the wheel and axle, (in which the power is applied to the circumference of the wheel, and the weight is raised by a rope, coiled around the axle, as the wheel turns,) is, *as the diameter of the axle is to the diameter of the wheel; so is the power applied to the wheel, to the weight on the axle,* or the weight to be raised.

The above rule is illustrated in the following EXAMPLE.

Suppose the diameter of the wheel to be 50 inches; what must be the diameter of the axle, so that 1 lb. on the wheel, will balance 10 lbs. on the axle?

Inversely as 1 : 50 :: 10 : 5 Ans.

### *The Screw.*

The screw gains power, in the proportion as the distance, between the threads of the screw, is to the circumference of the circle, described by the lever that turns the screw. The circumference is obtained by multiplying twice the length of the lever by 3.1416.

### RULE.

*Make the circumference of the circle the first term of a proportion, the width of the threads the 2d, and the weight to be raised the third; the required power will be the 4th.*

2. When it is required to know what a given power will raise, the order of the proportion is varied accordingly.

### *Cube, and Parallelopiped.*

The content of a cube is found by multiplying the length of one side 3 times into itself.

How many cubic feet in a block of wood 5 feet on each side? $5\times5\times5=125$ feet, Ans.

The content of a parallelopiped is found by multiplying the length, breadth, and height together.

In a block of wood 8 feet long, 4 feet wide, and 2 feet thick, how many feet? $8\times4\times2=64$ ft. Ans.

### *Cylinder, and Prism.*

To find the content of a cylinder or prism, find the area of the end, and then multiply that product by the length.

### *Pyramid, and Cone.*

To find the solid content, Multiply the area of the base by the height, and $\frac{1}{3}$ of the product will be the answer.

### *Globe or Sphere.*

To find the area of the surface of a globe;—Multiply the circumference by the diameter. To find the solid content;—Cube the diameter and multiply that number by the decimal .5236.

### *Cube contained in a Sphere.*

To find how large a cube may be cut from any given sphere;—Square the diameter of the sphere, divide that product by 3, and extract the square root of the quotient for the answer.

### *Guaging.*

By guaging, the content of barrels, hogsheads, &c. is obtained.

### Rule.

Measure the diameter at the bung—square it; measure the head diameter and square it; add these together, and multiply the sum by the length of the barrel, pipe, &c. Multiply this product by the decimal .0014, and the answer will be the number of gallons of ale it will contain. Multiply it by .0017 and it will give the wine gallons.

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## ERRATA.

The rapidity, with which it was necessary a part of the preceding sheets should pass through the press, has made it impossible to avoid several errors, either in the examples or answers. The following have been noticed in Part I. The learner may omit the examples in which they occur, as the others may be sufficiently numerous without them. In a *few examples* these are remainders which are not marked.

Sec. 3. No. 17. the last figures in the Ans. should be 611.—Sec. 5. No. 18. the word *barrel* on top of p. 29. should be *bushels*. Sec. 7. No. 7. Ans. 92 cwt. 0 qrs. &c. No. 8. in the Ans. 6 lbs. 7 oz. &c. No. 20. Ans. 29 cords, &c. No. 30. Ans. 9 bar. 2 gal.—Sec. 8. No. 6. Ans. 22 cords, 75 feet, &c.—Sec. 15. No. 14. Ans. .032409615. No. 15. Ans. 177.2107. No. 17. Ans. 10.3826.—Sec. 16. No. 12. Ans. $4\frac{31}{168}$. No. 21. Ans. $13\frac{3}{7}$.—Sec. 17. No. 7. Ans. 207 days, &c. No. 16. change 1 £. &c.—Sec. 18. No. 17. Ans. 211 £. &c. No. 25. Ans. $31\frac{867}{1001\,8}$. No. 29. Ans. $21\frac{1}{3}$.—Sec. 19. No. 28. Ans. $290\frac{270}{823}$.—Sec. 20. No. 1. Ans. $105\frac{25}{106}$. No. 10. Ans. 1000.—Sec. 21. No. 2. Ans. 49.233. No. 15. Ans. $51.53. No. 19. Ans. $9.45.—Sec. 25. in the definition the sign + should be ×.—Sec. 26. No. 1. Ans. 45+.

END.